Effect of Heat of Transfer on Unsteady Mhd Couette Flow Between Two Infinite Parallel Porous Plates In An Inclined Magnetic Field.

Daniel Simon

Department of Mathematics, Kaduna State University – Nigeria

ABSTRACT : The effect of heat transfer on unsteady MHD couette flow between two infinite parallel porous plates in an inclined magnetic field has been investigated. The lower plate is considered porous. The governing equations of the flow field are solved by variable perturbation technique and the expression for the velocity **u** and temperature $\boldsymbol{\theta}$ are obtained. The effects of various parameters such as Hartman number H**a**, Grashof number Gr, Radiation parameter N and Prandtl number P**r** on the flow field have been studied, the results are presented graphically and are discussed quantitatively.

KEYWORDS: Unsteady, MHD, Couette Flow, Porous Plate, Heat Transfer,.

I. INTRODUCTION

The Effect of Heat Transfer on Steady MHD Couette Flow between Two Infinite Parallel Porous Plates in an Inclined Magnetic Field has many applications in different field of engineering and technology. The interaction between the conduction fluid and the magnetic field radically modifies the flow, with effects on such important flow properties as heat transfer, the detail nature of which is strongly dependent on the orientation of the magnetic field. When fluid moves through a magnetic field, an electric field or consequently a current may be induced, and in turn the current interacts with the magnetic field to produce a body force on fluid. The production of this current has led to MHD power generators, MHD devices, nuclear engineering and the possibility of thermonuclear power has created a great practical need for understanding the dynamics of conducting. The influence of a magnetic field in viscous incompressible flow of electrically conducting fluid is of use in extrusion of plastics in the manufacture of rayon, nylon etc.

Hannes Alfven (1942), a Swedish electrical engineer first initiated the study of MHD. Shercliff (1956) considered the steady motion of an electrically conducting fluid in pipes under transverse magnetic fields. Sparrow and Cess (1961) observed that the free convection heat transfer to liquid metals may be significantly affected by the presence of magnetic field. Drake (1965) considered flow in a channel due to periodic pressure gradient and solved the resulting equation by separation of variables methods. Singh and Ram (1978) studied Laminar flow of an electrically conducting fluid through a channel in the presence of a transverse magnetic field under the influence of a periodic pressure gradient and solved the resulting differential equation by the method of Laplace transform. More to this, Ram et al (1984) have analyzed Hall effects on heat and mass transfer flow through porous media. Soundelgekar and Abdulla Ali (1986) studied the flow of viscous incompressible electrically conducting fluid past an impulsively started infinite vertical isothermal plate. Singh (1993) considered steady MHD fluid flow between two parallel plates. John Mooney and Nick Stokes (1997) considered the numerical requirements for MHD flows with free surfaces. Raptis and Perdikis (1999) considered the effects of thermal radiation and free convection flow past a moving vertical plate. Al-Hadhrami (2003) discussed flow through horizontal channels of porous material and obtained velocity expressions in terms of the Reynolds number. Ganesh (2007) studied unsteady MHD Stokes flow of a viscous fluid between two parallel porous plates. Stamenkovic et al (2010) investigates MHD flow of two immiscible and electrically conducting fluids between isothermal, insulated moving plates in the presence of applied electric and magnetic fields. He matched the solution at the interface and it was found that decrease in magnetic field inclination angle flattens out the velocity and temperature profiles. Rajput and Sahu (2011) studied the effect of a uniform transverse magnetic field on the unsteady transient free convection flow of an incompressible viscous electrically conducting fluid between two infinite vertical parallel porous plates with constant temperature and variable mass diffusion. Manyonge et al (2012) studied steady MHD Poiseuille flow between two infinite parallel porous plates in an inclined magnetic field and discover that high magnetic field strength decreases the velocity. Sandeep and Sugunamma (2013) analysed the effect of an inclined magnetic field on unsteady free convection flow of a dusty viscous fluid between two infinite flat plates filled by a porous medium.

Heat transfer effects on rotating MHD coquette flow in a channel partially field by a porous medium with hall current has been discussed by Singh and Rastogi (2012). Joseph et al (2014) studied the unsteady MHD couetteflow between two infinite parallel porous plates in an inclined magnetic field with heat transfer. The unsteady MHD poiseuille flow between two infinite parallel plates in an inclined magnetic field with heat transfer has been studied by Idowu et al (2014). In this paper, we investigated the effect of heat transfer on unsteady MHD Couette flow between two infinite parallel porous plates in an inclined magnetic field.

