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Deteriorating Items Inventory Models for Two Warehouses with Linear Demand, Time Varying Holding Cost under Inflation and Permisibble Delay in Payments

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ABSTRACT: *A two-warehouse inventory model for deteriorating items with linear trend in demand with time varying holding cost and inflationary conditions under permissible delay in payments is developed. Shortages are not allowed. A rented warehouse (RW) is used to store the excess units over the capacity of the own warehouse. Numerical examples are provided to illustrate the model and sensitivity analysis is also carried out for parameters.*

KEYWORDS: *Inventory model, Two-warehouse, Deterioration, Inflation, Permissible delay in payments*

I. INTRODUCTION

Deteriorating items inventory models have been studied by many authors in past. It is well known that certain products such as medicine, volatile liquids, food stuff decrease under deterioration during their normal storage period. Therefore while determining the optimal inventory policy of such types of products the loss due to deterioration must be considered. Ghare and Schrader [9] first developed an EOQ model with constant rate of deterioration. Covert and Philip [8] extended this model by considering variable rate of deterioration. Shah [23] further extended the model by considering shortages. The related work are found in (Nahmias [18], Raffat [20], Goyal and Giri [11], Wu et al. [27], Ouyang et al. [19]).Most of the existing literature in classical inventory model deals with single storage facility with the assumption that the available warehouse of the organization has unlimited capacity. But in actual practice many times the supplier provide price discounts for bulk purchases and the retailer may purchase more goods than can be stored in single warehouse (own warehouse). Therefore a rented warehouse (RW) is used to store the excess units over the fixed capacity W of the own warehouse. The rented warehouse is charged higher unit holding cost then the own warehouse, but offers a better preserving facility with a lower rate of deterioration. Hartley [12] first developed a two warehouse inventory model. An inventory model with infinite rate of replenishment with two warehouse was considered by Sarma [22]. Other research work related to two warehouse can be found in, for instance [Benkherouf [2], Bhunia and Maiti [3], Kar et al. [13], Chung and Huang [7], Rong et al. [21]]. An economic order quantity model under condition of permissible delay in payments was first considered by Goyal [10]. The model was extended by Aggarwal and Jaggi [1] for deteriorating items. An inventory model with varying rate of deterioration and linear trend in demand under trade credit was considered by Chang et al. [5]. Teng et al. [25] developed an optimal pricing and lot sizing model by considering price sensitive demand under permissible delay in payments. A literature review on inventory model under trade credit is given by Chang et al. [6]. Min et al. [15] developed an inventory model for exponentially deteriorating items under conditions of permissible delay in payments.

The effect of inflation and time value of money play important role in practical situations. Buzacott [4] and Mishra [16] simultaneously developed inventory model with constant demand and single inflation rate for all associated costs. Mishra [17] considered different inflation rate for different costs associated with inventory model with constant rate of demand. Singh et al. [24] considered a two-warehouse inventory model for deteriorating items under the condition of permissible delay in payments. Liang and Zhou [14] developed a two-warehouse inventory model for deteriorating items with constant rate of demand under conditionally permissible delay in payments. Tyagi and Singh [26] considered a two warehouse inventory model with time dependent demand, varying rate of deterioration and variable holding cost. In this paper we have developed a two-warehouse inventory model under time varying holding cost and linear demand under inflation and permissible delay in payments. Shortages are not allowed. Numerical examples are provided to illustrate the model and sensitivity analysis of the optimal solutions for major parameters is also carried out.

II. NOTATIONS AND ASSUMPTIONS:

The following notations and assumptions are used here:

NOTATIONS:

D(t) : Demand rate is a linear function of time t (a+bt, a>0, 0<b<1)
A : Replenishment cost per order for two warehouse system
c : Purchasing cost per unit
p : Selling price per unit
HC(OW): Holding cost per unit time is a linear function of time t (x_1+y_1t, $x_1>0$, $0<y_1<1$) in OW
HC(RW): Holding cost per unit time is a linear function of time t (x_2+y_2t, $x_2>0$, $0<y_2<1$) in RW
I_e : Interest earned per year
I_p : Interest charged per year
M : Permissible period of delay in settling the accounts with the supplier
T : Length of inventory cycle
I(t) : Inventory level at any instant of time t, $0 \le t \le T$
W : Capacity of owned warehouse
$I_0(t)$: Inventory level in OW at time t
$I_r(t)$: Inventory level in RW at time t
Q : Order quantity
R : Inflation rate
t_r : Time at which the inventory level reaches zero in RW in two warehouse system
$\theta_1 t$: Deterioration rate in OW, $0< \theta_1<1$
$\theta_2 t$: Deterioration rate in RW, $0< \theta_2<1$
TC_i : Total relevant cost per unit time (i=1,2,3)

ASSUMPTIONS:

The following assumptions are used in the development of the model:

- The demand of the product is declining as a linear function of time.
- Replenishment rate is infinite and instantaneous.
- Lead time is zero.
- Shortages are not allowed.
- OW has a fixed capacity W units and the RW has unlimited capacity.
- The goods of OW are consumed only after consuming the goods kept in RW.
- The unit inventory costs per unit in the RW are higher than those in the OW.
- During the time, the account is not settled; generated sales revenue is deposited in an interest bearing account. At the end of the credit period, the account is settled as well as the buyer pays off all units sold and starts paying for the interest charges on the items in stocks.

