International Journal of Mathematics and Statistics Invention (IJMSI)
E-ISSN: 2321 – 4767 P-ISSN: 2321 - 4759
www.ijmsi.org Volume 1 Issue 2 ǁ December. 2013ǁ PP-25-30

Multidimensional Mellin Transforms Involving I-Functions of Several Complex Variables

[1,]Prathima J , [2,]T.M.Vasudevan Nambisan
Research scholar, SCSVMV, Kanchipuram & Department of Mathematics, Manipal Institute of Technology, Manipal, Karnataka, India
[2,]Department of Mathematics, College of Engineering, Trikaripur, Kerala, INDIA.

ABSTRACT : *This paper provides certain multiple Mellin transforms of the product of two I-functions of r variables with different arguements. In the first instance, a basic integral formula for multiple Mellin transform of I-function of "r" variables is obtained. Then\double Mellin transform due to Reed has been employed in evaluating the integrals involving the product of two I- functions of r variables. This formula is very much useful in the evaluation of some integrals and integral equations involving I-functions.*
2010 Mathematics Subject Classification: 33E20, 44A20

KEYWORDS: *I-function, Mellin Barnes contour integral, H-function, Mellin transform.*

I. THE I-FUNCTION OF SEVERAL COMPLEX VARIABLES

The generalized Fox's H-function, namely I-function of 'r' variables is defined and represented in the following manner [3].

$$I[z_1,\ldots,z_r]= I^{0,n:m_1,n_1;\ldots;m_r,n_r}_{p,q:p_1,q_1;\ldots;p_r,q_r}\left[\begin{array}{c|c} z_1 & \left(a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)};A_j\right):\left(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)}\right);\ldots;\left(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)}\right) \\ \vdots & \vdots \\ z_r & \left(b_j;\beta_j^{(1)},\ldots,\beta_j^{(r)};B_j\right):\left(d_j^{(1)},\delta_j^{(1)};D_j^{(1)}\right);\ldots;\left(d_j^{(r)},\delta_j^{(r)};D_j^{(r)}\right)\end{array}\right]$$

$$=\frac{1}{(2\pi i)^r}\int_{L_1}\ldots\int_{L_r}\theta_1(s_1)\ldots\theta_r(s_r)\,\varphi(s_1,\ldots,s_r)\,z_1^{s_1}\ldots z_r^{s_r}\,ds_1\ldots ds_r$$

(1.1)

where $\varphi(s_1,\ldots,s_r)$ and $\theta_i(s_i)$, I = 1,…,r are given by

$$\varphi(s_1,\ldots,s_r)=\frac{\prod_{j=1}^{n}\Gamma^{A_j}\left(1-a_j+\sum_{i=1}^{r}\alpha_j^{(i)}s_i\right)}{\prod_{j=1}^{q}\Gamma^{B_j}\left(1-b_j+\sum_{i=1}^{r}\beta_j^{(i)}s_i\right)\prod_{j=n+1}^{p}\Gamma^{A_j}\left(a_j-\sum_{i=1}^{r}\alpha_j^{(i)}s_i\right)}$$

(1.2)

$$\theta_i(s_i)=\frac{\prod_{j=1}^{m_i}\Gamma^{D_j^{(i)}}\left(d_j^{(i)}-\delta_j^{(i)}s_i\right)\prod_{j=1}^{n_i}\Gamma^{C_j^{(i)}}\left(1-c_j^{(i)}+\gamma_j^{(i)}s_i\right)}{\prod_{j=m_i+1}^{q_i}\Gamma^{D_j^{(i)}}\left(1-d_j^{(i)}+\delta_j^{(i)}s_i\right)\prod_{j=n_i+1}^{p_i}\Gamma^{C_j^{(i)}}\left(c_j^{(i)}-\gamma_j^{(i)}s_i\right)}$$

(1.3)

i=1, 2,…,r

Also

- $z_i \neq 0$ for I = 1,…,r;
- $i=\sqrt{-1}$;
- an empty product is interpreted as unity;
- the parameters m_j, n_j, p_j, q_j (j=1,…,r), n, p, q qj are non-negetive integers such that

$0 \le n \le p,\ q \ge 0, 0 \le n_j \le p_j, 0 \le m_j \le q_j$ (j=1,…,r) (not all zero simultaneously).

