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Some Results on ϵ-Trans-Sasakian Manifolds

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ABSTRACT : *In this paper, we have studied* 3*-dimensional ε-trans-Sasakian manifold. Some basic results regarding* 3*-dimensional trans-Sasakian manifolds have been obtained. Locally φ-recurrent, locally φ-symmetric and φ-quasi conformally symmetric 3-dimensional ε-trans-Sasakian manifolds are also studied. Further some results on generalized Ricci-recurrent ε-trans-Sasakian manifold were given.*

KEYWORDS: *ε-trans-Sasakian manifold, locally φ-symmetric, locally φ-recurrent, quasi conformal curvature tensor, generalized Ricci-recurrent manifold.*

I. INTRODUCTION

In the Gray-Hervella classification of almost Hermitian manifolds [10], there appears a class W_4, of Hermitian manifolds which are closely related to locally conformal Kaehler manifolds. An almost contact metric structure on a manifolds *M* is called a *trans-Sasakian structure* [17] if the product manifold $M \times R$ belongs to the class W_4. The class $C_6 \oplus C_5$ ([14], [15]) coincides with the class of trans-Sasakian structures of type (α, β). In [15], local nature of the two subclasses, namely C_5 and C_6 structure of trans-Sasakian structures are characterized completely. Further trans-Sasakian structures of type (0, 0), (0, β) and (α, 0) are cosymplectic [2], β-Kenmotsu [11] and α-Sasakian [11] respectively. In 2003, U. C. De and M. M. Tripathi [7] obtained the explicit formulae for Ricci operator, Ricci tensor and curvature tensor in a 3-dimensional trans-Sasakian manifold. In 2007, C. S. Bagewadi and Venkatesha [1] studied some curvature tensors on a trans-Sasakian manifold. And in 2010, S. S. Shukla and D. D. Singh [19] studied ε-trans-Sasakian manifold. In their paper they have obtained fundamental results on ε-trans-Sasakian manifold.A Riemannian manifold is called locally symmetric due to Cartan if its Riemannian curvature tensor *R* satisfies the relation $\nabla R=0$, where ∇ denotes the operator of covariant differentiation [13]. Similarly the Riemannian manifold is said to be locally φ-symmetric if $\varphi^2(\nabla_W R)(X,Y)Z=0$, for all vector fields *X, Y, Z* and *W* orthogonal to *ξ*. This notion was introduced by T. Takahashi [20] for Sasakian manifolds. As a proper generalization of locally φ-symmetric manifolds, φ-recurrent manifolds were introduced by U. C. De and et al. [8]. Further locally φ-Quasiconformally symmetric manifolds were introduced and studied in [5]. In 2002, J. S. Kim and et al, [12] studied generalized Ricci-recurrent trans-Sasakian manifolds. A non-flat Riemannian manifold *M* is called a generalized Ricci-recurrent manifold [6], if its Ricci tensor *S* satisfies the condition,

$$(\nabla_X S)(Y,Z) = A(X)S(Y,Z) + B(X)g(Y,Z).$$

Where ∇ is the Levi-Civita connection of the Riemannian metric *g* and *A*, *B* are 1-forms on *M*. In particular, if the 1-form *B* vanishes identically, then *M* reduces to Ricci – recurrent manifold [18] introduced by E. M. Patterson.The paper is organized as follows: In section 2, preliminaries about the paper are provided. In section 3, the expressions for scalar curvature and Ricci tensor are obtained for three-dimensional ε-trans-Sasakian manifolds. In section 4, three-dimensional locally φ-recurrent ε-trans-Sasakian manifold are studied. Here we proved that 3-dimensional ε-trans-Sasakian manifold with α and β constant is locally φ-recurrent if and only if the scalar curvature is constant. Further in section 5, three-dimensional φ-Quasi conformally symmetric ε-trans-Sasakian manifold are studied and proved that a 3-dimensional ε-trans-Sasakian manifold with α and β constant is locally φ-Quasi conformally symmetric if and only if the scalar curvature is constant. Finally in section 6, some results on generalized Ricci-recurrent ε-trans-Sasakian manifold were given.

