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Fixed Point Theorem in Nonarchimedean Fuzzy Metric Space Using Weak Compatible of Type (A)

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ABSTRACT: *The purpose of this paper is to prove a common fixed point theorem for weak compatible mapping of type (A) in non Archimedean fuzzy metric space.*

KEYWORDS: *Non Archimedean fuzzy metric space, weak compatible mapping of type (A).*
MATHEMATICAL SUBJECT CLASSIFICATION: *47H10, 54H25.*

I. INTRODUCTION:

The concept of fuzzy sets was introduced by Zadeh [6].It was developed extensively by many authors and used in various fields. To use this concept in topology and analysis several researcher have defined fuzzy metric space in various ways. George and Veeramani [2] modified the concept of fuzzy metric space introduced by O. Kramosil and Michalek [9]. M.Grebiec [7] has proved fixed point results for fuzzy metric space. S.Sessa defined generalization of commutativity which called weak commmutativity. G.Jungck [5] introduced more generalized commutativity so called compatibility. The concepts of weak compatibility in fuzzy metric space are given by B.Singh and S.Jain [3].fuzzy metric space is their application in engineering problems, in information systems and in quantum particle physics, particularly in concern with both string and E-infinity theory which were given and studied to filtering, color image, improving some filters when replacing some classical metrics[8].Fixed point theorems also play a centre role also in proof of existence of general equilibrium in market economics as developed in the 1950's by Nobel prize winners in economics Gerard Debrew and Kenneth Arrow[10]. In fact, an equilibrium price is a fixed point in a stable market. In1975, V.I.Istratescu [13] first studied the non-Archimedean menger-spaces. They presented some basic topological preliminaries of non Archimedean fuzzy metric space. D.Mihet [4] introduced the concept of Non-Archimedean fuzzy metric space. In this paper we prove common fixed point theorem in Non-Archimedean fuzzy metric space using the concept of weak compatible mapping of type (A).

II. PRELIMINARIES:

Definition 2.1[12] A binary operation $*: [0, 1] \times [0, 1] \to [0, 1]$ is a continuous t-norm if it satisfies the following conditions:

[1] $*$ is associative and commutative,

[2] $*$ is continuous,

[3] $a * 1 = a$ for all $a \in [0, 1]$,

[4] $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, For each $a, b, c, d \in [0, 1]$

Two typical examples of continuous t-norm are $a * b = ab$ and $a * b = \min(a, b)$.

Definition 2.2[12] The 3-tuple (X, M, $*$) is called a non-Archimedean fuzzy metric space .If X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set in $X^2 \times [0,\infty)$ satisfying the following conditions: For all $x, y, z \in X$ and s, t > 0,

[1] M(x, y, 0) = 0,

[2] M(x, y, t) = 1, for all t > 0 if and only if x = y,

[3] M(x, y, t) = M(y, x, t),

[4] M(x, y, t) $*$ M(y, z, s) $\leq$ M(x, z, max {t, s})

[5] Or equivalently M(x, y, t) $*$ M(y, z, t) $\leq$ M(x, z, t)

[6] M(x, y .): $[0,\infty) \to [0,1]$ is left continuous.

Definition 2.3[12] Let (X, M, $*$) be a non-Archimedean fuzzy metric space:

[1] A sequence $\{x_n\}$ in X is said to be convergent to a point x $\in$ X denoted by
[2] $\lim_{n\to\infty} x_n = x$, if $\lim_{n\to\infty} M(x_n, x, t) = 1$ for all t > 0.

[3] A sequence $\{x_n\}$ in X is said to be Cauchy sequence if

[4] $\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$ for all t > 0, p>0.

[5] A non-Archimedean fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Definition 2.4[12] A non-Archimedean fuzzy metric space (X, M, $*$) is said to be of type (C) g if there exists a g$\in \Omega$ such that g(M(x,y,t)) $\leq$ {g(M(x,z,t)) +g(M(z,y,t))} for all x,y,z $\in$ X and t$\geq$0 Where Ω = {g: g: [0, 1] $\to$ $[0,\infty)$ is continuous, strictly decreasing {g(1) = 0 and g(0) < ∞ }

Definition 2.5[12] A non-Archimedean fuzzy metric space (X, M, $*$) is said to be of type (D) g if there exists a g$\in \Omega$ such that g($*$(s,t)) $\leq$ g(s)+g(t) for all s,t$\in$[0,1]