- $a_j\,(j=1,...,p)$, $b_j\,(j=1,...,q)$, $c_j^{(i)}\,(j=1,...,p_i,\ i=1,...,r)$ and $d_j^{(i)}\,(j=1,...,q_i,\ i=1,...,r)$ are complex numbers.
- $\alpha_j^{(i)}\,(j=1,...,p,\ i=1,...,r)$, $\beta_j^{(i)}\,(j=1,...,q,\ i=1,...,r)$, $\gamma_j^{(i)}\,(j=1,...,p_i,\ i=1,...,r)$ and $\delta_j^{(i)}\,(j=1,...,q_i,\ i=1,...,r)$ are assumed to be positive quantities for standardisation purpose.
- The exponents $A_j\,(j=1,...,p)$, $B_j\,(j=1,...,q)$, $C_j^{(i)}\,(j=1,...,p_i,\ i=1,...,r)$ and $D_j^{(i)}\,(j=1,...,q_i,\ i=1,...,r)$ of various gamma functions involved in (1.2) and (1.3) may take non-integer values.
- The contour L_i in the complex s_i - plane is of Mellin-Barnes type which runs from $c - i\infty$ to $c + i\infty$,(c real), with indentation, if necessary, in such a manner that all singularities of $\Gamma^{D_j^{(i)}}\left(d_j^{(i)} - \delta_j^{(i)} s_i\right), j=1,...,m_i$ lie to the right and $\Gamma^{C_j^{(i)}}\left(1 - c_j^{(i)} - \gamma_j^{(i)} s_i\right), j=1,...,n_i$ lie to the left of L_i .

Following the results of Braaksma[1] the I-function of 'r' variables is analytic if

$$\mu_i = \sum_{j=1}^{p} A_j \alpha_j^{(i)} - \sum_{j=1}^{q} B_j \beta_j^{(i)} + \sum_{j=1}^{p_i} C_j^{(i)} \gamma_j^{(i)} - \sum_{j=1}^{q_i} D_j^{(i)} \delta_j^{(i)} \le 0,\ i=1,...,r \tag{1.4}$$

II. CONVERGENCE CONDITIONS

The integral (1.1) converges absolutely if

$$\left|\arg\left(z_k\right)\right| < \frac{1}{2}\Delta_k \pi,\quad k=1,...,r \tag{2.1}$$

where
$$\Delta_k = \left[-\sum_{j=1}^{p} A_j \alpha_j^{(k)} - \sum_{j=1}^{q} B_j \beta_j^{(k)} + \sum_{j=1}^{m_k} D_j^{(k)} \delta_j^{(k)} - \sum_{j=m_k+1}^{q_k} D_j^{(k)} \delta_j^{(k)} + \sum_{j=1}^{n_k} C_j^{(k)} \gamma_j^{(k)} - \sum_{j=n_k+1}^{p_k} C_j^{(k)} \gamma_j^{(k)} \right] > 0 \tag{2.2}$$

And if $\left|\arg\left(z_k\right)\right| < \frac{1}{2}\Delta_k \pi,\ and\ \Delta_k > 0,\ k=1,...,r$ then integral (1.1) converges absolutely under the following conditions.

(i) $\mu_k = 0,\ \Omega_k < -1$ where μ_k is given by (1.4) and

$$\Omega_k = \sum_{j=1}^{p}\left[\frac{1}{2} - \Re\left(a_j\right)\right] A_j - \sum_{j=1}^{q}\left[\frac{1}{2} - \Re\left(b_j\right)\right] B_j + \sum_{j=1}^{p_k}\left[\frac{1}{2} - \Re\left(c_j^{(k)}\right)\right] C_j^{(k)} - \sum_{j=1}^{q_k}\left[\frac{1}{2} - \Re\left(d_j^{(k)}\right)\right] D_j^{(k)},\ k=1,...,r \tag{2.3}$$

(ii) $\mu_k \ne 0$ with $s_k = \sigma_k + it_k \left(\sigma_k \text{ and } t_k \text{ are real}, k=1,...,r\right)$, σ_k are so chosen that for $\left|t_k\right| \to \infty$ we have $\Omega_k + \sigma_k \mu_k < -1$

We have discussed the proof of convergent conditions in our earlier paper [3].