II. PRELIMINARIES

Let *M* be an ε- almost contact metric manifold [9] with an almost contact metric structure (φ, ξ, η, g, ε) that is, φ is a (1,1) tensor field, ξ is a vector field, η is a 1-form and *g* is an indefinite metric such that

(2.1) $$\phi^2(X) = -X + \eta(X)\xi, \quad \eta(\xi) = 1, \quad \phi(\xi) = 0, \quad \eta \circ \phi = 0.$$

(2.2) $$g(\xi,\xi) = \epsilon, \quad \eta(X) = \epsilon g(X,\xi)$$

(2.3) $$g(\phi X, \phi Y) = g(X,Y) - \epsilon\eta(X)\eta(Y),$$

for any vector fields *X, Y* on *M*, where ε is 1or -1 according as ξ is space like (or) time like.

An ε-almost contact metric manifold is called an ε-trans-Sasakian manifold [19], if

(2.4) $$(\nabla_X\phi)(Y) = \alpha\{g(X,Y)\xi - \epsilon\eta(Y)X\} + \beta\{g(\phi X,Y)\xi - \epsilon\eta(Y)\phi X\},$$

(2.5) $(\nabla_X \xi) = \epsilon\big(-\alpha\phi X + \beta(X - \eta(X)\xi)\big),$

(2.6) $(\nabla_X \eta)(Y) = -\alpha g(\phi X, Y) + \beta\big(g(X,Y) - \epsilon\eta(X)\eta(Y)\big),$

for any $X, Y \in \Gamma(TM)$, where ∇ is the Levi-Civita connection with respect to *g*.

Further in an ε-trans-Sasakian manifold, the following holds true:

(2.7) $$R(X,Y)\xi = (\alpha^2 - \beta^2)\{\eta(Y)X - \eta(X)Y\} + 2\alpha\beta\{\eta(Y)\phi X - \eta(X)\phi Y\} + \epsilon\{(Y\alpha)\phi X - (X\alpha)\phi Y + (Y\beta)\phi^2 X - (X\beta)\phi^2 Y\},$$

(2.8) $$R(\xi,Y)X = (\alpha^2 - \beta^2)\{\epsilon g(X,Y)\xi - \eta(X)Y\} + 2\alpha\beta\{\epsilon g(\phi X,Y)\xi + \eta(X)\phi Y\} + \epsilon(X\alpha)\phi Y + \epsilon g(\phi X,Y)(grad\alpha) - \epsilon g(\phi X,\phi Y)(grad\beta) + \epsilon(X\beta)(Y - \eta(Y)\xi),$$

(2.9) $$R(\xi,Y)\xi = \{\alpha^2 - \beta^2 - \epsilon(\xi\beta)\}(-Y + \eta(Y)\xi) - \{2\alpha\beta + \epsilon(\xi\alpha)\}(\phi Y),$$

(2.10) $$2\alpha\beta + \epsilon(\xi\alpha) = 0,$$

(2.11) $$S(X,\xi) = \big(2n(\alpha^2 - \beta^2) - \epsilon(\xi\beta)\big)\eta(X) - \epsilon(\phi X)\alpha - \epsilon(2n-1)(X\beta),$$

(2.12) $$Q\xi = \epsilon\big[\big(2n(\alpha^2 - \beta^2) - \epsilon(\xi\beta)\big)\xi + \phi(grad\alpha) - (2n-1)grad\beta\big],$$

(2.13) $$S(\xi,\xi) = 2n\big(\alpha^2 - \beta^2 - \epsilon(\xi\beta)\big).$$

Definition 2.1. *A non-flat Riemannian manifold M is called a generalized Ricci-recurrent manifold [12], if its Ricci tensor S satisfies the condition*

(2.14) $$(\nabla_X S)(Y,Z) = A(X)S(Y,Z) + B(X)g(Y,Z),$$

where ∇ denotes Levi-Civita connection of the Riemannian metric g and A and B are 1-forms on M.

Definition 2.2. *An* ε-*trans-Sasakian manifold is said to be locally φ-symmetric manifold* [4], *if*

(2.15) $$\phi^2\big((\nabla_W R)(X,Y)Z\big) = 0.$$

Definition 2.3. *An* ϵ*-trans-Sasakian manifold is said to be a φ-recurrent manifold* [3] *if there exist a non zero 1-form A such that*

(2.16) $$\phi^2\big((\nabla_W R)(X,Y)Z\big) = A(W)R(X,Y)Z,$$

for any arbitrary vector field X, Y, Z and W.
If X,Y,Z and W are orthogonal to ξ, then the manifold is called locally φ-recurrent manifold.
If the 1-form A vanishes, then the manifold reduces to a φ-symmetric manifold.

