International Journal of Mathematics and Statistics Invention (IJMSI)
E-ISSN: 2321 – 4767 P-ISSN: 2321 - 4759
www.ijmsi.org Volume 2 Issue 4 ǁ April. 2014ǁ PP-51-63

A Study on Star Intuitionistic Sets

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ABSTRACT. *The aim of this paper is to introduce a new type of Intuionistic sets known as the star Intuionistic sets and study some of its properties. 2000 Math. Subject Classi_cation: 54C10, 54C08.*
KEYWORDS *:and Phrases:Intuitionistic sets; Intuitionistic topological spaces;*

I. INTRODUCTION AND PRELIMINARIES

The concept of fuzzy sets were intoduced and investigated by "Zadeh[11]" in 1965.For the _rst time, the concept of a topological structures was generalized to fuzzy topological spaces[5] in 1968 and the concept of generalized Intuition- istic fuzzy sets was considered by K.Atanassov [2] in 1983."Intuitionistic fuzzy topological space"were introduced by Coker in [7]. Intuitionistic sets in point set was also de_ned by Coker[8] in 1996.In this paper, we de_ne a new operator on intuitionistic sets, which results again an intuitionistic set which we call it as a star intuitionistic set. We also study some of their properties

De_nition 1.1. [9]

Let X be a non empty fixed set. Then the set $A = < X, A^1, A^2 >$ Where A^1 and A^2 are subsets of X is called an intuionistic set if $A^1 \cap A^2 = \phi$ The set A^1 is called the set of member of A, A^2 is called the set of non member of A. Here after let us represent the intuionistic set as IS-sets.

De_nition 1.2. [9]

(a)Let X and Y are two non empty fixed sets. Let $A = \prec X, A^1, A^2 \succ$ and $B = \prec Y, B^1, B^2 \succ$ be two IS sets defined on X and Y respectively. Then the image of A under f, denoted by $f(A)$, is the IS in Y defined by $f(A) = \prec Y, f(A^1), f_-(A^2) \succ$, where $f_-(A^2) = (f(A^2)^c)^c$.

(b)If X and Y are two non empty fixed sets. Let $A = \prec X, A^1, A^2 \succ$ and $B = \prec Y, B^1, B^2 \succ$ be two IS sets defined on X and Y respectively. Then the preimage of B under f, denoted by $f^{-1}(B)$, is the IS in X defined by $f^{-1}(B) = \prec X, f^{-1}(B^1), f^{-1}(B^2) \succ$.

De_nition 1.3. [9]

An intuitionistic topology(IT for short) on a nonempty set X is a family τ of ISs in X satisfying the following axioms:

(T_1) $\widetilde{\phi}$, $\widetilde{X} \in \tau$

(T_2) $G_1 \cap G_2 \in \tau$ *for any* $G_1, G_2 \in \tau$.

(T_3) $\cup G_i \in \tau$ *for any arbitrary family* $\{G_i : i \in J\} \subseteq \tau$.

Definition 1.4. [9] *Let* (X,τ) *be an ITS and* $A=\prec X,A^1,A^2\succ$ *be an IS in* X. *Then the interior and closure of* A *are defined by*

$Cl(A)=\cap\{K : K$ *is an ICS in* X *and* $A\subseteq K\}$.

$int(A)=\cup\{G : G$ *is an IOS in* X *and* $G\subseteq A\}$.

Definition 1.5. [8] *Let* X *be a nonempty set and* $p\in X$ *a fixed element in* X. *Then the IS* $\widetilde{P}=\prec x,\{p\},\{p\}^c\succ$ *is called an intuitionistic point(IP for short) in* X.

IP's in X *can sometimes be inconvenient when express an IS in* X *in terms of IP's. This situation will occur if* $A=<X,A^1,A^2>$ *and* $p\notin A_1$. *Therefore we shall define vanishing IP's as follows:*

Definition 1.6. [8] *Let* X *be a nonempty set and* $p\in X$ *a fixed element in*X. *Then the IS* $p_{\approx}=\prec x,\phi,\{p\}^c\succ$ *is called a vanishing intuitionistic point(VIP for short) in* X

Definition 1.7. [8] *Let* $f:X\to Y$ *be a function.*

(a)Let $\widetilde{p}$ *be an IP in* X.*Then the image of* $\widetilde{p}$ *under f,denote by* $f(\widetilde{p})$,*is defined by* $f(\widetilde{p})=\prec Y,\{q\},\{q\}^c\succ$,*where* $q=f(p)$ *and*

(b)Let $p_{\approx}$ *be a VIP in* X. *Then the image of* $p_{\approx}$ *under f, denoted by* $f(p_{\approx})$ *,is defined by* $f(p_{\approx})=\prec Y,\phi,\{q\}^c\succ$,*where* $q=f(p)$.

It is easy to see that $f(\widetilde{p})$*is indeed an IP in* Y *,namely* $f(\widetilde{p})=\widetilde{q}$ *where* $q=f(p)$, *and it has exactly the same meaning of the image of an IS under the function* $f.f(p_{\approx})$ *is also a VIP in Y, namely*

$f(p_{\approx})=p_{\approx}$,*where* $q=f(p)$.

