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Some Stronger Forms of supra bT- Continuous Functions

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ABSTRACT: *The aim of this paper is to introduce new classes of functions called strongly supra bT-Continuous and perfectly supra bT - continuous functions and study some of their properties and relations among them.*

KEYWORD: *bT^{μ}-closed (open) sets, bT^{μ}- continuous, bT^{μ}-irresolute, strongly bT^{μ}- continuous and perfectly bT^{μ}- continuous.*

I. INTRODUCTION

In 1983 Mashhour et. al [2] introduced Supra topological spaces and studied S- continuous maps and S^*- continuous maps. In 2010, Sayed et. al [3] introduced and investigated several properties of supra b-open set and supra b-continuity. Recently Krishnaveni and Vigneshwaran [3] have introduced and investigated supra bT –closed sets. In this paper, we introduce a new class of function called strongly supra bT-continuous and perfectly supra bT - continuous functions.

II. PRELIMINARIES

Definition 2.1[4,6] A subfamily of μ of X is said to be a supra topology on X, if
(i) $X, \phi \in \mu$
(ii) If $A_i \in \mu$ for all $i \in J$ then $\cup A_i \in \mu$.

The pair (X,μ) is called supra topological space. The elements of μ are called supra open sets in (X,μ) and complement of a supra open set is called a supra closed set.

Definition 2.2[6]

(i) The supra closure of a set A is denoted by $cl^{\mu}(A)$ and is defined as $cl^{\mu}(A) = \cap\{B : B$ is a supra closed set and $A \subseteq B\}$.

(ii) The supra interior of a set A is denoted by $int^{\mu}(A)$ and defined as $int^{\mu}(A) = \cup\{B : B$ is a supra open set and $A \supseteq B\}$.

Definition 2.3[4] Let (X,τ) be a topological spaces and μ be a supra topology on X. We call μ a supra topology associated with τ if $\tau \subset \mu$.

Definition 2.4[6] Let (X,μ) be a supra topological space. A set A is called a supra b-open set if $A \subseteq cl^{\mu}(int^{\mu}(A)) \cup int^{\mu}(cl^{\mu}(A))$. The complement of a supra b-open set is called a supra b-closed set.

Definition 2.5[3] A subset A of a supra topological space (X,μ) is called bT^{μ}-closed set if $bcl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is T^{μ} - open in (X,μ).

Definition 2.6[3] Let (X,τ) and (Y,σ) be two topological spaces and μ be an associated supra topology with τ. A function $f : (X,\tau) \to (Y,\sigma)$ is called bT^{μ}- continuous if $f^{-1}(V)$ is bT^{μ}- closed in (X,μ) for every closed set V of (Y,σ).

Definition 2.7[3] Let (X,τ) and (Y,σ) be two topological spaces and μ be an associated supra topology with τ. A function $f : (X,\tau) \to (Y,\sigma)$ is called bT^{μ}- irresolute if $f^{-1}(V)$ is bT^{μ}-closed in (X,μ) for every bT^{μ}-closed set V of (Y,σ).

III. STRONGLY bT^{μ}- CONTINUOUS AND PERFECTLY bT^{μ}- CONTINUOUS.

Definition 3.1 Let (X,τ) and (Y,σ)be two topological spaces and μ be an associated supra topology with τ. A function $f:(X,\tau)\to(Y,\sigma)$ is called Strongly bT^{μ}-Continuous if the inverse image of every bT^{μ}-closed in Y is supra closed in X.

Theorem 3.2 Let (X,τ) be a topological spaces and μ be an associated supra topology with τ. A function $f:(X,\tau)\to(Y,\sigma)$is Strongly bT^{μ}-Continuous then it is bT^{μ}-Continuous.

Proof Assume that f is strongly bT^{μ}-Continuous. Let G be any supra closed set in Y. By the theorem 3.2[3] every supra closed set is bT^{μ}-closed in Y, G is bT^{μ}- closed in Y. Since f is strongly bT^{μ}- continuous, $f^{-1}(G)$ is supra closed in X. Therefore f is bT^{μ}- continuous.

The converse of the above theorem need not be true as seen from the following example.

Example 3.3 Let X=Y={a,b,c}, τ= {X,ϕ,{a}} and σ={Y,ϕ ,{a},{a,b}}.Let $f:(X,\tau)\to(Y,\sigma)$ be an identity map. Then f is bT^{μ}- continuous but not strongly bT^{μ}- continuous, since for the bT^{μ}- closed set V = {c} in Y, $f^{-1}(V) =f^{-1}(\{c\}) = \{c\}$ is not supra closed in X.

Theorem 3.4 A function $f:(X,\tau)\to(Y,\sigma)$ be strongly bT^{μ}- continuous if and only if the inverse image of every bT^{μ}- closed set in Y is supra closed in X.