III. THE MATHEMATICAL MODEL AND ANALYSIS:

At time t=0, a lot size of certain units enter the system. W units are kept in OW and the rest is stored in RW. The items of OW are consumed only after consuming the goods kept in RW. In the interval $[0,t_r]$, the inventory in RW gradually decreases due to demand and deterioration and it reaches to zero at $t=t_r$. In OW, however, the inventory W decreases during the interval $[0,t_r]$ due to deterioration only, but during $[t_r, T]$, the inventory is depleted due to both demand and deterioration and by the time to T, both warehouses are empty. The figure describes the behaviour of inventory system.

Let I(t) be the inventory at time t ($0 \le t \le T$) as shown in figure.

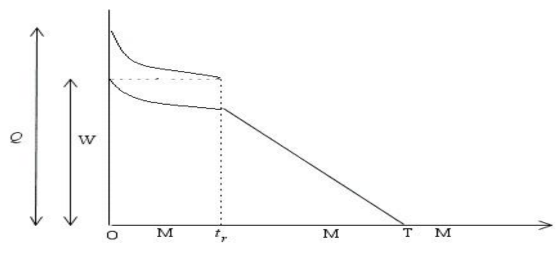


Figure 1

Hence, the inventory level at time t at RW and OW are governed by the following differential equations:

$$\frac{dI_r(t)}{dt} + \theta_2 t I_r(t) = -(a+bt), \qquad 0 \le t \le t_r \qquad (1)$$

with boundary conditions $I_r(t_r) = 0$ and

$$\frac{dI_0(t)}{dt} + \theta_1 t I_0(t) = 0, \qquad 0 \le t \le t_r \qquad (2)$$

with initial condition $I_0(0) = W$, respectively.

While during the interval (t_r, T), the inventory in OW reduces to zero due to the combined effect of demand and deterioration both. So the inventory level at time t at OW, $I_0(t)$, is governed by the following differential equation:

$$\frac{dI_0(t)}{dt} + \theta_1 t I_0(t) = -(a+bt), \qquad t_r \le t \le T \qquad (3)$$

with the boundary condition $I_0(T)=0$.

The solutions to equations (1) to (4) are given by:

$$I_r(t) = \begin{bmatrix} a(t_r - t) + \frac{1}{2}b(t_r^2 - t^2) + \frac{1}{6}a\theta_2(t_r^3 - t^3) \\ + \frac{1}{8}b\theta_2(t_r^4 - t^4) - \frac{1}{2}a\theta_2 t^2(t_r - t) - \frac{1}{4}b\theta_2 t^2(t_r^2 - t^2) \end{bmatrix}, \qquad 0 \le t \le t_r \qquad (4)$$

$$I_o(t) = W(1 - \theta_1 t^2), \qquad 0 \le t \le t_r \qquad (5)$$

$$I_o(t) = \begin{bmatrix} a(T - t) + \frac{1}{2}b(T^2 - t^2) + \frac{1}{6}a\theta_1(T^3 - t^3) \\ + \frac{1}{8}b\theta_1(T^4 - t^4) - \frac{1}{2}a\theta_1 t^2(T - t) - \frac{1}{4}b\theta_1 t^2(T^2 - t^2) \end{bmatrix}, \qquad t_r \le t \le T \qquad (6)$$

(by neglecting higher powers of θ_1, θ_2)

Using the condition $I_r(t) = Q – W$ at t=0 in equation (4), we have

$$Q - W = \left[at_r + \frac{1}{2}bt_r^2 + \frac{1}{6}a\theta_2 t_r^3 + \frac{1}{8}b\theta_2 t_r^4\right],$$

$$\therefore\ Q = W + \left[at_r + \frac{1}{2}bt_r^2 + \frac{1}{6}a\theta_2 t_r^3 + \frac{1}{8}b\theta_2 t_r^4\right]. \qquad (7)$$

