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On Intuitionistic Fuzzy Implicative Hyper Bck-Ideals of Hyper Bck-Algebras

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ABSTRACT: *In this paper, we define the notions of intuitionistic fuzzy (weak) implicative hyper BCK-ideals of hyper BCK-algebras and then we present some theorems which characterize the above notions according to the level subsets. Also we obtain the relationship among these notions, intuitionistic fuzzy (strong, weak, reflexive) hyper BCK-ideals, intuitionistic fuzzy positive implicative hyper BCK-ideals of types-1, 2 ...8 and obtain some related results.*

KEY WORDS: *Hyper BCK-algebras, Intuitionistic fuzzy sets, Intuitionistic fuzzy (weak) implicative hyper BCK-ideal.*

I. INTRODUCTION:

The hyperstructure theory was introduced in 1934 by Marty [8] at the 8th congress of Scandinavian Mathematicians. In the following years, around 40's several authors worked on this subject, especially in France, United States, Japan, Spain, Russia and Italy. Hyperstructers have many applications to several sectors of both pure and applied sciences. In [7], Jun, Zahedi, Xin and Borzooei applied the Hyperstructers to BCK-algebras and introduced the notion of hyper BCK-algebras which is a generalization of a BCK-algebra. After the introduction of the concept of fuzzy sets by Zadeh [11], several researchers were conducted on the generalization of the notion of fuzzy sets. The idea of "Intuitionistic fuzzy set" was first published by Atanassove [1, 2], as a generalization of the notion of fuzzy set. In [10], Jun et al. introduced the notion of implicative hyper BCK-ideals and obtain some related results. In this paper first we define the notions of intuitionistic fuzzy (Weak) implicative hyper BCK-ideals of hyper BCK-algebras and then we present some theorems which characterize the above notions according to the level subsets. Also we obtain the relationship among these notions, intuitionistic fuzzy (strong, weak, reflexive) hyper BCK-ideals, intuitionistic fuzzy positive implicative hyper BCK-ideals of types-1, 2 ...8 and related properties are investigated.

II. PRELIMINARIES:

Let H be a non-empty set endowed with hyper operation that is, $\circ$ is a function from $H \times H$ to $P^*(H) = P(H)\backslash\{\Phi\}$. For any two sub-sets A and B of H, denoted by $A \circ B$ the set $\bigcup_{a \in A,\ b \in B} a \circ b$. We shall use the $x \circ y$ instead of $x \circ \{y\}$, $\{x\} \circ y$, or $\{x\} \circ \{y\}$.

Definition 2.1 [5] By a hyper BCK-algebra, we mean a non-empty set H endowed with a hyper operation "$\circ$" and a constant 0 satisfying the following axioms:

(HK-1) $(x \circ z) \circ (y \circ z) << x \circ y$,

(HK-2) $(x \circ y) \circ z = (x \circ z) \circ y$,

(HK-3) $x \circ H << \{x\}$,

(HK-4) $x << y$ and $y << x \Rightarrow x = y$ for all $x, y, z \in H$.

We can define a relation "$<<$" on H by letting $x << y$ if and only if $0 \in x \circ y$ and for every $A, B \subseteq H$, $A << B$ is defined by for all $a \in A$ there exists $b \in B$ such that $a << b$. In such case, we call "$<<$" the hyper order in H. In the sequel H denotes hyper BCK-algebras. Note that the condition (HK3) is equivalently to the condition : (p1) $x \circ y << \{x\}$, for all $x, y \in H$. In any hyper BCK-algebra H the following hold:

(P2) $0 \circ 0 = \{0\}$, (P3) $0 << x$, (P4) $x << x$, (P5) $A \subseteq B \Rightarrow A << B$

(P6) $0 \circ x = \{0\}$, (P7) $x \circ 0 = \{x\}$, (P8) $0 \circ A = \{0\}$, (P9) $x \in x \circ 0$, for all $x,y,z \in H$ for any non empty A, B of H .

Let I be a non-empty subset of hyper BCK-algebra H and $0 \in I$. Then I is called a hyper BCK-sub algebra of H, if $x \circ y \subseteq I$, for all $x, y \in I$, weak hyper BCK-ideal of H if $x \circ y \subseteq I$ and $y \in I \Rightarrow x \in I, \forall x,y \in H$, a hyper BCK-ideal of H, if $x \circ y << I$ and $y \in I \Rightarrow x \in I, \forall x,y \in H$, a strong hyper BCK-ideal of H, if $x \circ y \cap I \neq \varphi$ and $y \in I \Rightarrow x \in I, \forall x,y \in H$, I is said to be reflexive if $x \circ x \subseteq I \ \forall x \in H$, S-reflexive if $(x \circ y) \cap I \neq \varphi \Rightarrow x \circ y << I, \forall x,y \in H$, closed, if $x << y$ and $y \in I. \Rightarrow x \in I, \forall x,y \in H$ It is easy to see that every S-reflexive sub-set of H is reflexive.