Definition 2.6 [12] Let $(X, M, *)$ be a complete non-Archimedean fuzzy metric space of type $(D)_g$ with a continuous strictly increasing t-norm $*$.Let $\phi:[0,\infty)\to[0,\infty)$ be a function satisfying the following condition $(\Phi):(\Phi)\phi$ is uppersemicontinious from the right and $\phi(t) < t$ for all $t > 0$

Definition 2.7 [15] Let $A, S : X \to X$ be mapping A and S are said to be compatible if

$$\lim_{n\to\infty} g(M(ASx_n, \mathrm{SAx}_n, \mathrm{t})) = 0 \quad ; \forall t > 0$$

When $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Ax_n = \mathrm{z} = \lim_{n\to\infty} Sx_n \text{ For some } z \in X$$

Definition 2.8 [15] Let $A, S : X \to X$ be mapping A and S are said to be compatible of type (A) if

$$\lim_{n\to\infty} g(M(ASx_n, \mathrm{SSx}_n, \mathrm{t})) = 0 = \lim_{n\to\infty} g(M(\mathrm{SA}\, x_n, \mathrm{AAx}_n, \mathrm{t}))\ ; \forall t > 0$$

When $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Ax_n = \mathrm{z} = \lim_{n\to\infty} Sx_n \text{ for some } z \in X$$

Definition 2.9 [15] Let $A,S : X \to X$ be mapping A and S are said to be weak compatible of type (A) if

$$\lim_{n\to\infty} g(M(ASx_n, \mathrm{SSx}_n, \mathrm{t})) \geq \lim_{n\to\infty} g(M(\mathrm{SA}\,x_n, \mathrm{SSx}_n, \mathrm{t}))$$

$$\lim_{n\to\infty} g(M(\mathrm{SAx}_n, \mathrm{AAx}_n, \mathrm{t})) \geq \lim_{n\to\infty} g(M(\mathrm{AS}\,x_n, \mathrm{AAx}_n, \mathrm{t}))\ ; \forall t > 0$$

whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Ax_n = \mathrm{z} = \lim_{n\to\infty} Sx_n \text{ for some } z \in X$$

Lemma 2.10 [15] If a function $\phi : [0,\infty) \to [0,\infty)$ satisfies the condition (Φ) then we have

[1] For all $t > 0$, $\lim_{n\to\infty} \phi^n(t) = 0$, where $\phi^n(\mathrm{t})$ is the n-th iteration of $\phi(t)$.

[2] If $\{\mathrm{t}_n\}$ is a nondecresing sequence of real number and $t_{n+1} \leq \phi(t_n), \mathrm{n} = 1,2,3,--$
Then $\lim_{n\to\infty} t_n = 0$. In particular, if $t \leq \phi(t)$ for all $t > 0$, then $t = 0$.

Lemma 2.11[15] Suppose that $\{\mathrm{y}_n\}$ be a sequence in X such that $\lim_{n\to\infty} F(y_n, y_{n+1}, t) = 1$
For all $t > 0$.If the sequence $\{\mathrm{y}_n\}$ is not a Cauchy sequence in X, then there exists $\varepsilon_0 > 0$, $t_0 > 0$, two sequence $\{m_i\},\{n_i\}$ of positive integers such that

[1] $m_i > n_i + 1$ and $n_i \to \infty$ as $i \to \infty$

[2] $\mathrm{F}(y_{m_i}, y_{n_i}, t) < 1 - \varepsilon_0$ and $\mathrm{F}(y_{m_i - 1}, y_{n_i}, t) \geq 1 - \varepsilon_0, i = 1,2,---$

Proposition 2.12[15] Let $A,S : X \to X$ be continuous mapping A and S are said to be compatible of type (A), then they are weak compatible of type (A).
Proof .Suppose that A and S are compatible of type (A).Let $\{x_n\}$ be a sequence in X such that

$$\lim_{n\to\infty} Ax_n = z = \lim_{n\to\infty} Sx_n \text{ For some } z \in X \text{, Then}$$

$$\lim_{n\to\infty} g(M(\mathrm{SAx}_n, \mathrm{SSx}_n, \mathrm{t})) = 0$$

$$\leq \lim_{n\to\infty} g(M(ASx, SSx, t))$$

$$\Rightarrow \lim_{n\to\infty} g(M(ASx_n, SSx_n, t)) \geq \lim_{n\to\infty} g(M(SAx_n, SSx_n, t))$$

Similarly we can show that

$$\Rightarrow \lim_{n\to\infty} g(M(SAx_n, \mathrm{AAx}_n, t)) = 0$$

$$\geq \lim_{n\to\infty} g(M(ASx_n, AAx_n, t))$$

Therefore A and S are weak compatible of type (A).