Using the continuity of $I_0(t)$ at $t = t_r$ in equations (5) and (6), we have

$$I_o(t_r) = W(1 - \theta_1 t_r^2) = \begin{bmatrix} a(T - t_r) + \frac{1}{2}b(T^2 - t_r^2) + \frac{1}{6}a\theta_1(T^3 - t_r^3) \\ + \frac{1}{8}b\theta_1(T^4 - t_r^4) - \frac{1}{2}a\theta_1 t_r^2(T - t_r) - \frac{1}{4}b\theta_1 t_r^2(T^2 - t_r^2) \end{bmatrix} \qquad (8)$$

which implies that

$$T = \frac{-a + \sqrt{a^2 + 2bW - bW\theta_1 t_r^2 + b^2 t_r^2 + 2abt_r}}{b} \qquad (9)$$

(by neglecting higher powers of t_r and T)

From equation (9), we note that T is a function of t_r, therefore T is not a decision variable.

Based on the assumptions and descriptions of the model, the total annual relevant costs TC_i, include the following elements:

(i) Ordering cost (OC) = A (10)

(ii) $HC(RW) = \int_0^{t_r} (x_2 + y_2 t) I_r(t) e^{-Rt} dt$

$$= -\frac{1}{56}y_2R\theta_2bt_r^7 + \frac{1}{6}\left(\frac{1}{8}\left(y_2 - x_2R\right)\theta_2 b - \frac{1}{3}y_2R\theta_2 a\right)t_r^6$$
$$+ \frac{1}{5}\left(\frac{1}{8}x_2\theta_2 b + \frac{1}{3}\left(y_2 - x_2R\right)\theta_2 a - y_2R\left(-\frac{1}{2}\theta_2\left(\frac{1}{2}bt_r^2 + at_r\right) - \frac{1}{2}b\right)\right)t_r^5$$
$$+ \frac{1}{4}\left(\frac{1}{3}x_2\theta_2 a + \left(y_2 - x_2R\right)\left(-\frac{1}{2}\theta_2\left(\frac{1}{2}bt_r^2 + at_r\right) - \frac{1}{2}b\right) + y_2Ra\right)t_r^4$$
$$+ \frac{1}{3}\left(x_2\left(-\frac{1}{2}\theta_2\left(\frac{1}{2}bt_r^2 + at_r\right) - \frac{1}{2}b\right) - \left(y_2 - x_2R\right)a - y_2R\left(\frac{1}{8}b\theta_2 t_r^4 + \frac{1}{6}a\theta_2 t_r^3 + \frac{1}{2}bt_r^2 + at_r\right)\right)t_r^3 \quad (11)$$
$$+ \frac{1}{2}\left(-x_2 a + \left(y_2 - x_2R\right)\left(\frac{1}{8}b\theta_2 t_r^4 + \frac{1}{6}a\theta_2 t_r^3 + \frac{1}{2}bt_r^2 + at_r\right)\right)t_r^2$$
$$+ x_2\left(\frac{1}{8}b\theta_2 t_r^4 + \frac{1}{6}a\theta_2 t_r^3 + \frac{1}{2}bt_r^2 + at_r\right)t_r$$

(by neglecting higher powers of R)

(iii) $HC(OW) = \int_0^{t_1}(x_1 + y_1 t)I_0(t)e^{-Rt}dt = \int_0^{t_r}(x_1 + y_1 t)I_0(t)e^{-Rt}dt + \int_{t_r}^{T}(x_1 + y_1 t)I_0(t)e^{-Rt}dt$

$$= W\left(\frac{1}{10}y_1Rb\theta_1 t_r^5 + \frac{1}{8}\left(y_1 - x_1R\right)\theta_1 t_r^4 + \frac{1}{3}\left(-\frac{1}{2}x_1\theta_1 - y_1R\right)t_r^3 + \frac{1}{2}\left(y_1 - x_1R\right)t_r^2 + x_1 t_r\right)$$

$$+\left(\begin{array}{l} -\frac{1}{56}y_1Rb\theta_1 T^7 + \frac{1}{6}\left(\frac{1}{8}\left(y_1 - x_1R\right)b\theta_1 - \frac{1}{3}y_1Ra\theta_1\right)T^6 \\ + \frac{1}{5}\left(\frac{1}{8}x_1b\theta_1 + \frac{1}{3}\left(y_1 - x_1R\right)a\theta_1 - y_1R\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2 + aT\right) - \frac{1}{2}b\right)\right)T^5 \\ + \frac{1}{4}\left(\frac{1}{3}x_1a\theta_1 + \left(y_1 - x_1R\right)\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2 + aT\right) - \frac{1}{2}b\right) + y_1Ra\right)T^4 \\ + \frac{1}{3}\left(x_1\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2 + aT\right) - \frac{1}{2}b\right) - \left(y_1 - x_1R\right)a + y_1R\left(\frac{1}{8}b\theta_1 T^4 + \frac{1}{6}a\theta_1 T^3 + \frac{1}{2}bT^2 + aT\right)\right)T^3 \\ + \frac{1}{2}\left(-x_1 a - \left(y_1 - x_1R\right)\left(\frac{1}{8}b\theta_1 T^4 + \frac{1}{6}a\theta_1 T^3 + \frac{1}{2}bT^2 + aT\right)\right)T^2 + x_1\left(\frac{1}{8}b\theta_1 T^4 + \frac{1}{6}a\theta_1 T^3 + \frac{1}{2}bT^2 + aT\right)T \end{array}\right)$$