Definition 1.8. [9]

Let X *be a nonempty fixed set. Then the operators* $[\,],<>$ *are defined on an intuitionistic set as* $[\,]A=<X,A^1,(A^1)^c>$ *and* $<>A=<X,(A^2)^c,A^2>$.

Lemma 1.9. [9]

If $A=<X,A^1,A^2>$ *is an IS sets, then* $\overline{A}=<X,A^2,A^1>$.

Definition 1.10. [9]*Let* (X, τ) *be a ITS.*

(a) $\tau_1 = \{G^1 : \prec X, G^1, G^2 \succ \in \tau\}$ *is a topological space on* X*. similarly* $\tau_2 = \{G^2 : \prec X, G^1, G^2 \succ \in \tau\}$ *is a family of all closed sets of the topological space* $\tau^2 = \{(G^2)^c : \prec X, G^1, G^2 \succ \in \tau\}$ *on* X*.*

(b) Since$G^1 \cap G^2 = \phi$ *for each* $G = \prec X, G^1, G^2 \succ \in \tau$*, we obtain* $G^1 \subseteq (G^2)^c$ *and* $G^2 \subseteq (G^1)^c$*. Hence* (X, τ_1, τ_2) *is a bitopological space.*

II. STAR INTUITIONISTIC SETS

In this chapter, we de_ne a new IS namely star intuitionistic set and studied some of their basic properties.

Definition 2.1. *Let* X *be a non empty fixed set and* $A = < X, A^1, A^2 >$ *be an intuitionistic set.Then we define the star intuitionistic set* A^* *as* $A^* = < X, (A^2)^c - (A^1)^c, (A^2) \cap (A^1)^c >$*, where* A^1*and* A^2 *are the subsets of* X*.*

Lemma 2.2. *Let* X *be a non empty set and* $A = < X, A^1, A^2 >$ *be an intuitionistic set.Then* $A^* = < X, (A^2)^c - (A^1)^c, (A^2) \cap (A^1)^c >$ *is also an intuitionistic set.*

proof:

To Prove:$< X, (A^2)^c - (A^1)^c, (A^2) \cap (A^1)^c >$ *is an IS, we have to prove that* $((A^2)^c - (A^1)^c) \cap ((A^2) \cap (A^1)^c) = \phi$*, which is so obvious and so*

A^* *is also an intuitionistic set.*

Corollary 2.3. *Let* X *be a non empty set.Then* $\widetilde{\phi}^* = < X, \phi^c - X^c, \phi \cap X^c >$ *and* $\widetilde{X}^* = < X, X \cap \phi^c, X^c - \phi^c >$ *are also star intuitionistic set.*

Theorem 2.4. *Let* X *be a non empty set with* $A = < X, A^1, A^2 >$ *and* $B = < X, B^1, B^2 >$ *be two given intuitionistic sets with* A^i*,*B^i *(i=1,2)are subsets of* X*. If* $A^* = < X, (A^2)^c - (A^1)^c, (A^2) \cap (A^1)^c >$ *and* $B^* = < X, (B^2)^c - (B^1)^c, (B^2) \cap (B^1)^c >$ *are star intutionistic sets on* X*, then* $A \subseteq B$ *implies* $A^* \subseteq B^*$*.*

proof:

Given $A \subseteq B$*. Then* $A^1 \subseteq B^1$ *and* $B^2 \subseteq A^2$

It is easy to prove that $(A^2)^c - (A^1)^c \subseteq (B^2)^c - (B^1)^c$ *and* $(B^2) \cap (B^1)^c \subseteq A^2 \cap (A^1)^c$*. So,* $A^* \subseteq B^*$*.*

Remark 2.5. *$A^* = B^*$ iff $A^* \subseteq B^*$ and $B^* \subseteq A^*$.*

Corollary 2.6. *We can also prove the equalities*

(i)$\overline{A}^* = \overline{< X, A_2^c - A_1^c, (A^2) \cap (A^1)^c >} = < X, (A^2) \cap (A^1)^c, A_2^c - A_1^c >$.

(ii)$\cup A_i^* = < X, (\cap A_i^2)^c - (\cup A_i^1)^c, (\cap A_i^2) \cap (\cup A_i^1)^c >$.

(iii)$\cap A_i^* = < X, (\cup A_i^2)^c - (\cap A_i^1)^c, (\cup A_i^2) \cap (\cap A_i^1)^c >$.

(iv)$A^* - B^* = A^* \cap \overline{B^*}$.

and it is easy to show that each R.H.S is also a star intuitionistic sets.

Corollary 2.7. *The operators [],<> defined on an intuitionistic set can also be extended to star intuitionistic set as follows.*

(i)$[]A^* = < X, (A^2)^c - (A^1)^c, (A^2)^c - (A^1)^c)^c >$.

(ii)$<> A^* = < X, (A^2) \cap (A^1)^c, ((A^2) \cap (A^1)^c)^c >$.

