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On Semi-Generalized Recurrent LP-Sasakian Manifolds

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ABSTRACT: *In this paper we have studied the nature of 1-forms and scalar curvature r on semi-generalized recurrent LP-Sasakian manifolds.*

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I. INTRODUCTION

In 1950, A.G.Walker [3] introduced the idea of recurrent manifolds. On the otherhand De and Guha [2] introduced generalized recurrent manifold with the non-zero 1-form A and another non-zero associated 1-form B. Such a manifold has been denoted by GK_n.If the associated 1-form B becomes zero, then the manifold GK_n reduces to a recurrent manifold introduced by Ruse [4] which is denoted by K_n. In 1989, K. Matsumoto [8] introduced the notion of LP-Sasakian manifold. Then I. Mihai and R. Rosca [9] introduced the same notion independently and they obtained several results on this manifold. LP-Sasakian manifolds have also been studied by K.Matsumoto and I. Mihai [10], U.C. De and et. al., [11]. A Riemannian manifold (M^n, g) is called a semi-generalized recurrent manifold if its curvature tensor R satisfies the condition

$$(\nabla_X R)(Y,Z)W = \alpha(X)R(Y,Z)W + \beta(X)g(Z,W)Y,$$

where α and β are two 1-forms , β is non-zero, P and $Q$are two vector fields such that

$$g(X,P) = \alpha(X), \qquad g(X,Q) = \beta(X)$$

for any vector field X and ∇ denotes the operator of covarient differentiation with respect to the metric g.

Generalizing the notion of recurrency, the author khan [1] introduced the notion of generalized recurrent Sasakian manifolds. In the paper B. Prasad [12] introduced the notion of semi-generalized recurrent manifold and obtained few interesting results. Motivated by the above studies, in this paper we extend the study of semi-generalized recurrent to LP-Sasakian manifolds and obtain some interesting results.

An LP-Sasakian manifold (M^n, g) is said to be an Einstein manifold if its Ricci tensor S is of the form

$$S(X,Y) = kg(X,Y),$$

where k is any constant.

Let (M^n, g) be a contact Riemannian manifold with a contact form η, the associated vector field ξ, (1-1) tensor field φ and the associated Riemannian metric g. If ξ is a killing vector field, then M^n is called a K-contact Riemannian manifold ([5],[6]).A K-contact Riemannian manifold is called an LP-Sasakian manifold if

$$(\nabla_X\varphi)(X,Y) = g(X,Y)\xi + \eta(Y)X + 2\eta(X)\eta(Y)\xi \tag{1}$$

holds, where ∇ denotes the operator of covarient differentiation with respect to g.

II. PRELIMINARIES

Let S and r denote, respectively, the Ricci tensors of type (0, 2) and of type $(1,1)$ of M^n, besides the relation (1), the following relations also hold ([7],[6],[5]):

$$\varphi(\xi) = 0, \tag{2}$$
$$\eta(\xi) = -1, \tag{3}$$
$$g(X,\xi) = \eta(X), \tag{4}$$
$$S(X,\xi) = (n-1)\eta(X), \tag{5}$$
$$\nabla_X\xi = \varphi X, \tag{6}$$

$$g(\varphi X, \varphi Y) = g(X, Y) + \eta(X)\eta(Y), \tag{7}$$
$$R(\xi, X)Y = g(X, Y)\xi - \eta(Y)X, \tag{8}$$
$$R(X, \xi, \xi) = -X - \eta(X)\xi, \tag{9}$$
$$g(R(\xi, X)Y, \xi) = g(X, Y) - \eta(X)\eta(Y), \tag{10}$$
$$\eta(\varphi X) = 0, \tag{11}$$

for all vector fields X, Y.

The above result will be used in the next sections.

III. ON SEMI-GENERALIZED RECURRENT LP-SASAKIAN MANIFOLDS

Definition 3.1 An LP-Sasakian manifold (M^n, g) is called semi-generalized recurrent if its curvature tensor R satisfies the condition

$$(\nabla_X R)(Y, Z, W) = \alpha(X)R(Y, Z, W) + \beta(X)g(Z, W)Y, \tag{12}$$

where α and β are two 1-forms, β is non-zero and these are defined by

$$\alpha(X) = g(X, A), \qquad \beta(X) = g(X, B), \tag{13}$$

and A and B are vector fields associated with 1-forms α and β respectively.