$$+\left(\begin{array}{l} \frac{1}{56}y_1Rb\theta_1 t_r^7 - \frac{1}{6}\left(\frac{1}{8}\left(y_1 - x_1R\right)b\theta_1 - \frac{1}{3}y_1Ra\theta_1\right)t_r^6 \\ - \frac{1}{5}\left(\frac{1}{8}x_1b\theta_1 + \frac{1}{3}\left(y_1 - x_1R\right)a\theta_1 - y_1R\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2 + aT\right) - \frac{1}{2}b\right)\right)t_r^5 \\ - \frac{1}{4}\left(\frac{1}{3}x_1a\theta_1 + \left(y_1 - x_1R\right)\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2 + aT\right) - \frac{1}{2}b\right) + y_1Ra\right)t_r^4 \\ - \frac{1}{3}\left(x_1\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2 + aT\right) - \frac{1}{2}b\right) - \left(y_1 - x_1R\right)a - y_1R\left(\frac{1}{8}b\theta_1 T^4 + \frac{1}{6}a\theta_1 T^3 + \frac{1}{2}bT^2 + aT\right)\right)t_r^3 \\ - \frac{1}{2}\left(-x_1 a + \left(y_1 - x_1R\right)\left(\frac{1}{8}b\theta_1 T^4 + \frac{1}{6}a\theta_1 T^3 + \frac{1}{2}bT^2 + aT\right)\right)t_r^2 - x_1\left(\frac{1}{8}b\theta_1 T^4 + \frac{1}{6}a\theta_1 T^3 + \frac{1}{2}bT^2 + aT\right)t_r \end{array}\right) \quad (12)$$

(iv) Deterioration cost

The amount of deterioration in both RW and OW during [0,T] are:

$$\int_0^{t_r}\theta_2 tI_r(t)dt \text{ and } \int_0^{T}\theta_1 tI_0(t)dt$$

So deterioration cost

$$DC = c\left[\int_0^{t_r}\theta_2 tI_r(t)e^{-Rt}dt + \int_0^{T}\theta_1 tI_0(t)e^{-Rt}dt\right]$$

$$= c\left[\int_0^{t_r}\theta_2 tI_r(t)e^{-Rt}dt + \int_0^{t_r}\theta_1 tI_0(t)e^{-Rt}dt + \int_{t_r}^{T}\theta_1 tI_0(t)e^{-Rt}dt\right]$$

$$= c\theta_2\left[\begin{array}{l} -\frac{1}{56}R\theta_2 bt_r^7 + \frac{1}{6}\left(\frac{1}{8}b\theta_2 - \frac{1}{3}Ra\theta_2\right)t_r^6 + \frac{1}{5}\left(\frac{1}{3}a\theta_2 - R\left(-\frac{1}{2}\theta_2\left(\frac{1}{2}bt_r^2 + at_r\right) - \frac{1}{2}b\right)\right)t_r^5 \\ + \frac{1}{4}\left(-\frac{1}{2}\theta_2\left(\frac{1}{2}bt_r^2 + at_r\right) - \frac{1}{2}b + Ra\right)t_r^4 \\ + \frac{1}{3}\left(-a - R\left(\frac{1}{8}b\theta_2 t_r^4 + \frac{1}{6}a\theta_2 t_r^3 + \frac{1}{2}bt_r^2 + at_r\right)\right)t_r^3 + \frac{1}{2}\left(\frac{1}{8}b\theta_2 t_r^4 + \frac{1}{6}a\theta_2 t_r^3 + \frac{1}{2}bt_r^2 + at_r\right)t_r^2 \end{array}\right]$$

$$+ c\theta_1 W\left[\frac{1}{10}R\theta_1 t_r^5 - \frac{1}{8}\theta_1 t_r^4 - \frac{1}{3}Rt_r^3 + \frac{1}{2}t_r^2\right]$$