Here are some of the basic properties of inclusion and complementation of star IS.

Corollary 2.8. *Let A_i be IS's in X where $i \in J$,where J is an index set and A_i^* are corresponding star IS sets defined on X then*

(a)$A_i^* \subseteq B^*$ *for each* $i \in J \Rightarrow \cup A_i^* \subseteq B^*$.

(b)$B^* \subseteq A_i^*$ *for each* $i \in J \Rightarrow B^* \subseteq \cup A_i^*$.

(c)$\overline{\cup A_i^*} = \cap \overline{A_i^*}$; $\overline{\cap A_i^*} = \cup \overline{A_i^*}$

(d)$A^* \subseteq B^* \Leftrightarrow \overline{B^*} \subseteq \overline{A^*}$.

(e)$\overline{(\overline{A^*})} = A^*$.

(f)$\overline{\widetilde{\phi}^*} = \widetilde{X}^*$;$\overline{\widetilde{X}^*} = \widetilde{\phi}^*$.

Now we shall define the image and preimage of star ISs. Let X,Y be two nonempty fixed sets and f:$X \rightarrow Y$ be a function.

Let A and B be the IS sets on X and Y respectively.

Definition 2.9. *(a)If $B^* = \prec Y, (B^2)^c - (B^1)^c, B^2 \cap (B^1)^c \succ$ is a star IS in Y, then the preimage of B under f, denoted by $f^{-1}(B)$, is the star IS in X defined by $f^{-1}(B^*) = \prec X, f^{-1}((B^2)^c - (B^1)^c), f^{-1}(B^2 \cap (B^1)^c) \succ$.*

(b)If $A^* = \prec X, (A^2)^c - (A^1)^c, A^2 \cap (A^1)^c \succ$ is a star IS in X, then the image of A under f, denoted by $f(A^*)$, is the star IS in X defined by $f(A^*) = \prec Y, f((A^2)^c - (A^1)^c), f_-(A^2 \cap (A^1)^c) \succ$. Where $f_-(A^2 \cap (A^1)^c) = (f(A^2 \cap (A^1)^c)^c)^c = Y - f(X - (A^2 \cap (A^1)^c))$.

Lemma 2.10. *Let* $A^* = \prec X, (A^2)^c - (A^1)^c, A^2 \cap (A^1)^c \succ$ *is an Intuitionistic set. Then* $A^2 \cap (A^1)^c \supseteq f^{-1}(f_-(A^2 \cap (A^1)^c)$.

proof:

$f^{-1}(f_-(A^2 \cap (A^1)^c) = f^{-1}(Y - f(X - (A^2 \cap (A^1)^c))$

$= f^{-1}(Y) - f^{-1}(f(X - (A^2 \cap (A^1)^c))$

$\subseteq X - (X - (A^2 \cap (A^1)^c))$

$= A^2 \cap (A^1)^c$

$f^{-1}(f_-(A^2 \cap (A^1)^c) \subseteq A^2 \cap (A^1)^c$

Theorem 2.11. *Let* $A_i^*(i \in J)$ *be star IS sets corresponding to the IS sets* A_i *in* X *and* $B_j^*(j \in k)$ *be star IS's corresponding to the IS sets* B_j *in* Y, *and* $f : X \to Y$ *be a function. Then*

(a) $A_1^* \subseteq A_2^* \Rightarrow f(A_1^*) \subseteq f(A_2^*)$.

(b) $B_1^* \subseteq B_2^* \Rightarrow f^{-1}(B_1^*) \subseteq f^{-1}(B_2^*)$.

(c) $A^* \subseteq f^{-1}(f(A^*))$ *and if f is injective, then* $A^* = f^{-1}(f(A^*)$.

(d) $f(f^{-1}(B^*)) \subseteq B^*$ *and if f is surjective, then* $f(f^{-1}(B^*)) = B^*$.

(e) $f^{-1}(\cup B_i^*) = \cup f^{-1}(B_i^*)$,

(f) $f^{-1}(\cap B_i^*) = \cap f^{-1}(B_i^*)$.

(g) $f(\cup A_i^*) = \cup f(A_i^*)$,

(h) $f(\cap A_i^*) \subseteq \cap f(A_i^*)$, *and if f is injective, then* $f(\cap A_i^*) = \cap f(A_i^*)$.

(i) $f^{-1}(\widetilde{Y^*}) = \widetilde{X^*}$,

(j) $f^{-1}(\widetilde{\phi^*}) = \widetilde{\phi^*}$.

(k) $f(\widetilde{\phi^*}) = \widetilde{\phi^*}$,

(l) $f(\widetilde{X^*}) = \widetilde{Y^*}$, *if f is surjective.*

(m)If f is surjective, then $\overline{f(A^*)} \subseteq f(\overline{A^*})$. *If furthermore, f is injective, then have* $\overline{f(A^*)} = f(\overline{A^*})$.