A hyper BCK-algebra H is called a positive implicative hyper BCK-algebra, if for all $x,y,z \in H$, $(x \circ y) \circ z = (x \circ z) \circ (y \circ z)$

Let I be a non-empty sub-set of H and $0 \in I$. Then I is said to be a weak implicative hyper BCK-ideal of H if $(x \circ z) \circ (y \circ x) \subseteq I$ and $z \in I \Rightarrow x \in I$ an implicative hyper BCK-ideal of H, if $(x \circ z) \circ (y \circ x) << I$ and $z \in I \Rightarrow x \in I$ for all $x,y,z \in X$.

A fuzzy set in a set X is a function $\mu : X \to [0,1]$., and the complement of μ , denoted by $\bar{\mu}$, is the fuzzy set in X given by $\bar{\mu}(x) = 1 - \mu(x)$, for all $x \in X$. Let μ and λ be the fuzzy sets of X. For $s, t \in [0,1]$ the set $U(\mu_A ; s) = \{x \in X / \mu_A(x) \geq s\}$ is called upper s- level cut of μ and the set $L(\lambda_A ; t) = \{x \in X / \lambda_A(x) \leq s\}$ is called lower t-level cut of λ..

Let μ be a fuzzy subset of H and $\mu(0) \geq \mu(x)$, for all $x \in H$. Then μ is called a

(i). fuzzy weak implicative hyper BCK-ideal of H if $\mu(x) \geq \min\{\inf_{a \in (x \circ z) \circ (y \circ x)} \mu(a), \mu(z)\}$ and

(ii) fuzzy implicative hyper BCK-ideal of H**,** if $\mu(x) \geq \min\{\sup_{a \in (x \circ z) \circ (y \circ x)} \mu(a), \mu(z)\}$

for all $x, y, z \in H$.

Definition 2.2 [1, 2] As an important generalization of the notion of fuzzy sets in X, Atanassov [1, 2] introduced the concept of an intuitionistic fuzzy set (IFS for short) defined by "An intuitionist fuzzy set in a non-empty set X is an object having the from $A = \{(x, \mu_A(x), \lambda_A(x)) / x \in X\}$ where the function $\mu_A : X \to [0,1]$ and $\lambda_A : X \to [0,1]$ denoted the degree of membership (namely $\mu_A(x)$) and the degree of non membership (namely $\lambda_A(x)$) of each element $x \in X$ to the set A respectively and $0 \leq \mu_A(x) + \lambda_A(x) \leq 1, \forall x \in X$. For the sake of simplicity, we use the symbol form $A = (X, \mu_A, \lambda_A)$ or $A = (\mu_A, \lambda_A)$ ".

Definition 2.3 [4] An IFS $A = (\mu_A, \lambda_A)$ in H is called an intuitionistic fuzzy hyper BCK-ideal of H if it satisfies

(k1) $x << y \Rightarrow \mu_A(x) \geq \mu_A(y)$ and $\lambda_A(x) \leq \lambda_A(y)$,

(k2) $\mu_A(x) \geq \min\{\inf_{a \in x \circ y} \mu_A(a), \mu_A(y)\}$,

(k3) $\lambda_A(x) \leq \max\{\sup_{b \in x \circ y} \lambda_A(b), \mu_A(y)\}$ for all $x, y \in H$.

Definition 2.4 [4] An IFS $A = (\mu_A, \lambda_A)$ in H is called an intuitionistic fuzzy strong hyper BCK-ideal of H if it satisfies

(sh1) $\inf_{a \in x \circ x} \mu_A(a) \geq \mu_A(x) \geq \min\{\sup_{b \in x \circ y} \mu_A(b), \mu_A(y)\}$ and

(sh2) $\sup_{c \in x \circ x} \lambda_A(c) \leq \lambda_A(x) \leq \max\{\inf_{d \in x \circ y} \lambda_A(d), \lambda_A(y)\} \quad \forall\ x, y \in H$.

Definition 2.5 [4] An IFS $A = (\mu_A, \lambda_A)$ in H is called an intuitionistic fuzzy s-weak hyper BCK-ideal of H if it satisfies

(s1) $\mu_A(0) \geq \mu_A(y)$ and $\lambda_A(0) \leq \lambda_A(y)$, for all $x, y \in H$.

(s2) for every $x, y \in H$ there exists $a, b \in x \circ y$, $\mu_A(x) \geq \min\{\mu_A(a), \mu_A(y)\}$ and $\lambda_A(x) \leq \max\{\lambda_A(b), \lambda_A(y)\}$.