Proposition 2.13[15] Let $A,S : X \to X$ be weak compatible mapping of type (A).If one of A and S is continuous, then A and S are compatible of type (A).
Proof. Let $\{x_n\}$ be a sequence in X such that

$$\lim_{n\to\infty} Ax_n = \mathrm{z} = \lim_{n\to\infty} Sx_n \text{ For some } z \in X$$

Suppose that S is continuous so $SSx_n, SAx_n \to Sz$ as $n \to \infty$, Since A and S are weak compatible of type (A), so we have

$$\lim_{n\to\infty} g(M(\mathrm{ASx}_n, \mathrm{SSx}_n, \mathrm{t})) \geq \lim_{n\to\infty} g(M(\mathrm{SAx}_n, \mathrm{SSx}_n, \mathrm{t}))$$

$$= \lim_{n\to\infty} g(M(Sz, Sz, t)) = 0$$

Thus

$$\lim_{n\to\infty} g(M(\mathrm{ASx}_n, \mathrm{SSx}_n, \mathrm{t})) = 0$$

Similarly $$\lim_{n\to\infty} g(M(\mathrm{SAx}_n, \mathrm{AAx}_n, \mathrm{t})) = 0$$

Hence A and S are compatible of type (A).

Proposition 2.14[15] Let $A, S : X \to X$ be continuous mapping then A and S are compatible of type (A) if and only if A and S are weak compatible of type (A).

Proposition 2.15 [15] Let $A, S : X \to X$ be mapping if A and S are weak compatible of type (A) and $Az = Sz$ for some $z \in X$ .Then $SAz = AAz = ASz = SSz$.

Proof. Let $\{x_n\}$ is a sequence in X defined by $x_n = z, n = 1, 2, ---$ and $Az = Sz$ for some $z \in X$ .Then we have $Ax_n, Sx_n \to Sz$ as $n \to \infty$ .Since A and S are weak compatible of type (A).

So we have

$$\lim_{n\to\infty} g(M(\mathrm{ASx}_n, \mathrm{SSx}_n, \mathrm{t})) \geq \lim_{n\to\infty} g(M(\mathrm{SA}\, x_n, \mathrm{SSx}_n, \mathrm{t}))$$

$$\lim_{n\to\infty} g(M(\mathrm{SAx}_n, \mathrm{AAx}_n, \mathrm{t})) \geq \lim_{n\to\infty} g(M(\mathrm{AS}\, x_n, \mathrm{AAx}_n, \mathrm{t}))$$

Now

$$\lim_{n\to\infty} g(M(\mathrm{SAz}, \mathrm{AAz}, \mathrm{t})) = \lim_{n\to\infty} g(M(\mathrm{SA}\, x_n, \mathrm{AAx}_n, \mathrm{t})) \geq \lim_{n\to\infty} g(M(\mathrm{AS}\, x_n, \mathrm{AAx}_n, \mathrm{t}))$$
$$= \lim_{n\to\infty} g(M(\mathrm{ASz}, \mathrm{SSz}, \mathrm{t}))$$

Since $Sz = Az$, then $SAz = AAz$ .Similarly we have $ASz = SSz$ but $Az = Sz$ for some $z \in X$ implies that $AAz = ASz = SAz = SSz$

Proposition 2.16 [15] Let $A, S : X \to X$ be weak compatible of type (A) and let $\{x_n\}$ be a sequence in X such that $\lim_{n\to\infty} Ax_n = z = \lim_{n\to\infty} Sx_n$ for some $z \in X$ then

(1) $\lim_{n\to\infty} ASx_n = Sz$ if S is continuous at z.

(2) $SAz = ASz$ and $Az = Sz$ if A and S are continuous at z.