Taking $Y = W = \xi$ in (12), we have

$$(\nabla_X R)(\xi, Z, \xi) = \alpha(X)R(\xi, Z, \xi) + \beta(X)g(Z, \xi)\xi \tag{14}$$

The left hand side of (14), clearly can be written in the form

$$(\nabla_X R)(\xi, Z, \xi) = XR(\xi, Z, \xi) - R(\nabla_X \xi, Z, \xi)$$
$$-R(\xi, \nabla_X Z, \xi) - R(\xi, Z, \nabla_X \xi)$$

which in view of (4),(6),(8) and (11) gives

$$\begin{aligned} & X[-Z - \eta(Z)\xi] + R(\varphi X, Z, \xi) - [-\nabla_X Z - \eta(\nabla_X Z)\xi] + R(\xi, Z, \varphi X) \\ &= -Xg(Z, \xi)\xi + R(Z, \varphi X, \xi) - g(\nabla_X Z, \xi)\xi - g(Z, \varphi X)\xi + \eta(\varphi X)Z \\ &= g(\nabla_X Z, \xi)\xi + g(Z, \nabla_X \xi)\xi + R(Z, \varphi X, \xi) - g(\nabla_X Z, \xi)\xi - g(Z, \varphi X)\xi \\ &= g(Z, \varphi X)\xi + R(Z, \varphi X, \xi) - g(Z, \varphi X)\xi \\ &= R(Z, \varphi X, \xi), \end{aligned}$$

while the right hand side of (14) equals

$$\begin{aligned} & \alpha(X)R(\xi, Z, \xi) + \beta(X)g(Z, \xi)\xi \\ &= -\alpha(X)[-Z - \eta(Z)\xi] + \beta(X)\eta(Z)\xi \\ &= \alpha(X)Z + [\alpha(X) + \beta(X)]\eta(Z)\xi. \end{aligned}$$

Hence,

$$R(Z, \varphi X, \xi) = \alpha(X)Z + [\alpha(X) + \beta(X)]\eta(Z)\xi. \tag{15}$$

Taking $Z = \xi$ in (15) and then using (3) and (9), we get

$$\varphi X + \eta(\varphi X)\xi = -\beta(X)\xi .$$

By virtue of (11), we have

$$\varphi X = -\beta(X)\xi .$$

In view of (6), we have

$$\nabla_X \xi = -\beta(X)\xi .$$

Hence we can state the following theorem:

Theorem 3.1 In a semi-generalized recurrent LP-Sasakian manifold the associated vector field ξ is not constant.

Permutting equation (12) twice with respect to X, Y, Z; adding the three equations and using Bianchi's identity, we have

$$\begin{aligned} & \alpha(X)R(Y, Z, W) + \beta(X)g(Z, W)Y \\ &+ \alpha(Y)R(Z, X, W) + \beta(Y)g(X, W)Z \\ &+ \alpha(Z)R(X, Y, W) + \beta(Z)g(Y, W)X = 0 . \end{aligned} \tag{16}$$

Contracting (16) with respect to Y, we get

$$\alpha(X)S(Z, W) + n\beta(X)g(Z, W) + {}'R(Z, X, W, A) \tag{17}$$

$$+\beta(Z)g(X,W) - \alpha(Z)S(X,W) + \beta(Z)g(X,W) = 0.$$

In view of $S(Y,Z) = g(QY,Z)$, the equation (17) reduces to

$$\alpha(X)g(QZ,W) + n\beta(X)g(Z,W) - g(R(Z,X,A)W) \\ +\beta(Z)g(X,W) - \alpha(Z)g(QX,W) + \beta(Z)g(X,W) = 0. \quad (18)$$

Factoring off W, we get from (18)

$$\alpha(X)Q(Z) + n\beta(X)Z - R(Z,X,A) \\ +\beta(Z)X - \alpha(Z)Q(X) + \beta(Z)X = 0. \quad (19)$$