Remark 1

If $D_j^{(i)} = 1\left(j = 1,...,m_i, i = 1,...,r\right)$ in (1.1), then the function will be denoted by

$\bar{I}\left[z_1,...,z_r\right]$

$$= I^{0,n:m_1,n_1;\dots;m_r,n_r}_{p,q:p_1,q_1;\dots;p_r,q_r}\left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix}\middle| \begin{matrix} {}_1\left(a_j;\alpha_j^{(1)},\dots,\alpha_j^{(r)};A_j\right)_p : \\ {}_1\left(b_j;\beta_j^{(1)},\dots,\beta_j^{(r)};B_j\right)_q : {}_1\left(d_j^{(1)},\delta_j^{(1)};1\right)_{m_1}, {}_{m_1+1}\left(d_j^{(1)},\delta_j^{(1)};D_j^{(1)}\right)_{q_1};\dots; \end{matrix} \right.$$

$$\left.\begin{matrix} {}_1\left(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)}\right)_{p_1};\dots;{}_1\left(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)}\right)_{p_r} \\ {}_1\left(d_j^{(r)},\delta_j^{(r)};1\right)_{m_r}, {}_{m_r+1}\left(d_j^{(r)},\delta_j^{(r)};D_j^{(r)}\right)_{q_r} \end{matrix}\right]$$

$$= \frac{1}{(2\pi i)^r}\int_{L_1}\dots\int_{L_r}\bar{\theta}_1(s_1)\dots\bar{\theta}_r(s_r)\,\varphi(s_1,\dots,s_r)\,z_1^{s_1}\dots z_r^{s_r}\,ds_1\dots ds_r \tag{2.4}$$

where

$$\bar{\theta}_i(s_i) = \frac{\prod_{j=1}^{m_i}\Gamma\left(d_j^{(i)}-\delta_j^{(i)}s_i\right)\prod_{j=1}^{n_i}\Gamma^{C_j^{(i)}}\left(1-c_j^{(i)}+\gamma_j^{(i)}s_i\right)}{\prod_{j=m_i+1}^{q_i}\Gamma^{D_j^{(i)}}\left(1-d_j^{(i)}+\delta_j^{(i)}s_i\right)\prod_{j=n_i+1}^{p_i}\Gamma^{C_j^{(i)}}\left(c_j^{(i)}-\gamma_j^{(i)}s_i\right)},\quad i=1,\dots,r \tag{2.5}$$

Remark 2

If $C_j^{(i)}=1\left(j=1,\dots,n_i, i=1,\dots,r\right)$, $D_j^{(i)}=1\left(j=1,\dots,m_i, i=1,\dots,r\right)$ and n=0 in (1.1), then the function will be denoted by

$$\bar{I}_1\left[z_1,\dots,z_r\right]$$

$$= I^{0,0:m_1,n_1;\dots;m_r,n_r}_{p,q:p_1,q_1;\dots;p_r,q_r}\left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix}\middle| \begin{matrix} {}_1\left(a_j;\alpha_j^{(1)},\dots,\alpha_j^{(r)};A_j\right)_p : {}_1\left(c_j^{(1)},\gamma_j^{(1)};1\right)_{n_1}, {}_{n_1+1}\left(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)}\right)_{p_1};\dots; \\ {}_1\left(b_j;\beta_j^{(1)},\dots,\beta_j^{(r)};B_j\right)_q : {}_1\left(d_j^{(1)},\delta_j^{(1)};1\right)_{m_1}, {}_{m_1+1}\left(d_j^{(1)},\delta_j^{(1)};D_j^{(1)}\right)_{q_1};\dots; \end{matrix} \right.$$