II. **PROBLEM FORMULATION**

A magnetic field of field strength represented by the vector B at right angle to the flow of an electrically conducting fluid moving with velocity V was introduced. Here, an electric field vector denoted by Eis induced at right angle to both V and B because of their interaction. We assume that the conducting fluid exhibits adiabatic flow in spite of magnetic field, then we denote the electrical conductivity of the fluid by a scalar σ . Lorentz force comes in place because the conducting fluid cuts the lines of the magnetic field in electric generator. This vector \mathbf{F} is parallel to \mathbf{V} but in opposite direction but is perpendicular to the plane of both] and B. Laminar flow through a channel under uniform transverse magnetic field is important because of the use of MHD generator, MHD pump and electromagnetic flow meter. Here, we consider an electrically conducting, viscous, unsteady, incompressible fluid moving between two infinite parallel plates both kept at a constant distance 2h.

The equations of motion are the continuity equation

$$\nabla . V = 0 \tag{2.1}$$

And the Navier-Stokes equation

$$\rho\left[\left(\frac{\partial}{\partial t} + V.\nabla\right)\right]V = f_{B} - \nabla P + \mu\nabla^{2}V \qquad (2.2)$$

Where ρ is the fluid density, f_{B} is the body force per unit mass of the fluid, μ is the fluid viscosity and P is the pressure acting on the fluid. If one dimensional flow is assumed, so that we choose the axis of the channel formed by the two plates as the x - axis and assume that flow is in this direction. Observed that $\overline{u}, \overline{v}$ and \overline{w} are the velocity components in \overline{x} , \overline{y} and \overline{z} directions respectively. Then this implies $\overline{v} = \overline{w} = 0$ and $\overline{u} \neq 0$, then the continuity equation is satisfied. From this we infer that \overline{u} is independent of \overline{x} and this will make $[(V,\nabla)V]$ in the Navier-stokes equation to vanish. The body force f_{B} is neglected and replace with Lorentz force and from the assumption that the flow is one dimensional, it means that the governing equation for this flow is

$$\frac{\partial \alpha}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + v \frac{\partial^2 \alpha}{\partial y^2} + \frac{F_x}{\rho}$$
(2.3)
Where $v = \frac{\mu}{r}$ is the kinematics viscosity and F_x is the component of the magnetic force in the direction of x-axis.

Assuming unidirectional flow so that $\bar{v} = \bar{w} = 0$ and $B_x = B_z = 0$ since magnetic field is along y-direction so that $V = i\bar{u}$ and $B = B_{0j}$ where B_0 is the magnetic field strength component. Now,

(2.5)

$$F_{x} = \sigma[(i\bar{u} \times jB_{0})] \times jB_{0}$$
(2.4)

So that we have

$$\frac{F_x}{\rho} = -\frac{\sigma}{\rho} B_0^2 \bar{u} \tag{2}$$

Then
$$(2,3)$$
 becomes

$$\frac{\partial \sigma}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + v \frac{\partial^2 \sigma}{\partial y^2} - \frac{\sigma}{\rho} B_0^2 \bar{u}$$
(2.6)

From (2.6), when angle of inclination is introduced, we have

$$\frac{\partial \alpha}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + v \frac{\partial^2 \alpha}{\partial y^2} - \frac{\sigma}{\rho} B_0^2 \bar{u} \sin^2(\alpha)$$
(2.7)

Where α is the angle between V and B. Equation (2.7) is general in the sense that both field can be assessed at any angle α for $0 \leq \alpha \leq \pi$.

Because of the porosity of the lower plate, the characteristic velocity v_0 is taken as a constant so as to maintain the same pattern of flow against suction and injection of the fluid in which it is moving perpendicular to the fluid flow. The origin is taken at the centre of the channel and \bar{x}, \bar{y} coordinate axes are parallel and perpendicular to the channel walls respectively. The governing equation, that is, the momentum equation is as follows

$$\rho \frac{\partial \alpha}{\partial t} = -v_0 \frac{\partial \alpha}{\partial y} - \frac{\partial p}{\partial x} + \mu \frac{\partial^2 \alpha}{\partial y^2} - \frac{\sigma}{\rho} B_0^2 \bar{u} \sin^2(\alpha) + g\beta(\bar{T} - \bar{T}_{\infty})$$
(2.8)

Since the flow is isentropic, the energy equation is given as

$$\frac{\partial T}{\partial f} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial g^2} + \frac{1}{\rho c_p} \frac{\partial q}{\partial g}$$
(2.9)

Where k the thermal conductivity of the fluid, ρ is is the density, c_p is the specific heat constant pressure and \overline{T} the temperature.