Contracting (19) with respect to Z, we get

$$\alpha(X)r + n^2\beta(X) - S(X,A) \\ +\beta(X) - S(X,A) + \beta(X) = 0$$

or,

$$\alpha(X)r + (n^2+2)\beta(X) - 2S(X,A). \quad (20)$$

Taking $X = \xi$ and then using (5) and (13), we get

$$\eta(A)r + (n^2+2)\eta(B) + 2(n-1)\eta(A) = 0.$$

or,

$$r = {}^{-1}/_{\eta(A)}\,[2(n-1)\eta(A) = (n^2+2)\eta(B)]. \quad (21)$$

Hence we can state the following theorem:

Theorem 3.2 The scalar curvature r of a semi-generalized recurrent LP-Sasakian manifold is related in terms of contact forms $\eta(A)$ and $\eta(B)$ as given by (21).

IV. NATURE OF THE 1-FORMS α AND β ON A SEMI-GENERALIZED RICCI-RECURRENT LP-SASAKIAN MANIFOLD

A Riemannian manifold (M^n, g) is semi-generalized Ricci-recurrent manifold ([8],[9]) if

$$(\nabla_X S)(Y,Z) = \alpha(X)S(Y,Z) + n\beta(X)g(Y,Z). \quad (22)$$

Taking $Z = \xi$ in (22), we have

$$(\nabla_X S)(Y,\xi) = \alpha(X)S(Y,\xi) + n\beta(X)g(Y,\xi). \quad (23)$$

The left hand side of (23), clearly can be written in the form

$$(\nabla_X S)(Y,\xi) = XS(Y,\xi) - S(\nabla_X Y,\xi) - S(Y,\nabla_X \xi),$$

which in view of (4), (5) and (6) gives

$$(n-1)X\eta(Y) - (n-1)\eta(\nabla_X Y) + S(Y,\varphi X),$$

which can be written as

$$(n-1)g(Y,\varphi X) - S(Y,\varphi X).$$

while the right side of (23) equals

$$\alpha(X)S(Y,\xi) + n\beta(X)g(Y,\xi) \\ = (n-1)\alpha(X)\eta(Y) + n\beta(X)\eta(Y).$$

Hence,

$$(n-1)g(Y,\varphi X) - S(Y,\varphi X) = (n-1)\alpha(X)\eta(Y) + n\beta(X)\eta(Y). \quad (24)$$

Putting $Y = \xi$ in (24) and then using (3), (4) and (5), we get

$$(n-1)\eta(\varphi X) + (n-1)\eta(\varphi X) = -[(n-1)\alpha(X) + n\beta(X)]\eta(\xi),$$

or,

$$(n-1)\alpha(X) + n\beta(X)] = 0. \quad (25)$$

This leads to the following theorem:

Theorem 4.1 In a semi-generalized Ricci-recurrent LP-Sasakian manifold, the 1-form α and β are related as (25).

For an Einstein manifold, we have $S(Y,Z) = kg(Y,Z)$ and $(\nabla_U S) = 0$, where k is constant.

Hence from (22) we have

$$[k\alpha(X) + n\beta(X)]g(Y,Z) + [k\alpha(Y) + n\beta(Y)]g(Z,X) + k\alpha(Z) + n\beta(Z)g(X,Y) = 0. \tag{26}$$

Replacing Z by ξ in (26) and using (4) and (13), we have

$$[k\alpha(X) + n\beta(X)]\eta(Y) + [k\alpha(Y) + n\beta(Y)]\eta(X) + [k\eta(A) + n\eta(B)]g(X,Y) = 0. \tag{27}$$

Again, taking $X = \xi$ in (27) and using (3),(4) and (13), we have which in view of $Y = \xi$, (2) and (13) gives

$$k\eta(A) + n\eta(B) = 0. \tag{28}$$

Using (4) and (13) in the above relation, it follows that

$$k\alpha(Y) + n\beta(Y) = 0. \tag{29}$$

This leads to the following theorem:

Theorem 4.2 If a semi-generalized Ricci-recurrent LP-Sasakian manifold is an Einstein manifold then 1-forms α and β related as (29).

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