$$\left.\begin{matrix} {}_1\left(c_j^{(r)},\gamma_j^{(r)};1\right)_{n_r}, {}_{n_r+1}\left(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)}\right)_{p_r} \\ {}_1\left(d_j^{(r)},\delta_j^{(r)};1\right)_{m_r}, {}_{m_r+1}\left(d_j^{(r)},\delta_j^{(r)};D_j^{(r)}\right)_{q_r} \end{matrix}\right] \tag{2.6}$$

where

$$\varphi_1(s_1,\dots,s_r) = \frac{1}{\prod_{j=1}^{q}\Gamma^{B_j}\left(1-b_j+\sum_{i=1}^{r}\beta_j^{(i)}s_i\right)\prod_{j=n+1}^{p}\Gamma^{A_j}\left(a_j-\sum_{i=1}^{r}\alpha_j^{(i)}s_i\right)} \tag{2.7}$$

$$\bar{\theta}_i(s_i) = \frac{\prod_{j=1}^{m_i}\Gamma\left(d_j^{(i)}-\delta_j^{(i)}s_i\right)\prod_{j=1}^{n_i}\Gamma\left(1-c_j^{(i)}+\gamma_j^{(i)}s_i\right)}{\prod_{j=m_i+1}^{q_i}\Gamma^{D_j^{(i)}}\left(1-d_j^{(i)}+\delta_j^{(i)}s_i\right)\prod_{j=n_i+1}^{p_i}\Gamma^{C_j^{(i)}}\left(c_j^{(i)}-\gamma_j^{(i)}s_i\right)} \tag{2.8}$$

i=1,2,…,r

Definition

Mellin transform of a function f(x) is defined as

$$M\left[f(x);s\right]=\int_0^\infty x^{s-1}f(x)\,dx \tag{2.9}$$

III. RESULT USED

We shall use the following result given by Reed [4]

$$\int_0^\infty\int_0^\infty x^{s-1}y^{t-1}\left[\frac{1}{(2\pi i)^2}\int_{\sigma_1-i\infty}^{\sigma_1+i\infty}\int_{\sigma_2-i\infty}^{\sigma_2+i\infty} g(s,t)\,x^{-s}y^{-t}\,ds\,dt\right]dx\,dy=g(s,t)$$

(2.10)

Basic formula for the multiple Mellin transform of the I-function of r variables

$$\int_0^\infty\ldots\int_0^\infty x_1^{s_1-1}\ldots x_r^{s_r-1}\bar{I}_1\left[t_1x_1^{\lambda_1},\ldots,t_rx_r^{\lambda_r}\right]dx_1\ldots dx_r=\frac{1}{\lambda_1\ldots\lambda_1}\bar{\theta}_1\left(\frac{-s_1}{\sigma_1}\right)\ldots\bar{\theta}_r\left(\frac{-s_r}{\sigma_r}\right)\varphi_1\left(\frac{-s_1}{\lambda_1},\ldots,\frac{-s_r}{\lambda_r}\right)t_1^{\frac{-s_1}{\lambda_1}}\ldots t_r^{\frac{-s_r}{\lambda_r}} \tag{3.1}$$

where $\varphi_1\left(\frac{-s_1}{\lambda_1},\ldots,\frac{-s_r}{\lambda_r}\right)$ and $\bar{\theta}_i\left(\frac{-s_i}{\sigma_i}\right), i=1,2,\ldots,r$ are given by (2.7) and (2.8) respectively. The condition given by (1.4) and the convergence conditions given in section 2 are assumed to be satisfied and

$$-\lambda_i\min_{1\le j\le m_i}\left[\Re\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]<\Re(s_i)<\lambda_i\min_{1\le j\le n_i}\left[\Re\left(\frac{1-c_j^{(i)}}{\gamma_j^{(i)}}\right)\right],\quad i=1,2,\ldots,r,$$

Proof

Express the I-function of 'r' variables involved in the left hand side of (3.1) in terms of Mellin-Barnes type contour integral (1.1), $\lambda_i\xi_i=-u_i, i=1,2,\ldots,r$ and interpret the resulting integral with the help of Reeds theorem (2.10) to get the required result.