The q in (2.9) is called the radiative heat flux. It is given by, $\frac{\partial q}{\partial \tau} = 4\alpha^2 (\overline{T_{\infty}} - \overline{T})$ (2.10)The boundary conditions are $\bar{u}(y,t) = 0, \bar{T} = \bar{T}_{\infty} \text{ at } \bar{t} = 0,$ $\overline{u}(-L,\overline{t}) = 0, \overline{u}(L,\overline{t}) = \frac{v}{L}, \overline{T} = \overline{T}_w \text{ at } \overline{t} > 0$ (2.11)

In other to solve equations (2.8) and (2.9) subject to the boundary conditions (2.11), we introduce the following dimensionless parameters:

$$\bar{x} = xL, \ \bar{y} = yL, \ \bar{p} = p\rho \frac{v^2}{L^2}, \ \bar{u} = \frac{uv}{L}, \ \bar{t} = \frac{tL^2}{v}, P_r = \frac{\mu c_p}{k}, \theta = \frac{\tau - \tau_{\infty}}{\tau_w - \tau_{\infty}}, \ Ha^2 = \frac{\sigma L^2 B_0^2}{\mu}, Gr = \frac{\rho L^2 gb(\tau_w - \tau)}{\mu v}, N^2 = \frac{4a^2 L^2}{k}$$
(2.12)
Equations (2.8) and (2.9) now become
$$\frac{\partial u}{\partial t} = A \frac{\partial u}{\partial y} + \frac{\partial^2 u}{\partial y^2} - M^2 u + Gr\theta$$
(2.13)

Where $M = M^* \sin \alpha$ and $M^* = LB_0 \int_{\mu}^{0} = H\alpha$, $A = \frac{-\pi e}{\rho}$. We assume that the rate of $\frac{\partial p}{\partial x} = 0$ (since it is couette

flow)

$$P_r \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + N^2 \theta$$
(2.14)

The boundary conditions in dimensionless form are $u(y,t) = 0, \theta(-1,t) = 0 \text{ at } t = 0$

$$u(-1,t) = 0, u(1,t) = 1, \theta(1,t) = 1at t > 0$$
 (2.15)

III. METHOD OF SOLUTION/SOLUTION OF THE PROBLEM

The momentum equation and energy equation can be reduced to the set of ordinary differential equations, which are solved analytically. This can be done by representing the velocity, temperature and concentration of the fluid in the perturbation series as follows

$$u(y,t) = u_0(y) + \varepsilon u_1(y)e^{i\omega t} + o(\varepsilon^2)$$
(3.1)

$$\theta(y,t) = \theta_0(y) + \varepsilon \theta_1(y) e^{i\omega t} + o(\varepsilon^2)$$
(3.2)

Substituting equations (3.1) and (3.2) into equations (2.13)-(2.15), equating the coefficients of harmonic and non-harmonic term and neglecting the coefficients of higher order of ε^2 , we get:

$u_{0}''(y) - Bu_{0}'(y) - M^{2}u_{0}(y) = -G_{r}\theta_{0}(y)$	(3.3)
$u_{1}''(y) - Bu_{1}'(y) - bu_{1}(y) = -G_{r}\theta_{1}(y)$	(3.4)
Where $b = M^2 + i\omega$	

$$\theta_0''(y) + N^2 \theta_0(y) = 0 \tag{3.5}$$

$$\theta_1''(y) - a_1 \theta_1(y) = 0 \tag{3.6}$$

Where, $a_1 = i\omega Pr - N^2$

The corresponding boundary condition become

 $u_0(-1,t) = 0, \theta_0(-1,t) = 0, u_1(-1,t), \theta_1(-1,t) \text{ at } t < 0$

$$u_0(1,t) = 0, \theta_0(1,t) = 0, u_1(1,t), \theta_1(1,t) \text{ at } t > 0$$

(3.7)

We now solved equations (3.4) - (3.6) under the relevant boundary conditions for the mean flow and unsteady flow separately. The mean flows are governed by the equations (3.3), (3.5) where u_0 and θ_0 are respectively called the mean velocity and respectively. The unsteady flows are governed by equations (3.4) and (3.6) where u_1 and θ_1 are the unsteady components.

