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A New Method for Solving Exponential (Indicial) Equations

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***ABSTRACT*:** *In mathematics, there are different methods for solving a problem which yield the same result. For over a half century, one method is commonly used in solving exponential equation. In this write up, a set of rules has been discovered to reduce the archaic way of solving exponential equation for real roots. This research is written in the context of a new approach to a known fact or idea.*

***KEYWORDS*:** *exponential equation, real roots.*

I. INTRODUCTION

In [1] Halmos gave what constitutes a mathematical creation (or discovery) as follows: "It may be a new proof of an old fact or it may be a new approach to several facts at the same time. If the new proof establishes same previously unsuspected connections between two ideas; it often leads to a generalization."
This research follows the above view by providing a new approach to a known method in mathematics. The proposed method saves time, space and it is easy to understand. To the best of our knowledge, this method is not in any text book. For simple understanding, the method used is grouped into three cases. Each case opens a new view to the structure of an exponential equation.In [2] Stroud gave the definition of exponential (indicial) equation as follows: "An indicial equation is an equation where the variable appears as an index and the solution of such an equation requires the application of logarithms."Talbert [3] and Tuttuh [4] emphasize on converting the indicial equation to quadratic equation, yet this conventional method still seems clumsy. The old method gave us a challenge that one must have the idea of quadratic equation and know how to solve it, unlike the new method which does not require the knowledge of quadratic equation. The connection between the old method and the new method is to completely express the equation in form of indices.

The type of exponential (indicial) equation considered in this write up is of the form

$$MA^{f(x)} + NA^{g(x)} + K = 0$$

M, N are coefficients of the exponential terms; $A^{f(x)}, A^{g(x)}$ are the exponential terms, A is the exponential base, K is the constant term which must be expressible in terms of A, and $f(x)$ and $g(x)$ are exponents or indices of the exponential base. Where $f(x) = 2x + a$ and $g(x) = x + b$; a and b are integers. M, A, N, K are real numbers
Before any of the cases discussed in this paper can be used, we must fully express the equation in terms of A The considered exponential equation is given as:

$$MA^{f(x)} + NA^{g(x)} + K = 0 \quad (1)$$

If $M = 1$, then the equation becomes:

$$A^{f(x)} + NA^{g(x)} + K = 0 \quad (2)$$

Since K must be expressible in terms of A, then let $K = A^d$, d is an integer
If N is expressible in terms of A, then let $N = TA^r$, where T is a real number and r is integer
Equation (2) becomes:

$$A^{f(x)} + TA^r A^{g(x)} + A^d = 0 \quad (3)$$

$$A^{f(x)} + TA^{r+g(x)} + A^d = 0 \quad (4)$$

If N is not expressible in terms of A, then equation (2) remains the same
If $M \neq 1$, then dividing equation (1) by M we have:

$$A^{f(x)} + \frac{NA^{g(x)}}{M} + \frac{A^d}{M} = 0 \quad (5)$$

let $\frac{N}{M} = P$ and $\frac{A^d}{M} = C$
Then we have:

$$A^{f(x)} + PA^{g(x)} + C = 0 \quad (6)$$

Where P and C are real numbers

Since C is expressible in terms of A (constant term is always expressible in terms of the exponential base), then let $C = A^e$, e is an integer

If P is expressible in terms of A, then let $P = QA^s$ where Q is a real number and s is an integer

Equation (6) become:

$$A^{f(x)} + QA^s A^{g(x)} + A^e = 0 \quad (7)$$

$$A^{f(x)} + QA^{s+g(x)} + A^e = 0 \quad (8)$$

If P is not expressible in terms of A, then (6) remains the same

Where C and K are constant terms, $A^{f(x)}$ and $A^{g(x)}$ are exponential terms

II. CASES OF THE NEW METHOD

In the proposed method we shall consider three cases;

1.1.1 Case 1

(1) The constant term K is positive

(2) The coefficient of the exponential term $A^{g(x)}$ is negative

If the above are satisfied and the equation has been fully expressed, then one of the solutions is obtained by equating the powers of the expressed exponentials i.e. $f(x) = s + g(x)$ or $f(x) = r + g(x)$. The second solution is obtained by equating the expressed exponential term $A^{s+g(x)}$ or $A^{r+g(x)}$ to the constant term.