$$+ c\theta_1\left[\begin{array}{l} -\frac{1}{56}R\theta_1 bT^7 + \frac{1}{6}\left(\frac{1}{8}b\theta_1 - \frac{1}{3}Ra\theta_1\right)T^6 + \frac{1}{5}\left(\frac{1}{3}a\theta_1 - R\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2 + aT\right) - \frac{1}{2}b\right)\right)T^5 \\ + \frac{1}{4}\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2 + aT\right) - \frac{1}{2}b + Ra\right)T^4 \\ + \frac{1}{3}\left(-a - R\left(\frac{1}{8}b\theta_1 T^4 + \frac{1}{6}a\theta_1 T^3 + \frac{1}{2}bT^2 + aT\right)\right)T^3 + \frac{1}{2}\left(\frac{1}{8}b\theta_1 T^4 + \frac{1}{6}a\theta_1 T^3 + \frac{1}{2}bT^2 + aT\right)T^2 \end{array}\right]$$

$$-c\theta_1\left[\begin{array}{l} -\frac{1}{56}R\theta_1 bt_r^7 + \frac{1}{6}\left(\frac{1}{8}b\theta_1 - \frac{1}{3}Ra\theta_1\right)t_r^6 + \frac{1}{5}\left(\frac{1}{3}a\theta_1 - R\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2 + aT\right) - \frac{1}{2}b\right)\right)t_r^5 \\ + \frac{1}{4}\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2 + aT\right) - \frac{1}{2}b + Ra\right)t_r^4 \\ + \frac{1}{3}\left(-a - R\left(\frac{1}{8}b\theta_1 T^4 + \frac{1}{6}a\theta_1 T^3 + \frac{1}{2}bT^2 + aT\right)\right)t_r^3 + \frac{1}{2}\left(\frac{1}{8}b\theta_1 T^4 + \frac{1}{6}a\theta_1 T^3 + \frac{1}{2}bT^2 + aT\right)t_r^2 \end{array}\right] \quad (13)$$

(vi) Interest Earned: There are two cases:

Case I : M $\le$ T:

In this case interest earned is:

$$IE_1 = pI_e\int_0^M (a+bt)te^{-Rt}dt$$

$$= pI_e\left[-\frac{1}{4}bRM^4 + \frac{1}{3}(-Ra+b)M^3 + \frac{1}{2}aM^2\right] \quad (14)$$

Case II : M > T:

In this case interest earned is:

$$IE_2 = pI_e\left(\int_0^T (a+bt)te^{-Rt}dt + (a+bT)T(M-T)\right)$$

$$= pI_e\left[-\frac{1}{4}bRT^2 + \frac{1}{3}(-Ra+b)T^3 + \frac{1}{2}aT^2 + (a+bT)T(M-T)\right] \quad (15)$$

(vii) Interest Payable: There are three cases described as in figure:

Case I : M $\le$ t_r $\le$ T:

In this case, annual interest payable is:

$$IP_1 = cI_p\left[\int_M^{t_r} I_r(t)e^{-Rt}dt + \int_M^{t_r} I_0(t)e^{-Rt}dt + \int_{t_r}^{T} I_0(t)e^{-Rt}dt\right]$$

$$= cI_p\left[\begin{array}{l} -\frac{1}{48}R\theta_2 bt_r^6+\frac{1}{5}\left(\frac{1}{8}\theta_2 b-\frac{1}{3}R\theta_2 a\right)t_r^5+\frac{1}{4}\left(\frac{1}{3}\theta_2 a-R\left(-\frac{1}{2}\theta_2\left(\frac{1}{2}bt_r^2+at_r\right)-\frac{1}{2}b\right)\right)t_r^4 \\ +\frac{1}{3}\left(-\frac{1}{2}\theta_2\left(\frac{1}{2}bt_r^2+at_r\right)-\frac{1}{2}b+Ra\right)t_r^3 \\ +\frac{1}{2}\left(-a-R\left(\frac{1}{8}b\theta_2 t_r^4+\frac{1}{6}a\theta_2 t_r^3+\frac{1}{2}bt_r^2+at_r\right)\right)t_r^2+\frac{1}{8}\theta_2 bt_r^5+\frac{1}{6}a\theta_2 t_r^4+\frac{1}{2}bt_r^3+at_r^2 \end{array}\right]$$

$$-cI_p\left[\begin{array}{l} -\frac{1}{48}R\theta_2 bM^6+\frac{1}{5}\left(\frac{1}{8}\theta_2 b-\frac{1}{3}R\theta_2 a\right)M^5+\frac{1}{4}\left(\frac{1}{3}\theta_2 a-R\left(-\frac{1}{2}\theta_2\left(\frac{1}{2}bt_r^2+at_r\right)-\frac{1}{2}b\right)\right)M^4 \\ +\frac{1}{3}\left(-\frac{1}{2}\theta_2\left(\frac{1}{2}bt_r^2+at_r\right)-\frac{1}{2}b+Ra\right)M^3 \\ +\frac{1}{2}\left(-a-R\left(\frac{1}{8}b\theta_2 t_r^4+\frac{1}{6}a\theta_2 t_r^3+\frac{1}{2}bt_r^2+at_r\right)\right)M^2+\frac{1}{8}\theta_2 bt_r^4M+\frac{1}{6}a\theta_2 t_r^3M+\frac{1}{2}bt_r^2M+at_rM \end{array}\right]$$