Definition 2.6 [4] An IFS $A = (\mu_A, \lambda_A)$ in H is called an intuitionistic fuzzy weak hyper BCK-ideal of H if it satisfies

$$\mu_A(0) \geq \mu_A(x) \geq \min\{\inf_{a \in x \circ y} \mu_A(a), \mu_A(y)\} \text{ and}$$

$$\lambda_A(0) \leq \lambda_A(x) \leq \max\{\sup_{b \in x \circ y} \lambda_A(b), \lambda_A(y)\} \quad \forall\ x, y \in H.$$

Definition 2.7 [4] An IFS $A = (\mu_A, \lambda_A)$ in H is said to satisfy the "sup-inf" property if for any sub-set T of H there exist $x_0, y_0 \in T$ such that $\mu_A(x_0) = \sup_{x \in T} \mu_A(x)$ and $\lambda_A(y_0) = \inf_{y \in T} \mu_A(y)$.

Definition 2.8 [9] Let $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy sub-set of H and $\mu_A(0) \geq \mu_A(x)$ **,** $\lambda_A(0) \leq \lambda_A(y)$ for all $x, y \in H$. Then $A = (\mu_A, \lambda_A)$ is said to be an intuitionistic fuzzy positive implicative hyper BCK-ideal of

(i) type 1, if for all $t \in x \circ z$,

$$\mu_A(t) \geq \min\{\inf_{a \in (x \circ y) \circ z} \mu_A(a), \inf_{b \in y \circ z} \mu_A(b)\} \text{ and}$$

$$\lambda_A(t) \leq \max\{\sup_{c \in (x \circ y) \circ z} \lambda_A(c), \sup_{d \in y \circ z} \lambda_A(d)\}.$$

(ii) type 2, if for all $t \in x \circ z$,

$$\mu_A(t) \geq \min\{\sup_{a \in (x \circ y) \circ z} \mu_A(a), \inf_{b \in y \circ z} \mu_A(b)\} \text{ and}$$

$$\lambda_A(t) \leq \max\{\inf_{c \in (x \circ y) \circ z} \lambda_A(c), \sup_{d \in y \circ z} \lambda_A(d)\}$$

(iii) type 3, if for all $t \in x \circ z$,

$$\mu_A(t) \geq \min\{\sup_{a \in (x \circ y) \circ z} \mu_A(a), \sup_{b \in y \circ z} \mu_A(b)\} \text{ and}$$

$$\lambda_A(t) \leq \max\{\inf_{c \in (x \circ y) \circ z} \lambda_A(c), \inf_{d \in y \circ z} \lambda_A(d)\}.$$

(iv) type 4, if for all $t \in x \circ z$,

$$\mu_A(t) \geq \min\{\inf_{a \in (x \circ y) \circ z} \mu_A(a), \sup_{b \in y \circ z} \mu_A(b)\} \text{ and}$$

$$\lambda_A(t) \leq \max\{\sup_{c \in (x \circ y) \circ z} \lambda_A(c), \inf_{d \in y \circ z} \lambda_A(d)\} \quad \forall\ x, y, z \in H$$

Definition 2.9[9] Let $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy sub-set of H. Then $A = (\mu_A, \lambda_A)$ is said to be an intuitionistic fuzzy positive implicative hyper BCK-ideal of

(i) type 5, if there exists $t \in x \circ z$ such that

$$\mu_A(t) \geq \min\{\inf_{a \in (x \circ y) \circ z} \mu_A(a), \inf_{b \in y \circ z} \mu_A(b)\} \text{ and}$$

$$\lambda_A(t) \leq \max\{\sup_{c \in (x \circ y) \circ z} \lambda_A(c), \sup_{d \in y \circ z} \lambda_A(d)\}.$$

(ii) type 6, if there exists $t \in x \circ z$ such that

$$\mu_A(t) \geq \min\{\sup_{a \in (x \circ y) \circ z} \mu_A(a), \sup_{b \in y \circ z} \mu_A(b)\} \text{ and}$$

$$\lambda_A(t) \leq \max\{\inf_{c \in (x \circ y) \circ z} \lambda_A(c), \inf_{d \in y \circ z} \lambda_A(d)\}.$$

(iii) type 7, if there exists $t \in x \circ z$ such that

$$\mu_A(t) \geq \min\{\inf_{a \in (x \circ y) \circ z} \mu_A(a), \sup_{b \in y \circ z} \mu_A(b)\} \text{ and}$$

$$\lambda_A(t) \leq \max\{\sup_{c \in (x \circ y) \circ z} \lambda_A(c), \inf_{d \in y \circ z} \lambda_A(d)\}.$$

(iv) type 8, if there exists $t \in x \circ z$ such that

$$\mu_A(t) \geq \min\{\sup_{a \in (x \circ y) \circ z} \mu_A(a), \inf_{b \in y \circ z} \mu_A(b)\} \text{ and}$$

$$\lambda_A(t) \leq \max\{\inf_{c \in (x \circ y) \circ z} \lambda_A(c), \sup_{d \in y \circ z} \lambda_A(d)\}.$$