(n) $(f^{-1}(\overline{B^*})) = \overline{f^{-1}(B^*)}$.

proof:

(a)Given $A_1^* \subseteq A_2^*$, *where* $A_1^* = \prec X, (A_1^2)^c - (A_1^1)^c, A_1^2 \cap (A_1^1)^c \succ$

$A_2^* = \prec X, (A_2^2)^c - (A_2^1)^c, A_2^2 \cap (A_2^1)^c \succ$

To Prove: $f(A_1^*) \subseteq f(A_2^*)$

By definition $f(A_1^*) = \prec Y, f((A_1^2)^c - (A_1^1)^c), f_-(A_1^2 \cap (A_1^1)^c) \succ$. *Where* $f_-(A_1^2 \cap (A_1^1)^c) = (f(A_1^2 \cap (A_1^1)^c)^c)^c$.

$f(A_2^*) = \prec Y, f((A_2^2)^c - (A_2^1)^c), f_-(A_2^2 \cap (A_2^1)^c) \succ$. *Where* $f_-(A_2^2 \cap (A_2^1)^c) = (f(A_2^2 \cap (A_2^1)^c)^c)^c$.*Also we can prove that*

$f((A_1^2)^c - (A_1^1)^c) \subseteq f((A_2^2)^c - (A_2^1)^c)$ *and* $f_-(A_1^2 \cap (A_1^1)^c) \supseteq f_-(A_2^2 \cap (A_2^1)^c)$

$\Rightarrow f((A_1^2)^c - (A_1^1)^c) \subseteq f((A_2^2)^c - (A_2^1)^c)$.

Therefore $A_1^* \subseteq A_2^* \Rightarrow f(A_1^*) \subseteq f(A_2^*)$

(b)Given $B_1^* \subseteq B_2^*$, *where* $B_1^* = \prec X, (B_1^2)^c - (B_1^1)^c, B_1^2 \cap (B_1^1)^c \succ$. $B_2^* = \prec X, (B_2^2)^c - (B_2^1)^c, B_2^2 \cap (B_2^1)^c \succ$

To Prove: $f^{-1}(B_1^*) \subseteq f^{-1}(B_2^*)$

By definition $f^{-1}(B_1^*) = \prec X, f^{-1}((B_1^2)^c - (B_1^1)^c), f^{-1}(B_1^2 \cap (B_1^1)^c) \succ$.

$f^{-1}(B_2^*) = \prec X, f^{-1}((B_2^2)^c - (B_2^1)^c), f^{-1}(B_2^2 \cap (B_2^1)^c) \succ$. *One can very easily prove that* $f^{-1}((B_1^2)^c - (B_1^1)^c) \subseteq f^{-1}((B_2^2)^c - (B_2^1)^c)$ *and* $f^{-1}(B_1^2 \cap (B_1^1)^c) \supseteq f^{-1}(B_2^2 \cap (B_2^1)^c)$.

hence $B_1^* \subseteq B_2^* \Rightarrow f^{-1}(B_1^*) \subseteq f^{-1}(B_2^*)$.

(c)To prove $A^* \subseteq f^{-1}(f(A^*))$ *and if f is injective.*

To prove:$A^* \subseteq f^{-1}(f(A^*)$.

$(A^2)^c - (A^1)^c \subseteq f^{-1}(f((A^2)^c - (A^1)^c))$ *and*

$A^2 \cap (A^1)^c \subseteq f^{-1}(f_-(A^2 \cap (A^1)^c)$.*(By lemma 2.10)*

Hence $A^* \subseteq f^{-1}(f(A^*)$.

If f is injective then

$$f^{-1}(f(A^*)) \subseteq f^{-1}(f(\prec X, (A^2)^c - (A^1)^c, (A^2 \cap (A^1)^c) \succ))$$
$$\subseteq f^{-1}(\prec Y, f((A^2)^c - (A^1)^c), f_-(A^2 \cap (A^1)^c) \succ))$$
$$= \prec X, f^{-1}(f((A^2)^c - (A^1)^c)), f^{-1}(f_-(A^2 \cap (A^1)^c)) \succ$$

Hence $f^{-1}(f(A^*)) = \prec X, (A^2)^c - (A^1)^c, (A^2 \cap (A^1)^c) \succ$

$$= A^*.$$

$(d) f(f^{-1}(B^*)) \subseteq B^*$ *and if f is onto, then* $f(f^{-1}(B^*)) = B^*$

$$f(f^{-1}(B^*)) = f(f^{-1}(\prec Y, (B^2)^c - (B^1)^c, (B^2 \cap (B^1)^c) \succ))$$
$$= f(\prec X, f^{-1}(B^2)^c - (B^1)^c, f^{-1}(B^2 \cap (B^1)^c) \succ)$$
$$f(f^{-1}(B^*)) = \prec Y, f(f^{-1}(B^2)^c - (B^1)^c), f_-(f^{-1}(B^2 \cap (B^1)^c)) \succ)$$
$$\subseteq \prec Y, (B^2)^c - (B^1)^c, (B^2 \cap (B^1)^c) \succ$$
$$= B^*$$