$$+cI_pW\left[t_r+\frac{1}{8}R\theta_1 t_r^4-\frac{1}{6}\theta_1 t_r^3-\frac{1}{2}Rt_r^2\right]-cI_pW\left[M+\frac{1}{8}R\theta_1 M^4-\frac{1}{6}\theta_1 M^3-\frac{1}{2}RM^2\right]$$

$$+cI_p\left[\begin{array}{l} -\frac{1}{48}R\theta_1 bT^6+\frac{1}{5}\left(\frac{1}{8}\theta_1 b-\frac{1}{3}R\theta_1 a\right)T^5+\frac{1}{4}\left(\frac{1}{3}\theta_1 a-R\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2+aT\right)-\frac{1}{2}b\right)\right)T^4 \\ +\frac{1}{3}\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2+aT\right)-\frac{1}{2}b+Ra\right)T^3 \\ +\frac{1}{2}\left(-a-R\left(\frac{1}{8}b\theta_1 T^4+\frac{1}{6}a\theta_1 T^3+\frac{1}{2}bT^2+aT\right)\right)T^2+\frac{1}{8}\theta_1 bT^5+\frac{1}{6}a\theta_1 T^4+\frac{1}{2}bT^3+aT^2 \end{array}\right]$$

$$-cI_p\left[\begin{array}{l} -\frac{1}{48}R\theta_1 bt_r^6+\frac{1}{5}\left(\frac{1}{8}\theta_1 b-\frac{1}{3}R\theta_1 a\right)t_r^5+\frac{1}{4}\left(\frac{1}{3}\theta_1 a-R\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_r^2+at_r\right)-\frac{1}{2}b\right)\right)t_r^4 \\ +\frac{1}{3}\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2+aT\right)-\frac{1}{2}b+Ra\right)t_r^3 \\ +\frac{1}{2}\left(-a-R\left(\frac{1}{8}b\theta_1 T^4+\frac{1}{6}a\theta_1 T^3+\frac{1}{2}bT^2+aT\right)\right)t_r^2+\frac{1}{8}\theta_1 bT^4t_r+\frac{1}{6}a\theta_1 T^3t_r+\frac{1}{2}bT^2t_r+aTt_r \end{array}\right] \quad (16)$$

Case II : $t_r \leq M \leq T$:

In this case interest payable is:

$$IP_2 = cI_p\int_M^T I_0(t)e^{-Rt}dt$$

$$= cI_p\left[\begin{array}{l} -\frac{1}{48}R\theta_1 bT^6+\frac{1}{5}\left(\frac{1}{8}\theta_1 b-\frac{1}{3}R\theta_1 a\right)T^5+\frac{1}{4}\left(\frac{1}{3}\theta_1 a-R\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2+aT\right)-\frac{1}{2}b\right)\right)T^4 \\ +\frac{1}{3}\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2+aT\right)-\frac{1}{2}b+Ra\right)T^3 \\ +\frac{1}{2}\left(-a-R\left(\frac{1}{8}b\theta_1 T^4+\frac{1}{6}a\theta_1 T^3+\frac{1}{2}bT^2+aT\right)\right)T^2+\frac{1}{8}\theta_1 bT^5+\frac{1}{6}a\theta_1 T^4+\frac{1}{2}bT^3+aT^2 \end{array}\right]$$

$$-cI_p\left[\begin{array}{l}-\frac{1}{48}R\theta_1 bM^6+\frac{1}{5}\left(\frac{1}{8}\theta_1 b-\frac{1}{3}R\theta_1 a\right)M^5+\frac{1}{4}\left(\frac{1}{3}\theta_1 a-R\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2+aT\right)-\frac{1}{2}b\right)\right)M^4\\+\frac{1}{3}\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2+aT\right)-\frac{1}{2}b+Ra\right)M^3\\+\frac{1}{2}\left(-a-R\left(\frac{1}{8}b\theta_1 T^4+\frac{1}{6}a\theta_1 T^3+\frac{1}{2}bT^2+aT\right)\right)M^2+\frac{1}{8}\theta_1 bT^4M+\frac{1}{6}a\theta_1 T^3M+\frac{1}{2}bT^2M+aTM\end{array}\right] \quad (17)$$

Case III : M > T:

In this case, no interest charges are paid for the item. So,

$IP_3 = 0.$ (18)

The retailer's total cost during a cycle, $TC_i(t_r,T)$, i=1,2,3 consisted of the following:

$$TC_i = \frac{1}{T}\left[A + HC(OW) + HC(RW) + DC + IP_i - IE_i\right] \quad (19)$$

Substituting values from equations (10) to (13) and equations (14) to (18) in equation (19), total costs for the three cases will be as under:

$$TC_1 = \frac{1}{T}\left[A + HC(OW) + HC(RW) + DC + IP_1 - IE_1\right] \quad (20)$$

$$TC_2 = \frac{1}{T}\left[A + HC(OW) + HC(RW) + DC + IP_2 - IE_2\right] \quad (21)$$

$$TC_3 = \frac{1}{T}\left[A + HC(OW) + HC(RW) + DC + IP_3 - IE_2\right] \quad (22)$$

The optimal value of $t_r = t_r^*$ (say), which minimizes $TC_i(t_r)$ can be obtained by solving equation (20), (21) and (22) by differentiating it with respect to t_r and equate it to zero

i.e. $$\frac{dTC_i(t_r)}{dt_r} = 0,\ i=1,2,3 \quad (23)$$

provided it satisfies the condition

$$\frac{d^2TC_i(t_r)}{dt_r^2} > 0,\quad (i=1,2,3). \quad (24)$$

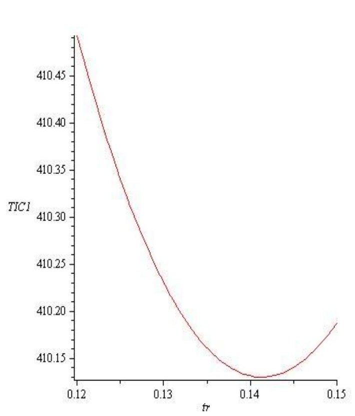
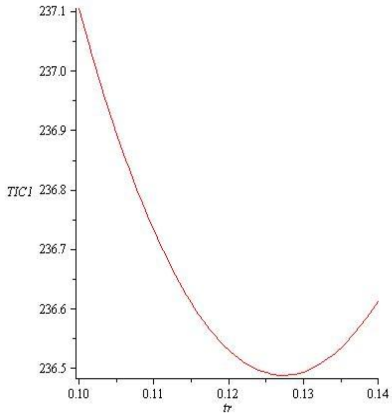
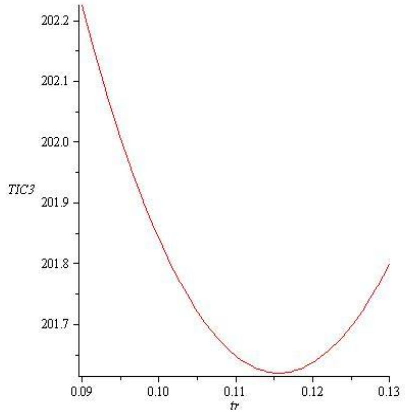
IV. NUMERICAL EXAMPLE

Case I: Considering A= Rs.150, W = 100, a = 200, b=0.05, c=Rs. 10, p= Rs. 15, θ_1=0.1, θ_2 =0.06, x_1 = Rs. 1, y_1=0.05, x_2= Rs. 3, y_2=0.06, Ip= Rs. 0.15, Ie= Rs. 0.12, R = 0.06, M=0.01 year, in appropriate units. The optimal value of t_r^* =0.1413, and TC_1^* = Rs. 410.1299.

Case II: Considering A= Rs.150, W = 100, a = 200, b=0.05, c = Rs. 10, p= Rs. 15, θ_1=0.1, θ_2 =0.06, x_1= Rs. 1, y_1=0.05, x_2= Rs. 3, y_2=0.06, Ip= Rs. 0.15, Ie = Rs. 0.12, M=0.55 year, in appropriate units. The optimal value of t_r^* =0.1272, and TC_2^* = Rs. 236.4879.

Case III: Considering A= Rs.150, W = 100, a = 200, b=0.05, c = Rs. 10, p= Rs. 15, θ_1=0.1, θ_2 =0.06, x_1= Rs. 1, y_1=0.05, x_2= Rs. 3, y_2=0.06, Ip= Rs. 0.15, Ie= Rs. 0.12, M = 0.65 year, in appropriate units. The optimal value of t_r^* =0.1155, and TC_3^* = Rs. 201.6199.

The second order conditions given in equation (24) are also satisfied. The graphical representation of the convexity of the cost functions for the three cases are also given.

Case I	Case II	Case I
t_r and cost	t_r and cost	t_r and cost
		
Graph - 1	Graph - 2	Graph - 3

V. SENSITIVITY ANALYSIS

On the basis of the data given in example above we have studied the sensitivity analysis by changing the following parameters one at a time and keeping the rest fixed.