Proof. (1) Suppose that S is continuous and $\{x_n\}$ is a sequence in X such that

$\lim_{n\to\infty} Ax_n = z = \lim_{n\to\infty} Sx_n$ for some $z \in X$

So $SSx_n \to Sz$ as $n \to \infty$

Since A and S are weak compatible of type (A) we have

$$\lim_{n\to\infty} g(M(\mathrm{ASx}_n, \mathrm{Sz}, \mathrm{t})) = \lim_{n\to\infty} g(M(\mathrm{AS}\, x_n, \mathrm{SSx}_n, \mathrm{t}))$$
$$\geq \lim_{n\to\infty} g(M(\mathrm{SAx}_n, \mathrm{SSx}_n, \mathrm{t})) \to 0 \text{ As } n \to \infty$$

For t>0 which implies that $ASx_n \to Sz$ as $n \to \infty$.

(2) Suppose that A and S are continuous at z .Since $Ax_n \to z$ as $n \to \infty$ and S is continuous at z by proposition 2.16 (1) $ASx_n \to Sz$ as $n \to \infty$, on the other hand, since $Sx_n \to z$ as $n \to \infty$ and A is also continuous at z, $ASx_n \to Az$ as $n \to \infty$ .Thus $Az = Sz$ by the unique of the limit and so by the proposition 2.15 $SAz = AAz = ASz = SSz$ .Therefore ,we have $ASz = SAz$.

III. Main Results:

Theorem 3.1 Let $A, B, S, T : X \to X$ be mapping satisfying

[1] $A(X) \subset T(X), B(X) \subset S(X)$

[2] The pair (A,S) and (B,T) are weak compatible of type (A),

[3] S and T is continuous ,

[4] $g(M(Ax,Bx,t)) \le \phi\left[\max\left\{\begin{array}{l} g(M(Sx,Ty,t)), g(M(Sx,\mathrm{Ax},t)), g(M(\mathrm{Ty},\mathrm{By},t)), \\ \frac{1}{2}(g(M(\mathrm{Sx},\mathrm{By},\mathrm{t})) + \mathrm{g}(\mathrm{M}(\mathrm{Ty},\mathrm{Ax},\mathrm{t}))) \end{array}\right\}\right]$

For all $t>0$ when a function $\phi:[0,\infty)\to[0,\infty)$ satisfy the condition (Φ).Then by (1) since $A(X)\subset T(X)$ for any $x_0 \in X$,there exists a point $x_1 \in X$ such that $Ax_0 = Tx_1$ .Since $B(X)\subset S(X)$ for any x_1 we can chose a point $x_2 \in X$ such that $Bx_1 = Sx_2$ and so on, inductively we can define a sequence $\{y_n\}$ in X such that

$$y_{2n} = Ax_{2n} = Tx_{2n+1}, \quad y_{2n+1} = Bx_{2n+1} = Sx_{2n+2}, \text{ for n=0, 1, 2, -----} \quad (1.1)$$

First we prove the following lemma

Lemma 3.2 Let $A,S: X\to X$ be mapping satisfying 3.1condition (1)& (4), then the sequence $\{y_n\}$ defined by (1.1) such that

$$\lim_{n\to\infty} g(\mathrm{M}(\mathrm{y}_n,\mathrm{y}_{n+1},\mathrm{t})) = 0, \forall t>0 \qquad (1.2)$$

is a Cauchy sequence in X.

Proof. Since $g\in\Omega$ .It follows that $\lim_{n\to\infty} g(\mathrm{M}(\mathrm{y}_n,\mathrm{y}_{n+1},\mathrm{t})) = 0$ for all $t>0$ .By lemma 2.11 if $\{y_n\}$ is not a Cauchy sequence in X there exists $\varepsilon_0>0, t_0>0$ and two sequence $\{m_i\},\{n_i\}$ of positive integer such that

(a) $m_i > n_i + 1$ and $n\to\infty$ as $i\to\infty$

(b) $g(M(y_{m_i}, y_{n_i}, t_0)) > \mathrm{g}(1-\varepsilon_0)$ and $g(M(y_{m_i-1}, y_{n_i}, t_0)) \le \mathrm{g}(1-\varepsilon_0)$, i=1,2,3,--