Notice that

$$f(f^{-1}(B^2)^c - (B^1)^c) \subseteq (B^2)^c - (B^1)^c$$
$$f_-(f^{-1}(B^2 \cap (B^1)^c) = Y - f(X - f^{-1}(B^2 \cap (B^1)^c))$$
$$= Y - f(f^{-1}(Y) - f^{-1}(B^2 \cap (B^1)^c)))$$
$$= Y - f(f^{-1}(Y - (B^2 \cap (B^1)^c))))$$
$$\supseteq Y - (Y - (B^2 \cap (B^1)^c))$$
$$= B^2 \cap (B^1)^c$$

$$f_-(f^{-1}(B^2 \cap (B^1)^c) \supseteq B^2 \cap (B^1)^c$$

(e) To prove $f^{-1}(\cup B_j)^*) = \cup(f^{-1}(B_j)^*)$

$$f^{-1}(\cup B_j) = f^{-1}(\prec Y, \cup B_j^1, \cap B_j^2 \succ)$$
$$f^{-1}(\cup B_j^*) = f^{-1}(\prec Y, (\cap B_j^2)^c - (\cup B_j^1)^c \ , \ (\cap B_j^2) \cap (\cup B_j^1)^c \succ)$$
$$= \prec X, f^{-1}((\cap B_j^2)^c - (\cup B_j^1)^c) \ , \ f^{-1}((\cap B_j^2) \cap (\cup B_j^1)^c \succ))$$
$$= \prec X, \cup(f^{-1}(B_j^2)^c - f^{-1}(B_j^1)^c) \ , \ \cap(f^{-1}(B_j^2) \cap f^{-1}(B_j^1)^c) \succ$$
$$= \cup f^{-1} \prec Y, (B_j^2)^c - (B_j^1)^c \ , \ (B_j^2) \cap (B_j^1)^c \succ$$
$$= \cup(f^{-1}(B_j)^*)$$

Therefore $f^{-1}(\cup B_j)^*) = \cup(f^{-1}(B_j)^*)$

(f) We need $f^{-1}(\cap B_j)^*) = \cap(f^{-1}(B_j)^*)$

$$f^{-1}(\cap B_j) = f^{-1}(\prec Y, \cap B_j^1, \cup B_j^2 \succ)$$

Now, $f^{-1}(\cap B_j^*) = f^{-1}(\prec Y, (\cup B_j^2)^c - (\cap B_j^1)^c \ , \ (\cup B_j^2) \cap (\cap B_j^1)^c \succ)$

$$= \cap \prec f^{-1}Y, f^{-1}((B_j^2)^c - (B_j^1)^c) \ , \ f^{-1}((B_j^2) \cap (B_j^1)^c) \succ$$
$$= \cap f^{-1} \prec Y, (B_j^2)^c - (B_j^1)^c \ , \ (B_j^2) \cap (B_j^1)^c \succ$$
$$= \cap(f^{-1}(B_j)^*)$$

$Therefore\ f^{-1}(\cap B_j)^*) = \cap(f^{-1}(B_j)^*)$

(g) To prove $f(\cup A_i)^*) = \cup(f(A_i)^*)$

$f(\cup A_i) = f(\prec X, \cup A_i^1, \cap A_i^2 \succ)$

$$f(\cup A_i^*) = f(\prec X, (\cap A_i^2)^c - (\cup A_i^1)^c\ ,\ (\cap A_i^2) \cap (\cup A_i^1)^c \succ)$$
$$= \prec f(X), f((\cap A_i^2)^c - (\cup A_i^1)^c)\ ,\ f_-((\cap A_i^2) \cap (\cup A_i^1)^c)) \succ ....(I)$$

Also

$$f((\cap A_i^2)^c - (\cup A_i^1)^c) = f(\cap A_i^2)^c - f(\cup A_i^1)^c$$
$$= \cup f(A_i^2)^c - \cap f(A_i^1)^c$$
$$= \cup(f(A_i^2)^c - f(A_i^1)^c)........(1)$$

$$f_-((\cap A_i^2) \cap (\cup A_i^1)^c) = Y - f(X - ((\cap A_i^2) \cap (\cup A_i^1)^c))$$
$$= Y - f(X) + f((\cap A_i^2) \cap (\cup A_i^1)^c)$$
$$= Y - f(X) + f(\cap A_i^2) \cap f((\cup A_i^1)^c)$$
$$= Y - f(X) + \cap f(A_i^2) \cap (\cap f(A_i^1)^c)$$
$$= Y - f(X) + \cap(f(A_i^2) \cap f(A_i^1)^c))$$
$$= \cap(f((A_i^2) \cap (A_i^1)^c)).........(2)$$

from (1) and (2) in (I)weget

$$= f(\cup A_i)^*) = \cup(f(A_i)^*)$$

$(h) f(\cap A_i)^*) = \cap(f(A_i)^*)$

$f(\cap A_i) = f(\prec X, \cap A_i^1, \cup A_i^2 \succ)$

$$f(\cap A_i^*) = f(\prec X, (\cup A_i^2)^c - (\cap A_i^1)^c\ ,\ (\cup A_i^2) \cap (\cap A_i^1)^c \succ)$$
$$= \prec f(X), f((\cup A_i^2)^c - (\cap A_i^1)^c)\ ,\ f_-((\cup A_i^2) \cap (\cap A_i^1)^c)) \succ ......(II)$$