III. INTUITIONISTIC FUZZY IMPLICATIVE HYPER BCK-IDEALS OF HYPER BCK-ALGEBRAS

Definition 3.1 Let $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy set on H and $\mu_A(0) \geq \mu_A(x)$ **,** $\lambda_A(0) \leq \lambda_A(y)$ $\forall\ x, y \in H$. Then $A = (\mu_A, \lambda_A)$ is called an

(i) intuitionistic fuzzy weak implicative hyper BCK-ideal of H if

$$\mu_A(x) \geq \min\{\inf_{a \in (x \circ z) \circ (y \circ x)} \mu_A(a), \mu_A(z)\} \text{ and}$$

$$\lambda_A(x) \leq \max\{\sup_{b \in (x \circ z) \circ (y \circ x)} \lambda_A(b), \lambda_A(z)\}.$$

(ii) intuitionistic fuzzy implicative hyper BCK-ideal of H**,** if

$$\mu_A(x) \geq \min\{\sup_{a \in (x \circ z) \circ (y \circ x)} \mu_A(a), \mu_A(z)\} \text{ and}$$

$$\lambda_A(x) \leq \max\{\inf_{b \in (x \circ z) \circ (y \circ x)} \lambda_A(b), \lambda_A(z)\} \text{ for all } x, y, z \in H.$$

Theorem 3.2 Every intuitionistic fuzzy implicative hyper BCK-ideal of H is an intuitionistic fuzzy weak implicative hyper BCK-ideal.

Proof: Suppose $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy implicative hyper BCK-ideal of H and for $x, y, z \in H$. we have $\inf_{a \in (x \circ z) \circ (y \circ x)} \mu_A(a) \leq \sup_{a \in (x \circ z) \circ (y \circ x)} \mu_A(a)$ and

$$\inf_{b \in (x \circ z) \circ (y \circ x)} \lambda_A(b) \leq \sup_{b \in (x \circ z) \circ (y \circ x)} \lambda_A(b).$$

Therefore

$\mu_A(x) \geq \min\{\sup_{a \in (x \circ z) \circ (y \circ x)} \mu_A(a), \mu_A(z)\} \geq \min\{\inf_{a \in (x \circ z) \circ (y \circ x)} \mu_A(a), \mu_A(z)\}$ and

$\lambda_A(x) \leq \max\{\inf_{b \in (x \circ z) \circ (y \circ x)} \lambda_A(b), \lambda_A(z)\} \leq \max\{\sup_{b \in (x \circ z) \circ (y \circ x)} \lambda_A(b), \lambda_A(z)\}$.

Thus $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy weak implicative hyper BCK-ideal of H.

Example 3.3 Let $H = \{0, a, b\}$. Consider the following cayley table

o	**0**	**a**	**b**
0	$\{0\}$	$\{0\}$	$\{0\}$
a	$\{a\}$	$\{0, a\}$	$\{0, a\}$
b	$\{b\}$	$\{a\}$	$\{0, a\}$

Then $(H, \circ)$ is a hyper BCK-algebra [5].

Define an IFS $A = (\mu_A, \lambda_A)$ on H by $\mu_A(0) = 1$, $\mu_A(a) = 0$, $\mu_A(b) = 0.6$ and $\lambda_A(0) = 0$, $\lambda_A(a) = 1$, and $\lambda_A(b) = 0.4$. Then $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy weak implicative hyper BCK-ideal of H but it is not an intuitionistic fuzzy implicative hyper BCK-ideal of H, because

$\mu_A(a) = 0 < 1 = \mu_A(0) = \min\{\sup_{t \in (a \circ 0) \circ (a \circ a)} \mu_A(t), \mu_A(0)\}$ and

$\lambda_A(a) = 1 > 0 = \lambda_A(0) = \max\{\inf_{t \in (a \circ 0) \circ (a \circ a)} \lambda_A(t), \lambda_A(0)\}$.

Theorem 3.4

(1). Every intuitionistic fuzzy implicative hyper BCK-ideal of H is an intuitionistic fuzzy strong hyper BCK-ideal.

(2). Every intuitionistic fuzzy weak implicative hyper BCK-ideal of H is an intuitionistic fuzzy weak hyper BCK-ideal.

Proof: (i) Let $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy implicative hyper BCK-ideal of H.

By putting $y = 0$ and $z = y$ in Definition 3.1(ii), we get

$\mu_A(x) \geq \min\{\sup_{a \in (x \circ y) \circ (0 \circ x)} \mu_A(a), \mu_A(y)\} = \min\{\sup_{a \in x \circ y} \mu_A(a), \mu_A(y)\}$ and

$\lambda_A(x) \leq \max\{\inf_{b \in (x \circ y) \circ (0 \circ x)} \lambda_A(b), \lambda_A(y)\} = \max\{\inf_{b \in x \circ y} \lambda_A(b), \lambda_A(y)\}$ ….. (i).