Table 1 Sensitivity Analysis

		Case I		**Case II**		**Case III**	
		($M \le t_r \le T$)		**($t_r \le M \le T$)**		**($t_r \le T \le M$**	
Parameter	%	t_r	Cost	t_r	Cost	t_r	Cost
a	+10%	0.1515	431.5340	0.1365	239.7043	0.1219	200.6922
	+5%	0.1467	420.9230	0.1322	238.2046	0.1219	201.2357
	-5%	0.1349	399.1463	0.1215	234.5446	0.1084	201.8053
	-10%	0.1276	387.9635	0.1146	232.3644	0.1001	201.7821
x_1	+10%	0.1374	416.1636	0.1234	242.4370	0.1120	207.4963
	+5%	0.1394	413.1495	0.1253	239.4654	0.1138	204.5610
	-5%	0.1432	407.1047	0.1292	233.5045	0.1174	198.6731
	-10%	0.1450	404.0740	0.1311	230.5152	0.1192	195.7206
x_2	+10%	0.1341	411.0189	0.1208	237.2251	0.1101	202.4168
	+5%	0.1341	410.5945	0.1239	236.8652	0.1128	201.9376
	-5%	0.1452	409.6529	0.1308	236.0919	0.1185	201.2876
	-10%	0.1493	409.1521	0.1345	235.6754	0.1216	200.9397
θ_1	+10%	0.1387	411.5305	0.1248	237.7977	0.1134	202.8847
	+5%	0.1400	410.8313	0.1260	237.1438	0.1145	202.2532
	-5%	0.1426	409.4264	0.1296	235.2360	0.1177	200.4112
	-10%	0.1438	408.7206	0.1297	235.1699	0.1178	200.3480
θ_2	+10%	0.1412	410.1389	0.1272	236.4946	0.1155	201.6250
	+5%	0.1412	410.1344	0.1272	236.4913	0.1155	201.6225
	-5%	0.1413	410.1254	0.1273	236.4846	0.1156	201.6174
	-10%	0.1414	410.1209	0.1274	236.4813	0.1157	201.6174
R	+10%	0.1418	409.8935	0.1278	236.5673	0.1153	201.7909
	+5%	0.1415	410.0118	0.1275	236.5277	0.1154	201.6225
	-5%	0.1410	410.2479	0.1270	236.4481	0.1157	201.5343
	-10%	0.1407	410.3659	0.1267	236.4081	0.1157	201.4487
M	+10%	0.1413	409.8291	0.1244	217.6154	0.1155	178.2163
	+5%	0.1413	409.9795	0.1259	227.0798	0.1156	189.9181
	-5%	0.1412	410.2802	0.1286	245.8406	0.1156	213.3037
	-10%	0.1412	410.4306	0.1289	255.1384	0.1160	225.0232
A	+10%	0.1644	433.1068	0.1508	259.9607	0.1375	225.5609
	+5%	0.1529	412.7200	0.1391	248.3323	0.1266	213.8051
	-5%	0.1294	398.3251	0.1151	224.4147	0.1043	189.4347
	-10%	0.1173	386.2931	0.1028	212.0985	0.0928	177.2495

From the table we observe that as parameter a increases/ decreases average total cost increases/ decreases in case I and case II, whereas there very slight decrease/ increase in average total cost due to increase/ decrease in parameter a in case III.

From the table we observe that with increase/ decrease in parameters A, x_1 and θ_1, there is corresponding increase/ decrease in total cost for case I, case II and case III respectively.

From the table we observe that with increase/ decrease in parameter x_2, there is corresponding increase/ decrease in total cost for case I and there is very slight increase/ decrease in total cost for case II and case III respectively.

Also, we observe that with increase and decrease in the value of θ_2, there is corresponding very slight increase/ decrease in total cost for case I, case II and case III.

Also, we observe that with increase and decrease in the value of R, there is corresponding very slight decrease/ increase in total cost for case I, and there is very slight increase/ decrease in total cost for case II and case III respectively.

Also, we observe that with increase and decrease in the value of M, there is corresponding very slight decrease/ increase in total cost for case I, and there is decrease/ increase in total cost for case II and case III respectively.

VI. CONCLUSION

In this paper, we have developed a two warehouse inventory model for deteriorating items with linear demand under inflationary conditions and permissible delay in payments. It is assume that rented warehouse holding cost is greater than own warehouse holding cost but provides a better storage facility and there by deterioration rate is low in rented warehouse. Sensitivity with respect to parameters have been carried out. The results show that with the increase/ decrease in the parameter values there is corresponding increase/ decrease in the value of cost.

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