Special cases

When all the exponents $A_j\,(j=1,\ldots,p)$, $B_j\,(j=1,\ldots,q)$, $C_j^{(i)}\,(j=n_i+1,\ldots,p_i,\ i=1,\ldots,r)$ and $D_j^{(i)}\,(j=m_i+1,\ldots,q_i,\ i=1,\ldots,r)$ are equal to unity in (3.1), we get the multiple Mellin transform of H-function of several variables where H-function of several variables is defined by Srivastava and Panda [6]. If we put r = 2 in (3.1) we get double Mellin transform of I-function of two variables where I-function of two variables is defined by Shantha,Nambisan and Rathie[5]. Further putting all exponents unity, it reduces to the double Mellin transform of H-function of two variables
where H-function of two variables is defined by Mittal and Gupta [2] .

IV. MULTIPLE MELLIN TRANSFORM OF THE PRODUCT OF TWO I-FUNCTIONS OF 'R' VARIABLES

$$\int_0^\infty\ldots\int_0^\infty x_1^{\rho_1-1}\ldots x_r^{\rho_r-1}\bar{I}_1\left[s_1x_1^{\lambda_1},\ldots,s_rx_r^{\lambda_r}\right]\times\bar{I}'_1\left[t_1x_1^{\mu_1},\ldots,t_rx_r^{\mu_r}\right]dx_1\ldots dx_r$$

$$=\frac{1}{\mu_1\ldots\mu_r}t_1^{-\frac{\rho_1}{\mu_1}}\ldots t_r^{-\frac{\rho_r}{\mu_r}}\,I^{\,0,0\,:\,m_1+n'_1,\,m'_1+n_1;\ldots;m_r+n'_r,\,m'_r+n_r}_{\,p+q',\,p'+q\,:\,p_1+q'_1,\,p'_1+q_1;\ldots;p_r+q'_r,\,p'_r+q_r}\left[\begin{matrix}s_1t_1^{-\frac{\lambda_1}{\mu_1}}\\ \vdots\\ s_rt_r^{-\frac{\lambda_r}{\mu_r}}\end{matrix}\left|\begin{matrix}C:C_1;\ldots;C_r\\ \\ D:\ D_1;\ldots;D_r\end{matrix}\right.\right] \tag{4.1}$$

where

$$C=\left(a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)};A_j\right)_{1,p},\ \left(1-b'_j-\sum_{i=1}^{r}\frac{\rho_i}{\mu_i}\beta_j'^{(i)},\frac{\lambda_1}{\mu_1}\beta_j'^{(1)},\ldots,\frac{\lambda_r}{\mu_r}\beta_j'^{(r)};B'_j\right)_{1,q'}$$

$$C_i = {}_1\left(c_j^{(i)},\gamma_j^{(i)};1\right)_{n_i},\ {}_1\left(1\text{-}d_j'^{(i)}\text{-}\delta_j'^{(i)}\frac{\rho_i}{\mu_i},\delta_j'^{(i)}\frac{\lambda_i}{\mu_i};1\right)_{m_i'},\ {}_{m'+1}\left(1\text{-}d_j'^{(i)}\text{-}\delta_j'^{(i)}\frac{\rho_i}{\mu_i},\delta_j'^{(i)}\frac{\lambda_i}{\mu_i};D_j'^{(i)}\right)_{q_i'}$$

$${}_{n_i+1}\left(c_j^{(i)},\gamma_j^{(i)};C_j^{(i)}\right)_{p_i},\quad i=1,\dots,r$$

$$D = {}_1\left(b_j;\beta_j^{(1)},\dots,\beta_j^{(r)};B_j\right)_q,\ {}_1\left(1\text{-}a_j'\text{-}\sum_{i=1}^{r}\frac{\rho_i}{\mu_i}\alpha_j'^{(i)},\frac{\lambda_1}{\mu_1}\alpha_j'^{(1)},\dots,\frac{\lambda_r}{\mu_r}\alpha_j'^{(r)};A_j'\right)_{p'}$$