First we show that, for $x, y \in H$, if $x << y$ implies $\mu_A(x) \geq \mu_A(y)$ and $\lambda_A(x) \leq \lambda_A(y)$

For this, let $x, y \in H$ be such that $x << y$, then $0 \in x \circ y$ and so from (i**)**, we get

$\mu_A(x) \geq \min\{\sup_{a \in x \circ y} \mu_A(a), \mu_A(y)\} = \min\{\mu_A(0), \mu_A(y)\} = \mu_A(y)$ and

$\lambda_A(x) \leq \max\{\inf_{b \in x \circ y} \lambda_A(b), \lambda_A(y)\} = \max\{\lambda_A(0), \lambda_A(y)\} = \lambda_A(y)$ …..(ii).

Let $x \in H$ and $a \in x \circ x$. Since $x \circ x << x$ then $a << x$ for all $a \in x \circ x$ and so by (ii), we have $\mu_A(a) \geq \mu_A(x)$ and $\lambda_A(a) \leq \lambda_A(x)$ for all $a \in x \circ x$.

Hence $\inf_{a \in x \circ x} \mu_A(a) \geq \mu_A(x)$ and $\sup_{a \in x \circ x} \lambda_A(a) \leq \lambda_A(x)$…..(i ii) .

Combining (i) and (iii), we get

$$\inf_{a \in x \circ x} \mu_A(a) \geq \mu_A(x) \geq \min\{\sup_{b \in x \circ y} \mu_A(b), \mu_A(y)\}$$ and

$$\sup_{c \in x \circ x} \lambda_A(c) \leq \lambda_A(x) \leq \max\{\inf_{d \in x \circ y} \lambda_A(d), \lambda_A(y)\}$$ for all $x, y \in H$.

Thus $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy strong hyper BCK-ideal of H.

(ii) Let $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy weak implicative hyper BCK-ideal of H.

By putting $y = 0$ and $z = y$ in Definition 3.1(i), we get

$$\mu_A(x) \geq \min\{\inf_{a \in (x \circ y) \circ (0 \circ x)} \mu_A(a), \mu_A(y)\} = \min\{\inf_{a \in x \circ y} \mu_A(a), \mu_A(y)\}$$ and

$$\lambda_A(x) \leq \max\{\sup_{b \in (x \circ y) \circ (0 \circ x)} \lambda_A(b), \lambda_A(y)\} = \max\{\sup_{b \in x \circ y} \lambda_A(b), \lambda_A(y)\}.$$

Therefore, $\mu_A(0) \geq \mu_A(x) \geq \min\{\inf_{a \in x \circ y} \mu_A(a), \mu_A(y)\}$ and

$$\lambda_A(0) \leq \lambda_A(x) \leq \max\{\sup_{b \in x \circ y} \lambda_A(b), \lambda_A(y)\}$$ for all $x, y \in H$.

Thus $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy weak implicative hyper BCK-ideal of H.

Example 3.5 Let $H = \{0, a, b, c\}$. Consider the following cayley table:

$\circ$	**0**	**a**	**b**	**c**
0	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$
a	$\{a\}$	$\{0\}$	$\{0\}$	$\{0\}$
b	$\{b\}$	$\{b\}$	$\{0\}$	$\{0\}$
c	$\{c\}$	$\{c\}$	$\{b, c\}$	$\{0, b, c\}$

Then $(H, \circ)$ is a hyper BCK-algebra [5].

Define IFS $A = (\mu_A, \lambda_A)$ in H by $\mu_A(0) = \mu_A(a) = 1$, $\mu_A(b) = \mu_A(c) = 0.4$ and $\lambda_A(0) = \lambda_A(a) = 0$, $\lambda_A(b) = \lambda_A(c) = 0.5$.

Then $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy strong hyper BCK-ideal (and so is an intuitionistic fuzzy weak hyper BCK-ideal) but it is not an intuitionistic fuzzy implicative (and so is not an intuitionistic fuzzy weak implicative) hyper BCK-ideal of H. Because

$$\mu_A(b) = 0.4 < 1 = \mu_A(0) = \min\{\inf_{t \in (b \circ 0) \circ (c \circ b)} \mu_A(t), \mu_A(0)\}$$ and

$$\lambda_A(b) = 0.5 > 0 = \lambda_A(0) = \max\{\sup_{t \in (b \circ 0) \circ (c \circ b)} \lambda_A(t), \lambda_A(0)\}$$

This implies that the converse of the Theorem 3.4 is not correct in general.