III. THREE DIMENSIONAL ϵ-TRANS-SASAKIAN MANIFOLD

Since conformal curvature tensor vanishes in a three dimensional Riemannian manifold, we get

(3.1) $$R(X,Y)Z = g(Y,Z)QX - g(X,Z)QY + S(Y,Z)X - S(X,Z)Y - \frac{r}{2}\big(g(Y,Z)X - g(X,Z)Y\big),$$

where *r* is the scalar curvature.

Theorem 3.1. *In a three dimensional ε-trans-Sasakian manifold, the Ricci operator is given by*

(3.2) $$QX = \Big[\frac{r}{2} - \epsilon(\alpha^2 - \beta^2) - \epsilon(\xi\beta)\Big]X - \Big[\frac{r}{2} + \xi\beta - 3\epsilon(\alpha^2 - \beta^2)\Big]\eta(X)\xi + \epsilon\big(\phi(grad\alpha) - grad\beta\big)\eta(X) - (\phi X)\alpha\xi - (X\beta)\xi.$$

Proof: Substitute Z by ξ in (3.1), we get

(3.3) $$R(X,Y)\xi = g(Y,\xi)QX - g(X,\xi)QY + S(Y,\xi)X - S(X,\xi)Y - \frac{r\epsilon}{2}\big(\eta(Y)X - \eta(X)Y\big).$$

Putting *Y*=*ξ* in (3.3), we get

(3.4) $$\epsilon QX = R(X,\xi)\xi + g(X,\xi)Q\xi - S(\xi,\xi)X + S(X,\xi)\xi + \frac{r\epsilon}{2}\big(X - \eta(X)\xi\big).$$

Using (2.2), (2.7) and (2.11) in (3.4), we get (3.2).

Theorem 3.2. *In a three dimensional ε-trans-Sasakian manifold, the Ricci tensor and curvature tensor are given by*

(3.5) $$S(X,Y) = \Big[\frac{r}{2} - \epsilon(\alpha^2 - \beta^2) + \xi\beta\Big]g(X,Y) - \Big[\frac{r}{2} + \xi\beta - 3\epsilon(\alpha^2 - \beta^2)\Big]\epsilon\eta(X)\eta(Y)$$

$$-\epsilon\eta(X)[(\phi Y)\alpha + Y\beta] - \epsilon\eta(Y)[(\phi X)\alpha + X\beta],$$

and

(3.6)
$$R(X,Y)Z = \left[\frac{r}{2} - 2\epsilon(\alpha^2-\beta^2) + 2(\xi\beta)\right](g(Y,Z)X - g(X,Z)Y)$$
$$-\left[\frac{r}{2} + \xi\beta - 3\epsilon(\alpha^2-\beta^2)\right][g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi$$
$$+\epsilon\eta(Y)\eta(Z)X - \epsilon\eta(X)\eta(Z)Y]$$
$$+\epsilon(\phi(grad\alpha) - grad\beta)[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)]$$
$$-(X\beta + (\phi X)\alpha)[g(Y,Z)\xi - \epsilon\eta(Z)Y]$$
$$+(Y\beta + (\phi Y)\alpha)[g(X,Z)\xi - \epsilon\eta(Z)X] - \epsilon((\phi Z)\alpha + Z\beta)[\eta(Y)X - \eta(X)Y].$$

Proof: Equation (3.5) follows from (3.2). Using (3.5) and (3.2) in (3.1), we get (3.6).

THREE DIMENSIONAL LOCALLY Φ--RECURRENT ϵ--TRANS-SASAKIAN MANIFOLD

Theorem 4.3. A three dimensional ε-trans-Sasakian manifold with α and β constants is locally φ-recurrent if and only if the scalar curvature is constant.