$$D_i = {}_1\left(d_j^{(i)},\delta_j^{(i)};1\right)_{m_i},\ {}_1\left(1\text{-}c_j'^{(i)}\text{-}\gamma_j'^{(i)}\frac{\rho_i}{\mu_i},\gamma_j'^{(i)}\frac{\lambda_i}{\mu_i};1\right)_{n_i'},\ {}_{n_i'+1}\left(1\text{-}c_j'^{(i)}\text{-}\gamma_j'^{(i)}\frac{\rho_i}{\mu_i},\gamma_j'^{(i)}\frac{\lambda_i}{\mu_i};C_j'^{(i)}\right)_{p_i'},$$

$${}_{m_i+1}\left(d_j^{(i)},\delta_j^{(i)};D_j^{(i)}\right)_{q_i},\quad i=1,\dots,r$$

The integral (4.1) is valid under the following sets of conditions.

(a) The conditions, modified appropriately(if necessary), given in section 3 are satisfied by both the I-functions of r

variables in the integrand of (4.1).

(b) $\lambda_i > 0,\ \mu_i > 0,\ i=1,2,\dots,r$

and $$-\lambda_i \min_{1\le j\le m_i}\left[\Re\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]-\mu_i\min_{1\le j\le m_i'}\left[\Re\left(\frac{d_j'^{(i)}}{\delta_j'^{(i)}}\right)\right]<\Re(s_i)$$

$$<\lambda_i\min_{1\le j\le n_i}\left[\Re\left(\frac{1-c_j^{(i)}}{\gamma_j^{(i)}}\right)\right]-\mu_i\min_{1\le j\le n_i'}\left[\Re\left(\frac{1-c_j'^{(i)}}{\gamma_j'^{(i)}}\right)\right],\quad i=1,\dots,r$$

Proof

Express the Express the function $\bar{I}_1\left[s_1x_1^{\lambda_1},\dots,s_rx_r^{\lambda_r}\right]$ involved in the integrand of (4.1) as Mellin-Barnes type contour integral using (2.6), change the order of integrations, change the order of integrations, evaluate the inner integrals using (3.1) and interpret the result with the help of (1.1) to obtain the right hand side of (4.1).

Special cases

$$\int_0^\infty\dots\int_0^\infty x_1^{\rho_1-1}\dots x_r^{\rho_r-1}\bar{I}_1\left[s_1x_1^{-\lambda_1},\dots,s_rx_r^{-\lambda_r}\right]\times\bar{I}_1'\left[t_1x_1^{\mu_1},\dots,t_rx_r^{\mu_r}\right]dx_1\dots dx_r$$

$$=\frac{1}{\mu_1\dots\mu_r}t_1^{-\frac{\rho_1}{\mu_1}}\dots t_r^{-\frac{\rho_r}{\mu_r}}\,I^{\,0,0:m_1+m_1',n_1+n_1';\dots;m_r+m_r',n_r'+n_r}_{\,p+p',q+q':p_1+p_1',q_1+q_1';\dots;p_r+p_r',q_r+q_r'}\left[\begin{array}{c|c} s_1t_1^{\frac{\lambda_1}{\mu_1}} & C:C_1;\dots;C_r\\ \vdots & \\ s_rt_r^{\frac{\lambda_r}{\mu_r}} & D:\ D_1;\dots;D_r\end{array}\right]\qquad(4.2)$$

where

$$C = {}_1\left(a_j;\alpha_j^{(1)},\dots,\alpha_j^{(r)};A_j\right)_p,\ {}_1\left(a_j'+\sum_{i=1}^{r}\frac{\rho_i}{\mu_i}\alpha_j'^{(i)},\frac{\lambda_1}{\mu_1}\alpha_j'^{(1)},\dots,\frac{\lambda_r}{\mu_r}\alpha_j'^{(r)};A_j'\right)_{p'}$$