Since $g(t)=1-t$

Thus we have

$$\begin{aligned} \mathrm{g}(1-\varepsilon_0) &= g(M(y_{m_i}, y_{n_i}, t_0)) \\ &\le g(M(y_{m_i}, y_{n_i}, \mathrm{y}_{m_i-1}, t_0)) + g(M(y_{m_i}, \mathrm{y}_{m_i-1}, t_0)) + g(M(\mathrm{y}_{m_i-1}, y_{n_i}, t_0)) \qquad (1.3) \\ &\le g(M(y_{m_i}, y_{n_i}, \mathrm{y}_{m_i-1}, t_0)) + g(M(y_{m_i}, \mathrm{y}_{m_i-1}, t_0)) + g(1-\varepsilon_0) \end{aligned}$$

As $i\to\infty$ in (1.3) we have

$$\lim_{n\to\infty} g(M(y_{m_i}, y_{n_i}, t_0)) = \mathrm{g}(1-\varepsilon_0) \quad (1.4)$$

On the other hand we have

$$\begin{aligned} g(1-\varepsilon_0) &< g(M(y_{m_i}, y_{n_i}, t_0)) \\ &\le g(M(y_{m_i}, y_{n_i}, \mathrm{y}_{n_i+1}, t_0)) + g(M(y_{m_i}, \mathrm{y}_{n_i+1}, t_0)) + g(M(\mathrm{y}_{n_i+1}, y_{n_i}, t_0)) \quad (1.5) \end{aligned}$$

Now consider $g(M(y_{m_i}, y_{n_i+1}, t_0))$ in (1.5) assume that both m_i and n_i are even then by 3.1 (4)

We have

$$g(M(y_{m_i}, \mathrm{y}_{n_i+1}, t_0)) = \mathrm{g}(\mathrm{M}(\mathrm{Ax}_{m_i}, \mathrm{Bx}_{n_i+1}, \mathrm{t}_0))$$

$$\le \phi\left[\max\left\{\begin{array}{l} g(M(Sx_{m_i}, Tx_{n_i+1}, t_0)), g(M(Sx_{m_i}, \mathrm{A}\,x_{m_i}, t_0)), \\ g(M(\mathrm{T}\,x_{n_i+1}, \mathrm{B}\,x_{n_i+1}, t_0)), \frac{1}{2}(g(M(\mathrm{S}\,x_{m_i}, \mathrm{B}\,x_{n_i+1}, t_0)) + \mathrm{g}(M(\mathrm{T}\,x_{n_i+1}, \mathrm{B}\,x_{n_i+1}, t_0))) \end{array}\right\}\right]$$

$$= \phi\left[\max\left\{\begin{array}{l} g(M(y_{m_i-1}, y_{n_i}, t_0)), g(M(y_{m_i-1}, y_{m_i}, t_0)), \\ g(M(y_{n_i}, y_{n_i+1}, t_0)), \frac{1}{2}(g(M(y_{m_i-1}, y_{n_i+1}, t_0)) + \mathrm{g}(M(y_{n_i}, y_{m_i}, t_0))) \end{array}\right\}\right] \quad (1.6)$$

By (1.4), (1.5) and (1.6) let $i\to\infty$ in (1.6), we get

$$g(1-\varepsilon_0) \le \phi\left[\max\left\{g(1-\varepsilon_0), 0, 0, g(1-\varepsilon_0)\right\}\right] = \phi(g(1-\varepsilon_0)) < g(1-\varepsilon_0) \quad (1.7)$$

Which is contradiction .Therefore $\{y_n\}$ is a Cauchy sequence in X.

Theorem 3.3 If $\lim_{n\to\infty} g(M(y_{n_i}, y_{n_i+1}, t)) = 0$ for all $t > 0$.Then the sequence $\{y_n\}$ defined by (1.1) is a Cauchy sequence in X.