Notice that

$$f((\cup A_i^2)^c - (\cap A_i^1)^c) = f(\cup A_i^2)^c - f(\cap A_i^1)^c$$
$$= \cap f(A_i^2)^c - \cup f(A_i^1)^c$$
$$= \cap(f(A_i^2)^c - f(A_i^1)^c)..........(1)$$

$$f_-((\cup A_i^2) \cap (\cap A_i^1)^c) = Y - f(X - ((\cup A_i^2) \cap (\cap A_i^1)^c))$$
$$= Y - f(X) + f((\cup A_i^2) \cap (\cap A_i^1)^c)$$
$$= \cup(f((A_i^2) \cap (A_i^1)^c))...............(2)$$

from (1) and (2) in(I) we get

$=\prec f(X), \cap(f(A_i^2)^c - f(A_i^1)^c)\ , \cup(f((A_i^2) \cap (A_i^1)^c) \succ$

$=\cap \prec f(X), f(A_i^2)^c - f(A_i^1)^c\ , f((A_i^2) \cap f(A_i^1)^c \succ$

$=f(\cap A_i)^*) = \cap(f(A_i)^*)$

$(i) f^{-1}(\widetilde{Y^*}) = f^{-1} \prec Y, \phi^c - Y^c, \phi \cap Y^c \succ$

$=\prec f^{-1}(Y), f^{-1}(\phi^c - Y^c), f^{-1}(\phi \cap Y^c) \succ$

$=\prec X, X - \phi, \phi \cap \phi \succ$

$=\prec X, \phi^c - X^c, \phi \cap X^c \succ$

$=\widetilde{X^*},$

$(j) f^{-1}(\widetilde{\phi^*}) = f^{-1} \prec Y, Y^c - \phi^c, Y \cap \phi^c \succ$

$=\prec f^{-1}(Y), f^{-1}(Y^c - \phi^c), f^{-1}(Y \cap \phi^c) \succ$

$=\prec X, \phi - X, X \cap X \succ$

$=\prec X, X^c - \phi^c, X \cap \phi^c \succ$

$=\widetilde{\phi^*},$

$(k)\ f(\widetilde{X^*}) = f \prec X, \phi^c - X^c, \phi \cap X^c \succ$

$=\prec f(X), f(\phi^c -| X^c), f_-(\phi \cap X^c) \succ ....(I)$

Notice that

$f(\phi^c - X^c) = f(\phi^c) - f(X^c)$

$= \phi^c - Y^c ......(1)$

$f_-(\phi \cap X^c) = Y - f(X - \phi \cap X^c)$

$= Y - f(X) + f(\phi \cap X^c)$

$= Y - f(X) + f(\phi) \cap f(X^c)$

$= Y - f(X) + f(\phi) \cap f(\phi)$

$=f(\phi) \cap f(\phi)..........(2)$

from (1) and (2)in (I) we get

$=\prec f(X), \phi^c - Y^c, f(\phi) \cap f(\phi) \succ$

$=\prec f(X), \phi^c - Y^c, \phi \cap \phi \succ$

$=\prec f(X), \phi^c - Y^c, \phi \cap Y^c \succ$

$=\widetilde{Y^*}$

$(l)\ f(\widetilde{\phi^*}) = \widetilde{\phi^*}$

$f(\widetilde{\phi^*}) = f \prec X, X^c - \phi^c, X \cap \phi^c \succ$

$=\prec f(X), f(X^c - \phi^c), f_-(X \cap \phi^c) \succ$

$=\prec Y, Y^c - \phi^c, Y \cap \phi^c \succ = \widetilde{\phi^*}$

Notice that

$$f(X^c-\phi^c)=f(\phi-X)=f(\phi)-f(X)=\phi-Y=Y^c-\phi^c$$

$$f_-(X\cap\phi^c)=Y-f(X-(X\cap\phi^c))$$
$$=Y-f(X)+f(X\cap\phi^c)$$
$$=f(X)\cap f(\phi^c$$
$$=Y\cap\phi^c$$

(m) $f(\overline{A^*})=f\overline{<X,(A^2)^c-(A^1)^c,A^2\cap(A^1)^c>}$

$$=f<X,A^2\cap(A^1)^c,(A^2)^c-(A^1)^c>$$
$$=<f(X),f(A^2\cap(A^1)^c),f_-((A^2)^c-(A^1)^c)>$$
$$=<Y,f(A^2\cap(A^1)^c),f_-((A^2)^c-(A^1)^c)>$$