Proof Assume that f is strongly bT^{μ}- continuous. Let F be any bT^{μ}- closed set in Y. Then F^c is bT^{μ}- open set in Y. Since f is strongly bT^{μ}- continuous, $f^{-1}(F^c)$ is supra open in X. But $f^{-1}(F^c)$ = X- $f^{-1}(F)$ and so $f^{-1}(F)$ is supra closed in X.

Conversely, assume that the inverse image of every bT^{μ}-closed set in Y is supra closed in X. Let G be any bT^{μ}- open set in Y. Then G^c is bT^{μ}- closed set in Y. By assumption, $f^{-1}(G^c)$ is supra closed in X. But $f^{-1}(G^c)$ = X-$f^{-1}(G)$ and so $f^{-1}(G)$ is supra open in X. Therefore f is strongly bT^{μ}- continuous.

Theorem 3.5 If a function $f:X\to Y$ is strongly bT^{μ}- continuous and a map g: $Y\to Z$ is bT^{μ}- continuous, then the composition gof : $X\to Z$ is strongly bT^{μ}- continuous.

Proof Let G be any supra closed in Z. Since g is bT^{μ}- continuous,$g^{-1}(G)$ is bT^{μ}- closed in Y. Since f is strongly bT^{μ}- continuous,$f^{-1}(g^{-1}(G))$ is supra closed in X. But $(gof)^{-1}(G) = f^{-1}(g^{-1}(G))$. Therefore gof is strongly bT^{μ}- continuous.

Theorem 3.6 If a function f: $X\to Y$ is strongly bT^{μ}- continuous and a map g: $Y\to Z$ is bT^{μ}- continuous, then the composition gof : $X\to Z$ is bT^{μ}- continuous.

Proof Let G be any supra closed in Z. Since g is bT^{μ}- continuous,$g^{-1}(G)$ is bT^{μ}- closed in Y. Since f is strongly bT^{μ}- continuous,$f^{-1}(g^{-1}(G))$ is supra closed in X. By the theorem (3.2)[3] every supra closed is bT^{μ}-closed,$f^{-1}(g^{-1}(G))$ is bT^{μ}- closed. But $f^{-1}(g^{-1}(G)) = (gof)^{-1}(G)$. Therefore gof is bT^{μ}- continuous.

Theorem 3.7 If a function $X\to Y$ is supra continuous then it is strongly bT^{μ}- continuous but not conversely.

Proof Let f: $X\to Y$ is supra continuous. Let F be a supra closed set in Y. Since f is continuous , $f^{-1}(F)$ is supra closed in X. By the theorem (3.2)[3] every supra closed set is bT^{μ} - closed set, $f^{-1}(F)$ is bT^{μ} -closed .Hence f is bT^{μ} – closed.

Converse of the above theorem need not be true as seen from the following example.

Example 3.8 Let X=Y={a,b,c }, τ= {X,ϕ,{a},{a,b}} and σ={Y,ϕ ,{a}, {c},{a,b},{a,c}}.Let $f:(X,\tau)\to(Y,\sigma)$ be an identity map. Then f is Strongly bT^{μ}- continuous, V = {b} is bT^{μ} -closed in Y, $f^{-1}(V) =f^{-1}(\{b\}) = \{b\}$ is supra closed in X. Since V={b} is not supra closed in Y, f is not supra continuous.

Definition 3.9 Let (X,τ) and (Y,σ) be two topological spaces and μ be an associated supra topology with τ. A function $f:(X,\tau)\to(Y,\sigma)$ is called perfectly bT^{μ}-Continuous if the inverse image of every bT^{μ}-closed in Y is both supra closed and supra open in X.

Theorem 3.10 Let (X,τ) be a topological spaces and μ be an associated supra topology with τ. A function $f:(X,\tau)\rightarrow(Y,\sigma)$ is perfectly bT^{μ}-Continuous then it is strongly bT^{μ}-Continuous.

Proof Assume that f is perfectly bT^{μ}-Continuous. Let G be any bT^{μ} - closed set in Y. Since f is perfectly bT^{μ}-continuous, $f^{-1}(G)$ is supra closed in X. Therefore f is strongly bT^{μ}- continuous.

The converse of the above theorem need not be true as seen from the following example.

Example 3.11 Let X=Y={a,b,c }, τ= {X,ϕ,{a},{a,b}} and σ={Y,ϕ ,{a}, {c},{a,b},{a,c}}.Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be an identity map. Then f is strongly bT^{μ}- continuous but not perfectly bT^{μ}- continuous, since for the bT^{μ}- closed set V = {b,c} in Y, $f^{-1}(V) = f^{-1}(\{b,c\})$ = {b,c} is not in both supra open and supra closed in X.