Example 1

Solve: $2^{2x} - 6(2^x) + 8 = 0$ (9)

Solution

The normal way of solving this exponential equation is illustrated below:

$$(2^x)^2 - 6(2^x) + 8 = 0 \quad (10)$$

Let $2^x = p$

Substitute p in equation (10), then we have:

$p^2 - 6p + 8 = 0$ (11)

By factorization:

$(p-2)(p-4) = 0$ (12)

Then, $p = 2$ or $p = 4$

Since $2^x = p$

$Thus: 2^x = 2$ or $2^x = 4$ (13)

$2^x = 2^1$ or $2^x = 2^2$

$x = 1$ or $x = 2$

Thus, $x = 1 \; or \; 2$

Using the new method we solve as follows

We express the equation as: $2^{2x} - 3 \times 2^1(2^x) + 2^3 = 0$ (14)

$2^{2x} - 3(2^{x+1}) + 2^3 = 0$ (15)

For the first solution we let:

$2x = x + 1$ (16)

$2x - x = 1$

$$x = 1$$

The second solution is obtained by setting:

$2^{x+1} = 2^3$ (17)

$x + 1 = 3$

$x = 2$

Therefore, $x = 1 \; or \; 2$

1.1.2 Case 2

(1) The constant term K is negative

(2) The coefficient of the exponential term $A^{g(x)}$ is negative

If the above are satisfied and the equation has been fully expressed, then the first solution is obtained by equating an expressed exponential term $A^{s+g(x)}$ or $A^{r+g(x)}$ to the negative of the constant term. The second solution has no real root.

Example 2

Solve: $3^{2(x-1)} - 8(3^{x-2}) - 1 = 0$ (18)

III. SOLUTION

We first proceed by using the old method

We write $3^{2(x-1)} - 8(3^{x-2}) - 1 = 0$ as:

$$\frac{(3^x)^2}{3^2} - 8\left(\frac{3^x}{3^2}\right) - 1 = 0 \quad (19)$$

Let $3^x = w$ Substitute w in equation (19):

$$\frac{w^2}{9} - 8\left(\frac{w}{9}\right) - 1 = 0 \quad (20)$$

$$w^2 - 8w - 9 = 0 \quad (21)$$

By factorization, we have:

$$(w+1)(w-9) = 0 \quad (22)$$

Thus, $w = -1$ or $w = -1$

Since $3^x = w$

Then: $3^x = -1$ or $3^x = 9$ (23)

$3^x = 3^2$

Thus, $x = 2$

The second root is not real

Using the new method we proceed as follows:

From $3^{2(x-1)} - 8(3^{x-2}) - 1 = 0$ we noticed that the equation cannot be expressed further

Thus, for the first solution

We let: $3^{x-2} = -(-1)$ (24)

$3^{x-2} = 1$

$3^{x-2} = 3^0$

$x - 2 = 0$

$x = 2$

There is no other real root solution for the second answer.

1.1.3 *Case 3*

(1) The constant term K is negative

(2) The coefficient of the exponential term $A^{g(x)}$ is positive

If the above are satisfied and the equation has been fully expressed then the first solutions is obtained by equating the powers of the expressed exponentials i.e. $f(x) = s + g(x)$ or $f(x) = r + g(x)$. The second solution is not a real root.

Example 3

Solve: $3(2^{2x}) + 12(2^x) - 96 = 0$

(25)

IV. SOLUTIO

The normal way of solving exponential equation is illustrated below

Divide the equation by 3 to give: $2^{2x} + 4(2^x) - 32 = 0$ (26)

Thus: $(2^x)^2 + 4(2^x) - 32 = 0$ (27)

Let $2^x = k$

Substitute k in equation (27):

$$k^2 + 4k - 32 = 0 \quad (28)$$

By factorization: $(k-4)(k+8) = 0$ (29)

Therefore $k = 4$ or $k = -8$

since $2^x = k$

Thus: $2^x = 4$ or $2^x = -8$ (30)

$2^x = 2^3$

$x = 2$

Thus, $x = 2$ while the other solution has no real root

The new method is used as follows:

From the question, we divide the equation by 3 to get:

$$2^{2x} + 4(2^x) - 32 = 0$$

(31)

Therefore we express the equation as:

$$2^{2x} + 2^2 \times (2^x) - 2^5 = 0$$

(32)

to be:

$$2^{2x} + (2^{2+x}) - 2^5 = 0$$ (33)

The first solution is obtained by equating the appropriate powers of the expressed exponentials i.e $f(x) = s + g(x)$:

$$2x = x + 2$$ (34)

$$2x - x = 2$$

$$x = 2$$

There is no other real root for the second solution. Hence, the problem has only one real root solution.

V. CONCLUSION

We have given an alternative method for solving an exponential equation of real roots. We demonstrated that an exponential equation of the form $MA^{f(x)} + NA^{g(x)} + K = 0$ will have two real root solutions if the constant term is positive and one real root if the constant term is negative.

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