$\overline{f(A^*)}=\overline{f<X,(A^2)^c-(A^1)^c,A^2\cap(A^1)^c>}$

$$=\overline{<f(X),f((A^2)^c-(A^1)^c),f_-(A^2\cap(A^1)^c)>}$$
$$=<Y,f_-(A^2\cap(A^1)^c),f((A^2)^c-(A^1)^c)>............(I)$$

since f *is onto and* $\overline{f(A^*)}\subseteq f(\overline{A^*})$

$f_-(A^2\cap(A^1)^c)\subseteq f(A^2\cap(A^1)^c)$ *and*

$Y-f(X-(A^2\cap(A^1)^c))\subseteq f(A^2\cap(A^1)^c)$

$Y-f(X)+f((A^2\cap(A^1)^c)))\subseteq f(A^2\cap(A^1)^c)$

$f((A^2\cap(A^1)^c)))\subseteq f(A^2\cap(A^1)^c)............(1)$

$f((A^2)^c-(A^1)^c)\supseteq f_-((A^2)^c-(A^1)^c)$

$f((A^2)^c-(A^1)^c)\supseteq Y-f(X-(A^2)^c-(A^1)^c)$

$f((A^2)^c-(A^1)^c)\supseteq Y-f(X)+f((A^2)^c-(A^1)^c))$

$f((A^2)^c-(A^1)^c)\supseteq f((A^2)^c-(A^1)^c))........................(2)$

from(1) and (2)in(I) we get

$f(\overline{A^*})=f\overline{<X,(A^2)^c-(A^1)^c,A^2\cap(A^1)^c>}$

$(n)f^{-1}(\overline{B^*})=f^{-1}\overline{<Y,(B^2)^c-(B^1)^c,B^2\cap(B^1)^c>}$

$$=f^{-1}<Y,B^2\cap(B^1)^c,(B^2)^c-(B^1)^c>$$
$$=<f^{-1}(Y),f^{-1}(B^2\cap(B^1)^c),f^{-1}((B^2)^c-(B^1)^c)>$$
$$=<X,f^{-1}(B^2\cap(B^1)^c),f^{-1}((B^2)^c-(B^1)^c)>$$

$\overline{f^{-1}(B^*)})=\overline{f^{-1}<Y,(B^2)^c-(B^1)^c,B^2\cap(B^1)^c>}$

$$=\overline{<f^{-1}(Y),f^{-1}((B^2)^c-(B^1)^c),f^{-1}(B^2\cap(B^1)^c)>}$$
$$=<X,f^{-1}(B^2\cap(B^1)^c),f^{-1}((B^2)^c-(B^1)^c)>$$

$f^{-1}(\overline{B^*})=\overline{f^{-1}(B^*)}$

III. STAR INTUITIONISTIC TOPOLOGICAL SPACES

Now we generalize the concept of `"Star intuitionistic topological space" by means of Star intuitionistic sets:In this case the pair $X,\tau)$ is always known as an intuitionistic topological space and any set in τ is known as an intuitionistic open set in X.

Definition 3.1. *Let (X,τ) be an IS topological space. Let $A_i^* = < X, (A_i^2)^c - (A_i^1)^c, A_i^2 \cap (A_i^1)^c >$ be a star IS set with $A_i \in \tau$*

Then $\tau^* = \{\widetilde{\phi}^*, \widetilde{X}^*, A_i^*\}$ is called as the star IS-topological space.

Example 3.2. *Let $X = \{a,b,c,d,e\}$ with the topology $\tau = \{\phi, \dot{X}, A_1, A_2, A_3, A_4\}$ where $A_1 = \prec X, \{a,b,c\}, \{d\} \succ$, $A_2 = \prec X, \{c,d\}, \{e\} \succ$, $A_3 = \prec X, \{c\}, \{d,e\} \succ$, $A_4 = \prec X, \{a,b,c,d\}, \{\phi\} \succ$. Then (X,τ) is an intuitionistic topological spaces in X.*

We define $A^* = < X, (A^2)^c - (A^1)^c, (A^2) \cap (A^1)^c >$ and $\tau^* = \{\widetilde{\phi^*}, \widetilde{X^*}, A_1^*, A_2^*, A_3^*, A_4^*\}$ where $A_1^* = \prec X, \{a,b,c\}, \{d\} \succ$, $A_2^* = \prec X, \{c,d\}, \{e\} \succ$, $A_3^* = \prec X, \{c\}, \{d,e\} \succ$, $A_4^* = \prec X, \{a,b,c,d\}, \{\phi\} \succ$. Then (X,τ^*) is an StarITS on X.

Definition 3.3. *Let (X,τ) be a ITS and $\tau = \{\phi, X\} \cup \{G_i^* : i \in J\}$.*

Then we construct two StarITS's on X as follows:

(a) $\tau^1 = \left\{\widetilde{\phi^*}, \widetilde{X^*}\right\} \cup \{\prec X, \phi^c - G_i^c, \phi \cap G_i^c \succ : i \in J\}$.

(b) $\tau^2 = \left\{\widetilde{\phi^*}, \widetilde{X^*}\right\} \cup \{\prec X, (G_i^c)^c - \phi^c, G_i^c \cap \phi^c \succ : i \in J\}$.