$$C_i = \left(c_j^{(i)}, \gamma_j^{(i)}; 1\right)_{1,n_i}, \left(c_j'^{(i)} + \gamma_j'^{(i)} \frac{\rho_i}{\mu_i}, \gamma_j'^{(i)} \frac{\lambda_i}{\mu_i}; 1\right)_{1,n_i'}, \left(c_j'^{(i)} + \gamma_j'^{(i)} \frac{\rho_i}{\mu_i}, \gamma_j'^{(i)} \frac{\lambda_i}{\mu_i}; C_j'^{(i)}\right)_{n_i'+1,p_i'}$$

$$\left(c_j^{(i)}, \gamma_j^{(i)}; C_j^{(i)}\right)_{n_i+1,p_i}, \quad i=1,\ldots,r$$

$$D = \left(b_j; \beta_j^{(1)}, \ldots, \beta_j^{(r)}; B_j\right)_{1,q}, \left(b_j' + \sum_{i=1}^{r} \frac{\rho_i}{\mu_i} \beta_j'^{(i)}, \frac{\lambda_1}{\mu_1} \beta_j'^{(1)}, \ldots, \frac{\lambda_r}{\mu_r} \beta_j'^{(r)}; B_j'\right)_{1,q'}$$

$$D_i = \left(d_j^{(i)}, \delta_j^{(i)}; 1\right)_{1,m_i}, \left(d_j'^{(i)} + \delta_j'^{(i)} \frac{\rho_i}{\mu_i}, \delta_j'^{(i)} \frac{\lambda_i}{\mu_i}; 1\right)_{1,m_i'}, \left(d_j'^{(i)} + \delta_j'^{(i)} \frac{\rho_i}{\mu_i}, \delta_j'^{(i)} \frac{\lambda_i}{\mu_i}; D_j'^{(i)}\right)_{m_i'+1,q_i'},$$

$$\left(d_j^{(i)}, \delta_j^{(i)}; D_j^{(i)}\right)_{m_i+1,q_i}, \quad i = 1,\ldots,r$$

The conditions of validity of (4.2) are

$\lambda_i > 0,\ \mu_i > 0,\ i=1,2,\ldots,r$

and $-\lambda_i \min\limits_{1\le j\le n_i} \left[\Re\left(\frac{1-c_j^{(i)}}{\gamma_j^{(i)}}\right)\right] - \mu_i \min\limits_{1\le j\le m_i'} \left[\Re\left(\frac{d_j'^{(i)}}{\delta_j'^{(i)}}\right)\right] < \Re\left(s_i\right) < \lambda_i \min\limits_{1\le j\le m_i} \left[\Re\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right] - \mu_i \min\limits_{1\le j\le n_i'} \left[\Re\left(\frac{1-c_j'^{(i)}}{\gamma_j'^{(i)}}\right)\right]$, i=1,…,r

Proof of (4.2) is similar to that of (4.1).

When all the exponents are equal to unity in (4.1)and (4.2), we get the multiple Mellin transform of product of two H-functions of several variables. In (4.1) and (4.2) if we put r = 2 we get double Mellin transforms of product of two I-functions of two variables.

Further putting all exponents unity it reduces to the double Mellin transforms of product of two H-functions of two variables.

REFERENCES

[1] Braaksma. B. L. J., Asymptotic expansions and analytic continuations for a class of Barnes-integrals, Compositio Math. 15,239-341, (1964)

[2] Mittal P.K and Gupta K.C., An integral involving generalized function of two variables, Proc.Indian Acad.Sci Sect. A 75, 117-123, (1972).

[3] Prathima J, Vasudevan Nambisan T.M., Shantha Kumari K A study of I-function of several complex variables, International journal of Engineering Mathematics, Hindawi Publications (In Press).

[4] Reed I.S, The Mellin type of double integral, Duke Math.J. 11(1944), 565-575.

[5] Shanthakumari.K,Vasudevan Nambisan T.M,Arjun K.Rathie: A study of the I-function of two variables,Le Matematiche(In Press),(2013). Also see arXiv:1212.6717v1 [math.CV].30Dec2012

[6] Srivastav H.M,Panda R:Some bilateral generating functions for a class of generalized hypergeometric polynomials,J.Reine Angew.Math.17,288 ,265-274,(1976).