Theorem 3.6 Let $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy sub-set of H, then the following statements hold:

(i). A is an intuitionistic fuzzy weak implicative hyper BCK-ideal of H if and only if for all $s, t \in [0,1]$, $U(\mu_A; s) \neq \Phi \neq L(\lambda_A; t)$ are weak implicative hyper BCK- ideals of H.

(ii). if A is an intuitionistic fuzzy implicative hyper BCK-ideal of H, then for all $s,t \in [0,1]$, $U(\mu_A;s) \neq \Phi \neq L(\lambda_A;t)$ are implicative hyper BCK- ideals of H.

(iii). If for all $s,t \in [0,1]$, $U(\mu_A;s) \neq \Phi \neq L(\lambda_A;t)$ are S-reflexive implicative hyper BCK- ideals of H, and A satisfies the "sup-inf" property, then A is an intuitionistic fuzzy implicative hyper BCK-ideal of H.

Proof: **(i)** Assume $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy weak implicative hyper BCK-ideal of H. Let $s, t \in [0,1]$ and $x, y, z \in X$ be such that $(x \circ z) \circ (y \circ x) \subseteq U(\mu_A; s)$ and $z \in U(\mu_A; s)$. Then $a \in U(\mu_A; s)$ for all $a \in (x \circ z) \circ (y \circ x)$ implies $\mu_A(a) \geq s$, for all $a \in (x \circ z) \circ (y \circ x)$ **a**nd $\mu_A(z) \geq s$, implies that $\inf_{a \in (x \circ z) \circ (y \circ x)} \mu_A(a) \geq s$ and $\mu_A(z) \geq s$. Thus by hypothesis,

$$\mu_A(x) \geq \min\left\{ \inf_{a \in (x \circ z) \circ (y \circ x)} \mu_A(a) \quad \Rightarrow \quad \mu_A(z) \right\} \geq \min\{s,s\} = s.$$

Imply $x \in U(\mu_A; s)$. Let $x, y, z \in X$ be such that $(x \circ z) \circ (y \circ x) \subseteq L(\lambda_A; t)$ and $z \in L(\lambda_A; t)$. Then $b \in L(\lambda_A; t)$, for all $b \in (x \circ z) \circ (y \circ x)$. Implies $\lambda_A(c) \leq t$, for all $c \in (x \circ z) \circ (y \circ x)$ and $\lambda_A(z) \leq t$, implies that $\sup_{b \in (x \circ z) \circ (y \circ x)} \lambda_A(b) \leq t$ and $\lambda_A(z) \leq t$. Thus by hypothesis,

$$\lambda_A(x) \leq \max\{\sup_{b \in (x \circ z) \circ (y \circ x)} \lambda_A(b), \ \lambda_A(z)\} \leq \max\{t, t\} = t \Rightarrow x \in L(\lambda_A;t).$$

Therefore $U(\mu_A; s)$ and $L(\lambda_A; t)$ are weak implicative hyper BCK-ideals of H, for all $s, t \in [0,1]$.

Conversely let for all $s, t \in [0,1]$, $U(\mu_A; t) \neq \varphi \neq L(\lambda_A; s)$ are weak implicative hyper BCK-ideals of H and let $x, y, z \in H$ and put $s = \min\left\{ \inf_{a \in (x \circ z) \circ (y \circ x)} \mu_A(a), \ \mu_A(z) \right\}$. Then $\inf_{a \in (x \circ z) \circ (y \circ x)} \mu_A(a) \geq s$ and $\mu_A(z) \geq s$. So that $\mu_A(a) \geq s$ for all $a \in (x \circ z) \circ (y \circ x)$ and $\mu_A(z) \geq s$.

Hence $a \in U(\mu_A; s)$ for all $a \in (x \circ z) \circ (y \circ x)$ and $z \in U(\mu_A; s)$. That is, $(x \circ z) \circ (y \circ x) \subseteq U(\mu_A; s)$, $z \in U(\mu_A; s)$ and so by hypothesis

$$x \in U(\mu_A; s) = \min\left\{ \inf_{a \in (x \circ z) \circ (y \circ x)} \mu_A(a), \ \mu_A(z) \right\}.$$

Thus $\mu_A(x) \geq s = \min\left\{ \inf_{a \in (x \circ z) \circ (y \circ x)} \mu_A(a), \ \mu_A(z) \right\}$.

Let $x,y,z \in H$ and put $t = \max\{\sup_{b \in (x \circ z) \circ (y \circ x)} \lambda_A(b), \ \lambda_A(z)\} \Rightarrow t \geq \sup_{b \in (x \circ z) \circ (y \circ x)} \lambda_A(b)$ and $\lambda_A(z) \leq t$ so that $\lambda_A(b) \leq t$ for all $b \in (x \circ z) \circ (y \circ x)$ and $\lambda_A(z) \leq t$ implies $b \in L(\lambda_A; t)$ for all $b \in (x \circ z) \circ (y \circ x)$ and $z \in L(\lambda_A; t)$. Hence $(x \circ z) \circ (y \circ x) \subseteq L(\lambda_A; t)$ and $z \in L(\lambda_A; t)$. By hypothesis $x \in L(\lambda_A; t)$. Thus $\lambda_A(x) \leq t = \max\{\sup_{b \in (x \circ z) \circ (y \circ x)} \lambda_A(b), \ \lambda_A(z)\}$.