Proposition 3.4. *let (X,τ) be a Intuitionistic topological space on X.Then we can also construct several ITS's on X in the following way:*

(a) $\tau_{0,1} = \{[]G^* : G^* \in J\}$ (b) $\tau_{0,2} = \{<> G^* : G^* \in J\}$.

Remark 3.5. *Let (X,τ^*) be a StarITS.*

(a) $\tau_1^* = \{(G^2)^c - (G^1)^c : \prec X, (G^2)^c - (G^1)^c, G^2 \cap (G^1)^c \succ \in \tau^*\}$is a topological space on X.

similarly $\tau_2^* = \{G^2 \cap (G^1)^c : \prec X, (G^2)^c - (G^1)^c \succ \in \tau\}$ is a family of all closed sets of the topological space $\tau_2^* = \{[G^2 \cap (G^1)^c]^c : \prec X, (G^2)^c - (G^1)^c \succ \in \tau^*\}$on X.

(b) Since$(G^2)^c - (G^1)^c \cap G^2 \cap (G^1)^c = \phi$ for each $G^* = \prec X, (G^2)^c - (G^1)^c, G^2 \cap (G^1)^c \succ \in \tau$, we obtain $(G^2)^c - (G^1)^c \subseteq [G^2 \cap (G^1)^c]^candG^2 \cap (G^1)^c \subseteq [(G^2)^c - (G^1)^c]^c$.

Example 3.6. *Let (X,τ^*) be a StarITS .Let $X = \{a,b\}$ and consider the family $\tau^* = \{\widetilde{\phi}^*, \widetilde{X}^*, A^*, B^*\}$ where $A^* = \prec X, \phi, \{a\} \succ$,$B^* = \prec X, \phi, \{b\} \succ$,$\widetilde{\phi}^* = \prec X, \phi, X \succ$, $\widetilde{X}^* = \prec X, X, \phi \succ$.Then $\tau_1^* = \{\phi : \prec X, \phi, \{a\} \succ \in \tau^*\}$is a topological space on X.*

Similarly $\tau_2^* = \{\{a\} : \prec X, \phi, \{a\} \succ \in \tau^*\}$ is the family of all closed sets of the topological space

$\tau_2^*=\{\{a\}^c :\prec X, \phi, \{a\}\succ \in \tau^*\}$ *on* X

(b) Since $\phi \cap \{a\}= \phi$ *for each* $G^*=\prec X, \phi, \{a\}\succ \in \tau^*$,

we obtained

$$\phi \subseteq \{a\}^c$$

$$\phi \subseteq \{b\} \text{ and}$$

$$\{a\} \subseteq \{\phi\}^c$$

$$\{a\} \subseteq \{a, b\}$$

Hence we conclde that (X, τ_1^*, τ_2^*) *is a bitopological space.*

Definition 3.7. *The complement* $\bar{A}^*$ *of an Star IOS* A^* *in an ITS* (X, τ) *is called an Star ICS in X. Now we define closure and interior operations in StarITS's.*

Definition 3.8. *Let* (X, τ) *be an ITS and* $A=\prec X, A^1, A^2 \succ$ *be an IS in X.*

Then the interior and closure of A are defined by

Let (X, τ) *be an ITS* $A^*=< X, (A^2)^c - (A^1)^c, (A^2) \cap (A^1)^c >$ *be an IS in X.*

Then the int and cl of A are defined by

$Cl(A^*)=\cap\{K^* : K^*$ *is an Star ICS in X and* $A^* \subseteq K^*\}$.

$int(A^*)=\cup\{G^* : G^*$ *is an Star IOS in X and* $G^* \subseteq A^*\}$.

It can be shown that $Cl(A^*)$ *is an StarICS and* $int(A^*)$ *is an StarIOS in X, and* A^* *is an StarICS in X iff* $Cl(A^*) = A^*$ *and A is an StarIOS in X iff* $int(A^*)=A^*$.

Example 3.9. *Consider the Star ITS* (X, τ) *in Examples 3.2 .If* $B^*=\prec X, \{a, c\}, \{d\} \succ$, *then we can write down*

$int(B^*)=\prec X, \{c\}, \{d, e\} \succ$ *and* $Cl(B^*)=\prec X, X, \phi \succ$

Proposition 3.10. *Let* (X, τ) *be an StarITS and A,B be IS's in X. Then the followig properties hold:*

(a)$int(A^*) \subseteq A^*$

(a^1) $A \subseteq cl(A^*)$

$(b) A \subseteq B \Rightarrow int(A^*) \subseteq int(B^*)$

$(b^1)\ A \subseteq B \Rightarrow Cl(A^*) \subseteq Cl(B^*)$

$(c)\ int(int(A^*) = int(A^*)$

$(c^1)\ cl(cl(A^*)=cl(A^*)$

$(d)\ int(A^* \cap B^*) = int(A^*) \cap int(B^*)$

$(d^*)\ cl(A^* \cap B^*) = cl(A^*) \cap cl(B^*)$

$(e)\ int(X^*) = X^*$

$(e^*)\ cl(\phi^*) = \phi^*$

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