Proof. Suppose that $\lim_{n\to\infty} g(M(y_{n_i}, y_{n_i+1}, t)) = 0$ for all $t > 0$.In fact by theorem 3.1 (4) and (3.2) We have

$$g(M(y_{2n}, y_{2n+1}, t)) = g(\mathrm{M}(\mathrm{A}x_{2n}, \mathrm{B}x_{2n+1}, \mathrm{t}))$$

$$\le \phi\left[\max\left\{\begin{array}{l} g(M(Sx_{2n}, Tx_{2n+1}, t)), g(M(Sx_{2n}, \mathrm{A}\,x_{2n}, t)), \\ g(M(\mathrm{T}\,x_{2n}, \mathrm{B}x_{2n+1}, t)), \frac{1}{2}(g(M(Sx_{2n}, \mathrm{B}\,x_{2n+1}, t)) + g(M(\mathrm{T}\,x_{2n+1}\,\mathrm{A}\,x_{2n}, t))) \end{array}\right\}\right]$$

$$= \phi\left[\max\left\{\begin{array}{l} g(M(y_{2n-1}, y_{2n}, t)), g(M(y_{2n-1}, y_{2n}, t)), \\ g(M(y_{2n}, y_{2n+1}, t)), \frac{1}{2}(g(M(y_{2n-1}, y_{2n+1}, t)), +g(1)) \end{array}\right\}\right] \quad (1.8)$$

$$\le \phi\left[\max\left\{g(M(y_{2n-1}, y_{2n}, t)), g(M(y_{2n}, y_{2n+1}, t)), g(M(y_{2n-1}, y_{2n}, t)) + g(M(y_{2n}, y_{2n+1}, t))\right\}\right]$$

If $g(M(y_{2n-1}, y_{2n}, t)) \le g(M(y_{2n}, y_{2n+1}, t))$ for all $t > 0$.then by theorem 3.1 (4) $g(M(y_{2n}, y_{2n+1}, t)) \le \phi(g(M(y_{2n}, y_{2n+1}, t)))$ and thus by lemma 2.10 $g(M(y_{2n}, y_{2n+1}, t)) = 0$ for all $t > 0$.On the other hand if $g(M(y_{2n-1}, y_{2n}, t)) \ge g(M(y_{2n}, y_{2n+1}, t))$ then by lemma 3.1 (4),

We have

$$g(M(y_{2n}, y_{2n+1}, t)) \le \phi(g(M(y_{2n-1}, y_{2n}, t))) \text{ For all } t > 0 \quad (1.9)$$

Similarly

$$g(M(y_{2n+1}, y_{2n+2}, t)) \le \phi(g(M(y_{2n}, y_{2n+1}, t))) \text{ For all } t > 0 \quad (1.10)$$

Hence

$$g(M(y_n, y_{n+1}, t)) \le \phi(g(M(y_{n-1}, y_n, t))) \text{ For all } t > 0 \;\;, n = 1, 2, 3, -- \quad (1.11)$$

Therefore by lemma 2.10

$$\lim_{n\to\infty} g(M(y_n, y_{n+1}, t)) = 0 \text{ For all } t > 0 \quad (1.12)$$

Which implies that $\{y_n\}$ is a Cauchy sequence in X. By lemma 3.2 since $(X, M, *)$ is a complete, the sequence $\{y_n\}$ converges to a point $z \in X$ and so the subsequence $\{Ax_{2n}\}, \{\mathrm{B}x_{2n+1}\}, \{\mathrm{S}x_{2n}\}$ and $\{T\,\mathrm{x}_{2n+1}\}$ of $\{y_n\}$ also converges to the limit z.

Now, suppose that T is continuous. Since B and T are weak compatible of type (A) by proposition 2.16 BTx_{2n+1}, TTx_{2n+1} tends to Tz as n tends to ∞.

Putting $x = x_{2n}$ and $y = Tx_{2n+1}$ in theorem 3.1(4), we have

$$g(M(Ax_{2n}, \mathrm{BT}\,x_{2n+1}, t)) \le \phi\left[\max\left\{\begin{array}{l} g(M(Sx_{2n}, \mathrm{TT}\,x_{2n+1}, t)), \\ g(M(Sx_{2n}, \mathrm{A}\,x_{2n}, t)), \\ g(M(\mathrm{TT}\,x_{2n+1}, \mathrm{BT}\,x_{2n+1}, t)), \\ \frac{1}{2}(g(M(Sx_{2n}, \mathrm{BT}\,x_{2n+1}, \mathrm{t})) + \mathrm{g}(\mathrm{M}(\mathrm{TT}\,x_{2n+1}, \mathrm{A}\,x_{2n}, \mathrm{t}))) \end{array}\right\}\right]$$

(1.13)