Therefore $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy weak implicative hyper BCK-ideal of H.

(ii) Suppose $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy implicative hyper BCK-ideal of H.

By Theorem 3.4(i), $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy strong hyper BCK-ideal of H and so it is an intuitionistic fuzzy hyper BCK-ideal of H by Theorem 3.17 [4], for all $s, t \in [0,1]$, $U(\mu_A; s) \neq \varphi \neq L(\lambda_A; t)$ are hyper BCK-ideals of H. By Theorem 4.6(ii) [5], it is enough to show that, let

$x, y, z \in H$ and if $x \circ (y \circ x) \ll U(\mu_A; s)$ and $x \circ (y \circ x) \ll L(\lambda_A; t)$, then $x \in U(\mu_A; s) \cap L(\lambda_A; t)$. For this, let $x \circ (y \circ x) \ll U(\mu_A; s)$ for $x, y \in H$. Then for all $a \in x \circ (y \circ x)$ there exists $b \in U(\mu_A; s)$ such that $a \ll b$, we have $\mu_A(a) \geq \mu_A(b) \geq s$ implies $\mu_A(a) \geq s$ for all $a \in x \circ (y \circ x)$ implies $\sup_{a \in x \circ (y \circ x)} \mu_A(a) \geq s$.

Hence by hypothesis,

$$\mu_A(x) \geq \min\{\sup_{a \in (x \circ 0) \circ (y \circ x)} \mu_A(a), \mu_A(0)\} = \sup_{a \in x \circ (y \circ x)} \mu_A(a) \geq s$$

That is $x \in U(\mu_A; s)$. Let $x \circ (y \circ x) \ll L(\lambda_A; t)$ for $x, y \in H$. Then for all $c \in x \circ (y \circ x)$ there exists $d \in L(\lambda_A; t)$ such that $c \ll d$. We have $\lambda_A(c) \leq \lambda_A(d) \leq t$ implies $\lambda_A(c) \leq t$ for all $c \in x \circ (y \circ x)$ implies $\inf_{c \in x \circ (y \circ x)} \lambda_A(c) \leq t$. Hence by hypothesis,

$$\lambda_A(x) \leq \max\{\inf_{c \in (x \circ 0) \circ (y \circ x)} \lambda_A(a), \lambda_A(0)\} = \inf_{c \in x \circ (y \circ x)} \lambda_A(c) \leq t.$$

That is, $x \in L(\lambda_A; t)$. Thus $U(\mu_A; s)$ and $L(\lambda_A; t)$ are implicative hyper BCK-ideals of H, for all $s, t \in [0,1]$.

(iii) Assume that, for all $s, t \in [0,1]$, $U(\mu_A; s)$ and $L(\lambda_A; t)$ are S-reflexive implicative hyper BCK-ideal of H. Let $x, y, z \in H$. Put $s = \min\{\sup_{a \in (x \circ z) \circ (y \circ x)} \mu_A(a), \mu_A(z)\}$ imply $\sup_{a \in (x \circ z) \circ (y \circ x)} \mu_A(a) \geq s$ and $\mu_A(z) \geq s$, Since μ_A satisfies the "sup" property, then there exists $a_0 \in (x \circ z) \circ (y \circ x)$ such that $\mu_A(a_0) = \sup_{a \in (x \circ z) \circ (y \circ x)} \mu_A(a) \geq s$ and so $a_0 \in U(\mu_A; s)$, hence by (HK2), we have $((x \circ (y \circ x)) \circ z) \cap U(\mu_A; s) = ((x \circ z) \circ (y \circ x)) \cap U(\mu_A; s) \neq \varphi$, then there exists $a \in (x \circ (y \circ x))$ such that $(a \circ z) \cap U(\mu_A; s) \neq \varphi$. Since by Theorem 4.6(i)[5], $U(\mu_A; s)$ is a hyper BCK-ideal of H and hypothesis $U(\mu_A; s)$ is S-reflexive, then it is reflexive hyper BCK-ideal of H, by Theorem 2.3(i) [5] and so it is strong hyper BCK-ideal of H. Since $\mu_A(z) \geq s \Rightarrow z \in U(\mu_A; s)$, $(a \circ z) \cap U(\mu_A; s) \neq \varphi$ and $z \in U(\mu_A; s)$, then $a \in U(\mu_A; s)$ and so $(x \circ (y \circ x)) \cap U(\mu_A; s) \neq \varphi$. Since $U(\mu_A; s)$ is reflexive, by Theorem 3.5(ii)[5], $x \circ (y \circ x) \ll U(\mu_A; s)$, $U(\mu_A; s)$ is an implicative hyper BCK-ideal of H then $x \in U(\mu_A; s)$ and so

$$\mu_A(x) \geq s = \min\{\sup_{a \in (x \circ z) \circ (y \circ x)} \mu_A(a), \mu_A(z)\}.$$