Proof: Taking the covariant differentiation of the equation (3.6), we have

$$(\nabla_W R)(X,Y)Z = \left[\frac{dr(W)}{2} - 4\epsilon(d\alpha(W) - d\beta(W) + 2(\nabla_W(\xi\beta))\right](g(Y,Z)X - g(X,Z)Y)$$
$$-\left[\frac{dr(W)}{2} + \nabla_W(\xi\beta) - 6\epsilon(d\alpha(W) - d\beta(W))\right][g(Y,Z)\eta(X)\xi$$
$$-g(X,Z)\eta(Y)\xi + \epsilon\eta(Y)\eta(Z)X - \epsilon\eta(X)\eta(Z)Y]$$
$$-\left[\frac{r}{2} + \xi\beta - 3\epsilon(\alpha^2-\beta^2)\right][g(Y,Z)\nabla_W\eta(X)\xi + g(Y,Z)\eta(X)\nabla_W\xi$$
$$-g(X,Z)\nabla_W\eta(Y)\xi - g(X,Z)\eta(Y)\nabla_W\xi + \epsilon\nabla_W\eta(Y)\eta(Z)X$$
$$+\epsilon\eta(Y)\nabla_W\eta(Z)X - \epsilon\nabla_W\eta(X)\eta(Z)Y - \epsilon\eta(X)\nabla_W\eta(Z)Y]$$
$$+\epsilon\nabla_W(\phi(grad\alpha) - grad\beta)[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)]$$
$$+\epsilon(\phi(grad\alpha) - grad\beta)[g(Y,Z)\nabla_W\eta(X) - g(X,Z)\nabla_W\eta(Y)]$$
$$-\nabla_W(X\beta + (\phi X)\alpha)\alpha[g(Y,Z)\xi - \epsilon\eta(Z)Y] - (X\beta + (\phi X)\alpha)[g(Y,Z)\nabla_W\xi$$
$$-\epsilon\nabla_W\eta(Z)Y] + \nabla_W(Y\beta + (\phi Y)\alpha)[g(X,Z)\xi - \epsilon\eta(Z)X]$$
$$+(Y\beta + (\phi Y)\alpha)[g(X,Z)\nabla_W\xi - \epsilon\nabla_W\eta(Z)X] - \epsilon\nabla_W((\phi Z)\alpha + Z\beta)(\eta(Y)X - \eta(X)Y)$$
$$-\epsilon((\phi Z)\alpha + Z\beta)(\nabla_W\eta(Y)X - \nabla_W\eta(X)Y).$$

Suppose α and β are constants and *X*, *Y*, *Z* and *W* orthogonal to ξ. Applying φ^2 on the above equation and using (2.16), we get

(4.1)
$$A(W)R(X,Y)Z = -\frac{dr(W)}{2}[g(Y,Z)X - g(X,Z)Y].$$

Putting $W=\{e_i\}$ in (4.1), where $\{e_i\}$, $i=1, 2, 3$ is an orthonormal basis of the tangent space at any point of the manifold and taking summation over i, $1 \le i \le 3$, we obtain

(4.2)
$$R(X,Y)Z = \lambda[g(Y,Z)X - g(X,Z)Y],$$

where $\lambda = -\frac{dr(e_i)}{2A(e_i)}$ is a scalar.

Corollary 4.1. *A three-dimensional ε-trans-Sasakian manifold with α and β constants is locally φ-symmetric if and only if the scalar curvature is constant.*

Theorem 4.4. *A three dimensional ε-trans-Sasakian manifold with α and β constants is locally φ-Ricci symmetric if and only if the scalar curvature is constant.*

Proof: Now differentiating (3.2) covariantly along *W* we obtain

(4.3)
$$(\nabla_W Q)(X) = \frac{dr(W)}{2}X - \frac{dr(W)}{2}\eta(X)\xi$$
$$-\left[\frac{r}{2} + \xi\beta - 3\epsilon(\alpha^2-\beta^2)\right]((\nabla_W\eta)(X)\xi + \eta(X)\nabla_W\xi)$$
$$+\epsilon(\phi(grad) - grad\beta)\nabla_W\eta(X) - (\phi X)\alpha\nabla_W\xi - (X\beta)\nabla_W\xi.$$

Applying φ^2 on both side of (4.3) and using (2.1) we have,

(4.4)
$$\phi^2(\nabla_W Q)(X) = \frac{dr(W)}{2}(-X + \eta(X)\xi)$$

$$-\left(\frac{r}{2}+\xi\beta-3\epsilon(\alpha^2-\beta^2)\right)(\eta(X)\phi^2\nabla_W\xi)$$
$$-(\phi X)\alpha\phi^2(\nabla_W\xi)-(X\beta)\phi^2(\nabla_W\xi).$$

If X is orthogonal to ξ, we get

(4.5) $$\phi^2(\nabla_W Q)(X) = -\frac{dr(W)}{2}X.$$

THREE DIMENSIONAL LOCALLY Φ-QUASICONFORMALLY SYMMETRIC ϵ--TRANS-SASAKIAN MANIFOLD

The quasiconformal curvature tensor on a Riemannian manifold is given by [5]

(5.1) $$C^*(X,Y)Z = aR(X,Y)Z + b[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY] - \frac{r}{3}\left[\frac{a}{2}+2b\right][g(Y,Z)X - g(X,Z)Y],$$

where a and b are constants and r is the scalar curvature of the manifold.