Letting $n \to \infty$ in (1.13), we get

$$g(\mathrm{M(z,Tz,t)}) \le \phi\left[\max\left\{\begin{array}{l} g(\mathrm{M(z,Tz,t)}), g(\mathrm{M(z,z,t)}), g(\mathrm{M(Tz,Tz,t)}), \\ \frac{1}{2}(g(\mathrm{M(z,Tz,t)}) + g(\mathrm{M(Tz,z,t)})) \end{array}\right\}\right] \quad (1.14)$$

$$= \phi(g(M(z,Tz,t))), \forall t > 0$$

i.e. M (z, Tz, 1) =1

Which means that $g(M(z,Tz,t)) = 0$ for all $t > 0$ by lemma 2.10 and so we have $Tz = z$

We replace x by x_{2n} and y by z in theorem 3.1 (iv), we have

$$g(M(Ax_{2n}, Bz,t)) \le \phi\left[\max\left\{\begin{array}{l} g(M(Sx_{2n}, \mathrm{T}z,t)), g(M(Sx_{2n}, \mathrm{A}x_{2n},t)), \\ g(M(\mathrm{T}z, \mathrm{B}z,t)), \\ \frac{1}{2}(g(M(Sx_{2n}, \mathrm{B}z,\mathrm{t})) + \mathrm{g}(\mathrm{M}(\mathrm{T}z, \mathrm{A}x_{2n}, \mathrm{t}))) \end{array}\right\}\right] \quad (1.15)$$

Letting $n \to \infty$ in (1.15), we get

$$g(\mathrm{M(z,Bz,t)}) \le \phi\left[\max\left\{\begin{array}{l} g(\mathrm{M(z,z,t)}), g(\mathrm{M(z,Bz,t)}), g(\mathrm{M(z,Bz,t)}), \\ \frac{1}{2}(g(\mathrm{M(z,Bz,t)}) + g(\mathrm{M(z,z,t)})) \end{array}\right\}\right] \forall t > 0 \quad (1.16)$$

Which implies that $g(\mathrm{M(z,Bz,t)}) \le \phi[g(\mathrm{M(z,Bz,t)})]$

$$\Rightarrow g(\mathrm{M(z,Bz,t)}) = 0,\ \forall t > 0$$

$$i.e. M(z,Bz,t) = 1$$

So we have $Bz = z$ .Since $B(X) \subset S(X)$, there exists a point $u \in X$ such that $Bz = Su = z$ .By using condition theorem 3.1 (4) again we have

$$g(M(Au,z,t)) = g(M(Au,Bz,t))$$

$$\le \phi\left[\max\left\{\begin{array}{l} g(M(Su,Tz,t)), g(M(Su,Au,t)), g(M(Tz,Bz,t)), \\ \frac{1}{2}(g(M(Su,Bz,t)) + g(M(Tz,Au,t))) \end{array}\right\}\right] \quad (1.17)$$

$$\le \phi[g(M(\mathrm{A}u,z,t))], \forall t > 0$$

$$\Rightarrow g(\mathrm{M}(Au,\mathrm{z,t})) = 0$$

$$\text{i.e. } M(Au,z,t) = 1$$

Which means that $Au = z$ .Since A and S are weak compatible mapping of type (A) and $Au = Su = z$ by proposition 2.15 $Az = ASu = SSu = Sz$ .Again by using theorem 3.1(4) we have $Az = z$ .Therefore $Az = Bz = Sz = Tz = z$, that is z is a common fixed point of the given mapping A, B, S and T.The uniqueness of the common fixed point z follows easily from theorem 3.1(4).

Remark 3.3 In theorem 3.1, if S and T are continuous, then by proposition 2.14, the theorem is true even though the pair A, S and B and T is compatible of type (A) instead of condition (2).

Corollary 3.4 Suppose that $A, B, S, T : X \to X$ be mapping satisfying condition (1), (4) and the following.

[1] The pair (A, S) is compatible type (A) and (B, T) is weakly compatible type (A).
[2] One of A or S is continuous.

Then A, B, S and T have a unique common fixed point in X.

Corollary 3.5 Suppose that $A, B, S, T : X \to X$ be mapping satisfying condition (1), (4) and the following.

[1] The pair (A, S) and (B, T) are compatible type (A),

[2] One of A.B, S or T is continuous.

Then A, B, S and T have a unique common fixed point in X.

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