Theorem 3.12 A function $f:(X,\tau)\rightarrow(Y,\sigma)$ be perfectly bT^{μ}- continuous if and only if the inverse image of every bT^{μ}- closed set in Y is both supra open and supra closed in X.

Proof Assume that f is perfectly bT^{μ}- continuous. Let F be any bT^{μ}- closed set in Y. Then F^c is bT^{μ}-open set in Y. Since f is perfectly bT^{μ}- continuous, $f^{-1}(F^c)$ is both supra open and supra closed in X. But $f^{-1}(F^c) = X- f^{-1}(F)$ and so $f^{-1}(F)$ is both supra open and supra closed in X.

Conversely, assume that the inverse image of every bT^{μ}-closed set in Y is both supra open and supra closed in X. Let G be any bT^{μ}- open set in Y. Then G^c is bT^{μ}- closed set in Y. By assumption, $f^{-1}(G^c)$ is supra closed in X. But $f^{-1}(G^c) = X-f^{-1}(G)$ and so $f^{-1}(G)$ is both supra open and supra closed in X. Therefore f is perfectly bT^{μ}- continuous.

Theorem 3.13 If a function $X\rightarrow Y$ is strongly bT^{μ} -continuous then it is bT^{μ}-irresolute but not conversely.

Proof Let f: $X\rightarrow Y$ is strongly bT^{μ} -continuous function. Let F be a bT^{μ} -closed set in Y. Since f is strongly bT^{μ} - continuous , $f^{-1}(F)$ is supra closed in X. By the theorem (3.2)[3] every supra closed set is bT^{μ} - closed set, $f^{-1}(F)$ is bT^{μ} -closed in X. Hence f is bT^{μ} –irresolute .

Converse of the above theorem need not be true as seen from the following example.

Example 3.14 Let X=Y={a,b,c }, τ= {X,ϕ,{a}} and σ={Y,ϕ ,{a},{a,b}}. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be an identity map. Then f is bT^{μ}-irresolute but not strongly bT^{μ}- continuous, since for the bT^{μ}-closed set V = {b} in Y, $f^{-1}(V) = f^{-1}(\{b\})$ = {b} is not supra closed in X.

Theorem 3.15 If a function $X\rightarrow Y$ is perfectly bT^{μ} -continuous then it is bT^{μ}-irresolute but not conversely.

Proof Let f: $X\rightarrow Y$ is perfectly bT^{μ} -continuous function. Let F be a bT^{μ} -closed set in Y. Since f is perfectly bT^{μ} - continuous , $f^{-1}(F)$ is both supra open and supra closed in X. By the theorem (3.2)[ ] every supra closed set is bT^{μ} - closed set, $f^{-1}(F)$ is bT^{μ} -closed in X. Hence f is bT^{μ} –irresolute .

Converse of the above theorem need not be true as seen from the following example.

Example 3.16 Let X=Y={a,b,c }, τ= {X,ϕ,{a}} and σ={Y,ϕ ,{a},{a,b}}. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be an identity map. Then f is bT^{μ}-irresolute but not perfectly bT^{μ}- continuous, since for the bT^{μ} - closed set V = {b} in Y, $f^{-1}(V) = f^{-1}(\{b\})$ = {b} is not in both supra open and supra closed in X.

From the above theorem and example we have the following diagram

Supra continuous   **Perfectly bT^{μ} - continuous**

$\downarrow$ $\downarrow$ $\downarrow$

Strongly bT^{μ} -continuous $\rightarrow$ bT^{μ} -irresolute

$\downarrow$ $\downarrow$

bT^{μ} - continuous

REFERENCES

[1] Arya .S .P. and Gupta.R, On Strongly continuous mappings, Kyungpook Math.J., 14(1974), 131-143.
[2] Jamal M. Mustafa, Totally supra b- continuous and slightly supra b- continuous
[3] functions, Stud. Univ. Babes-Bolyai Math. 57(2012), No. 1, 135-144.
[4] K.Krishnaveni and M.Vigneshwaran , On supra bT- closed sets in supra topological spaces , International journal of Mathematical Archive-4(2),2013,1-6.
[5] A.S.Mashhour, A.A.Allam,F.S.MohamoudandF.H.Khedr,Onsupra
[6] topologicalspaces,Indian J.PureandAppl.Math.No.4, 14(1983),502-510.
[7] N.Ramya and A.Parvathi, Strong forms of $\psi\hat{g}$ - continuous functions in
[8] topological spaces, J.Math.Comput.Sci. 2(2012), No. 1, 101-109
[9] O.R. Sayed and Takashi Noiri,On b-open sets and supra b- Continuity on
[10] topological spaces,European Journal of Pure and applied Mathematics,
[11] 3(2)(2010), 295-302.