Theorem 5.5. *A three dimensional ε-trans-Sasakian manifold with α and β constants is locally φ-quasiconformally symmetric if and only if the scalar curvature is constant.*

Proof: Using (3.2), (3.5) and (3.6) in (5.1) we have,

(5.2) $$\begin{aligned}C^*(X,Y)Z &= (a+b)[\left\{\frac{r}{3}-2\epsilon(\alpha^2-\beta^2)+2(\xi\beta)\right\}(g(Y,Z)X-g(X,Z)Y)\\ &-\left\{\frac{r}{2}+\xi\beta-3\epsilon(\alpha^2-\beta^2)\right\}(g(Y,Z)\eta(X)\xi-g(X,Z)\eta(Y)\xi+\epsilon\eta(Y)\eta(Z)X\\ &-\epsilon\eta(X)\eta(Z)Y)+\epsilon(\phi(grad\alpha)-grad\beta)(g(Y,Z)\eta(X)-g(X,Z)\eta(Y))\\ &-(X\beta+(\phi X)\alpha)(g(Y,Z)\xi-\epsilon\eta(Z)Y)+(Y\beta+(\phi Y)\alpha)(g(X,Z)\xi-\epsilon\eta(Z)X)\\ &-\epsilon((\phi Z)\alpha+Z\beta)(\eta(Y)X-\eta(X)Y)\end{aligned}$$

Taking the covariant differentiation of the above equation and assuming α and β as constants we have,

(5.3) $$\begin{aligned}(\nabla_W C^*)(X,Y)Z &= (a+b)[\left\{\frac{dr(W)}{3}\right\}(g(Y,Z)X-g(X,Z)Y)-\left\{\frac{dr(W)}{2}\right\}(g(Y,Z)\eta(X)\xi\\ &-g(X,Z)\eta(Y)\xi+\epsilon\eta(Y)\eta(Z)X-\epsilon\eta(X)\eta(Z)Y)\\ &-\left\{\frac{r}{2}+\xi\beta-3\epsilon(\alpha^2-\beta^2)\right\}[g(Y,Z)(\nabla_W\eta(X)\xi+\eta(X)\nabla_W\xi)\\ &-g(X,Z)(\nabla_W\eta(Y)\xi+\eta(Y)\nabla_W\xi)+\epsilon\nabla_W\eta(Y)\eta(Z)X\\ &+\epsilon\eta(Y)\nabla_W\eta(Z)X-\epsilon\nabla_W\eta(X)\eta(Z)Y-\epsilon\eta(X)\nabla_W\eta(Z)Y]\\ &+\epsilon(\phi(grad\alpha)-grad\beta)(g(Y,Z)\nabla_W\eta(X)-g(X,Z)\nabla_W\eta(Y))\\ &-(X\beta+(\phi X)\alpha)(g(Y,Z)\nabla_W\xi-\epsilon\nabla_W\eta(Z)Y)\\ &+(Y\beta+(\phi Y)\alpha)(g(X,Z)\nabla_W\xi-\epsilon\nabla_W\eta(Z)X)\\ &-\epsilon((\phi Z)\alpha+Z\beta)(\nabla_W\eta(Y)X-\nabla_W\eta(X)Y)]\end{aligned}$$

Now assume that X, Y and Z are horizontal vector fields. Using (2.1) in (5.3), we get

(5.4) $$\phi^2(\nabla_W C^*)(X,Y)Z = (a+b)\left[\left\{\frac{dr(W)}{3}\right\}(g(Y,Z)X-g(X,Z)Y)\right].$$

Suppose $\phi^2(\nabla_W C^*)(X,Y)Z = 0$ then either $a+b=0$ or $dr(W)=0$. If $a+b=0$ then substituting $a=-b$ in (5.1) we find

(5.5) $$\phi^2(\nabla_W C^*)(X,Y)Z = aC(X,Y)Z,$$

Where C is the Weyl conformal curvature tensor. But in a 3-dimensional Riemannian manifold $C=0$ which implies $C^*=0$ and so $a+b\neq 0$. Therefore $dr(W) = 0$.