Since $L(\lambda_A; t)$ is S-reflexive implicative hyper BCK-ideal of H and $x, y, z \in H$.
Put $t = \max\{\inf_{a \in (x \circ z) \circ (y \circ x)} \lambda_A(a), \lambda_A(z)\}$ implies $\inf_{a \in (x \circ z) \circ (y \circ x)} \lambda_A(a) \leq t$ and $\lambda_A(z) \leq t$

Since λ_A satisfies the "inf" property, then there exist $b_0 \in (x \circ z) \circ (y \circ x)$ such that $\lambda_A(b_0) = \inf_{a \in (x \circ z) \circ (y \circ x)} \lambda_A(a) \leq t$ and so $b_0 \in L(\lambda_A; t)$, hence by (HK2), we have $(x \circ (y \circ x)) \circ z) \cap L(\lambda_A; t) = ((x \circ z) \circ (y \circ x)) \cap L(\lambda_A; t) \neq \varphi$, then there exist $b \in (x \circ (y \circ x))$ such that $(b \circ z) \cap L(\lambda_A; t) \neq \varphi$. Since by the theorem 4.6(i)[5], $L(\lambda_A; t)$ is a hyper BCK-ideal of H and hypothesis $L(\lambda_A; t)$ is S-reflexive, then it is reflexive hyper BCK-ideal of H, by Theorem 2.3(i) [5] and so it is strong hyper BCK-ideal of H. Since $\lambda_A(z) \le t \Rightarrow z \in L(\lambda_A; t)$, $(b \circ z) \cap L(\lambda_A; t) \ne \varphi$ and $z \in L(\lambda_A; t)$ then $b \in L(\lambda_A; t)$ and so $(x \circ (y \circ x)) \cap L(\lambda_A; t) \ne \varphi$. $L(\lambda_A; t)$ is reflexive, by Theorem 3.5(ii)[5] then $x \circ (y \circ x) << L(\lambda_A; t)$, $L(\lambda_A; t)$ is a implicative hyper BCK-ideal of H, by theorem 4.6(ii)[5], we have $x \in L(\lambda_A; t)$ and so $\lambda_A(x) \le t = \max\{\inf_{a \in (x \circ z) \circ (y \circ x)} \lambda_A(a), \lambda_A(z)\}$

Therefore $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy implicative hyper BCK-ideal of H.

Theorem 3.7 Let $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy set on H.

(i). If A satisfies the "sup-inf" property and for all $s, t \in [0,1]$, $U(\mu_A; s)$ and $L(\lambda_A; t)$ are reflexive and A is an intuitionistic fuzzy implicative hyper BCK-ideal of H, then A is an intuitionistic fuzzy positive implicative hyper BCK-ideal of type 3.

(ii). Let H be a positive implicative hyper BCK-algebra. If $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy weak implicative hyper BCK-ideal of H, then A is an intuitionistic fuzzy positive implicative hyper BCK-ideal of type 1

Theorem 3.8 Let $A = (\mu_A, \lambda_A)$ be an IFS on H. Then A is an intuitionistic fuzzy weak implicative hyper BCK-ideal if and only if the fuzzy sets μ_A and $\overline{\lambda_A}$ are fuzzy weak implicative hyper BCK-ideals of H.

Theorem 3.9 Let $A = (\mu_A, \lambda_A)$ be an IFS on H. Then A is an intuitionistic fuzzy weak implicative hyper BCK-ideal if and only if the fuzzy sets $\nabla A = (\mu_A, \overline{\mu_A})$ and $\Diamond A = (\overline{\lambda_A}, \lambda_A)$ are intuitionistic fuzzy weak implicative hyper BCK-ideals of H.

Proof: The proof follows from the Theorem 3.8

Theorem 3.10 Let $A = (\mu_A, \lambda_A)$ be an IFS on H. Then A is an intuitionistic fuzzy implicative hyper BCK-ideal if and only if the fuzzy sets μ_A and $\overline{\lambda_A}$ are fuzzy implicative hyper BCK-ideals.

Theorem 3.11 Let $A = (\mu_A, \lambda_A)$ be an IFS on H. Then A is an intuitionistic fuzzy implicative hyper BCK-ideal if and only if the $\nabla A = (\mu_A, \overline{\mu_A})$ and $\Diamond A = (\overline{\lambda_A}, \lambda_A)$ are fuzzy implicative hyper BCK-ideals.

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