Using **Corollary 4.1** and **Theorem 5.5**, we state the following **Corollary:**

Corollary 5.2. A three-dimensional ε-trans-Sasakian manifold is locally φ-quasiconformally symmetric if and only if it is locally φ-symmetric.

IV. GENERALIZED RICCI-RECURRENT ϵ--TRANS-SASAKIAN MANIFOLD

Theorem 6.6. The 1-forms A and B of a generalized Ricci-recurrent (2n+1) dimensional ε-trans-Sasakian manifold are related by

(6.1)
$$B(X) = 2n\epsilon[X(\alpha^2 - \beta^2 - \epsilon(\xi\beta)) - (\alpha^2 - \beta^2 - \epsilon(\xi\beta))A(X)] - 2(2n-1)(\alpha\phi X + \beta\phi^2 X)\beta - 2(\alpha\phi^2 X - \beta\phi X)\alpha.$$

In particular, we get

(6.2)
$$B(\xi) = 2n\epsilon[\xi(\alpha^2 - \beta^2 - \epsilon(\xi\beta)) - (\alpha^2 - \beta^2 - \epsilon(\xi\beta))A(\xi)].$$

Proof: We have

(6.3)
$$(\nabla_X S)(Y,Z) = XS(Y,Z) - S(\nabla_X Y, Z) - S(Y, \nabla_X Z).$$

Using (2.14) in (6.3), we get

(6.4)
$$A(X)S(Y,Z) + B(X)g(Y,Z) = XS(Y,Z) - S(\nabla_X Y, Z) - S(Y, \nabla_X Z).$$

Putting $Y = Z = \xi$ in (6.4), we obtain

(6.5)
$$A(X)S(\xi,\xi) + \epsilon B(X) = XS(\xi,\xi) - 2S(\nabla_X \xi, \xi),$$

which in view of (2.5), (2.11) and (2.13) reduces to (6.1). The equation (6.2) is obvious from (6.1).

A Riemannian manifold is said to admit cyclic Ricci tensor if

(6.6)
$$(\nabla_X S)(Y,Z) + (\nabla_Y S)(Z,X) + (\nabla_Z S)(X,Y) = 0.$$

Theorem 6.7. In a (2n+1)-dimensional generalized Ricci-recurrent ε-trans-Sasakian manifold with cyclic Ricci tensor satisfies

(6.7)
$$\begin{aligned} A(\xi)S(X,Y) = {} & 2n\epsilon[(\alpha^2 - \beta^2 - \epsilon(\xi\beta))A(\xi) - (\alpha^2 - \beta^2 - \epsilon(\xi\beta))\xi]g(X,Y) \\ & - (2n(\alpha^2 - \beta^2) - \epsilon(\xi\beta))(A(X)\eta(Y) + A(Y)\eta(X)) \\ & + 2n\epsilon(\alpha^2 - \beta^2 - \epsilon(\xi\beta))(A(X)\eta(Y) + A(Y)\eta(X)) \\ & + \epsilon(2n-1)(A(X)Y\beta + A(Y)X\beta) + \epsilon(A(X)(\phi Y)\alpha + A(Y)(\phi X)\alpha) \\ & - 2n\epsilon(\alpha^2 - \beta^2 - \epsilon(\xi\beta))(\eta(Y)X + \eta(X)Y) \\ & + 2(2n-1)\{(\alpha\phi X + \beta\phi^2 X)\beta\eta(Y) - (\alpha\phi Y + \beta\phi^2 X)\beta\eta(X)\} \\ & + 2\{(\alpha\phi^2 X - \beta\phi X)\alpha\eta(Y) + (\alpha\phi^2 Y - \beta\phi Y)\alpha\eta(X)\}. \end{aligned}$$

Proof: From the definition of generalized Ricci-recurrent manifold and (6.6), we get

$$A(X)S(Y,Z) + B(X)g(Y,Z) + A(Y)S(Z,X) + B(Y)g(Z,X) + A(Z)S(X,Y) + B(Z)g(X,Y) = 0.$$

Putting $Z=\xi$ in the above equation we get,

$$A(\xi)S(X,Y) = -B(\xi)g(X,Y) - A(X)S(Y,\xi) - A(Y)S(X,\xi) - B(X)\eta(Y) - B(Y)\eta(X),$$

which in view of (2.11) and (6.2) gives (6.7).

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