These equations are solved analytically under the relevant boundary conditions (3.7) as follows;

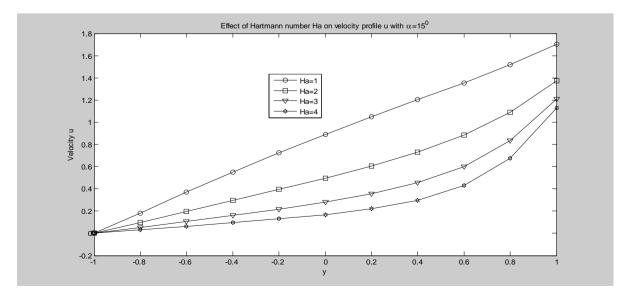
Solving equations (3.3) and (3.5) subject to the corresponding relevant boundary conditions in (3.7), we obtain the mean velocity and mean temperature as

$$\begin{aligned} u_0(y) &= C_5 e^{m_1 y} + C_6 e^{m_2 y} + K_1 cosNy + K_2 sinNy \end{aligned} (3.7) \\ \theta_0(y) &= C_1 cosNy + C_2 sinNy \end{aligned} (3.8) \\ \text{Similarly, solving equations (3.7) and (3.9) under the relevant boundary conditions in (3.10), the unsteady temperature becomes \\ u_1(y) &= C_7 e^{m_3 y} + C_6 e^{m_4 y} + K_3 e^{\sqrt{a}y} + K_4 e^{-\sqrt{a}y} \end{aligned} (3.9) \\ \theta_1(y) &= C_2 e^{\sqrt{a}y} + C_4 e^{-\sqrt{a}y} \end{aligned} (3.10)$$

Therefore, the solutions for the velocity, temperature and species concentration profiles are $u(y,t) = C_5 e^{m_1 y} + C_6 e^{m_2 y} + K_1 cosNy + K_2 sinNy + \varepsilon [C_7 e^{m_3 y} + C_6 e^{m_4 y} + K_2 e^{\sqrt{a}y} + K_4 e^{-\sqrt{a}y}] e^{i\omega t}$ (3.11) $\theta(y,t) = C_1 cosNy + C_2 sinNy + \varepsilon [C_2 e^{\sqrt{a}y} + C_4 e^{-\sqrt{a}y}] e^{i\omega t}$ (3.12)

IV. DISCUSSION OF RESULTS

To discuss the effect of Heat Transfer on Unsteady MHD Couette flow between two infinite parallel porous plates in an inclined magnetic field. The velocity profile u and the temperature distribution θ are shown graphically against y using Matlab for different values of the following parameters such as Hartmann number Ha, Grashof number Gr, Radiation parameter N and Prandtl number Pr. Figures 1, 2, 3, and 4 depict decrease in velocity u as Hartmann number Ha increases with effect of increase in the angle of inclination \propto on velocity. To this effect, the magnetic field suppresses the turbulence of flow. Figure 5, 6, 7, and 8 show the effect of Grashof number Gr and the angle of inclination α . Figure 9 shows the effects of Radiation parameter N on temperature distribution θ . It is shown that the temperature θ increases with increase in radiation parameter N. Figure 10 depicts the effect of Prandtl number Pr.



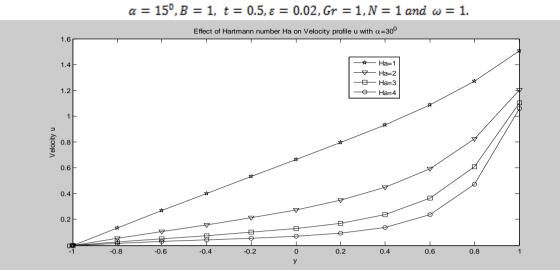
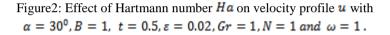


Figure 1: Effect of Hartmann number Ha on velocity profile u with $\alpha = 15^{\circ}$, B = 1, t = 0.5, $\varepsilon = 0.02$, Gr = 1, N = 1 and $\omega = 1$.



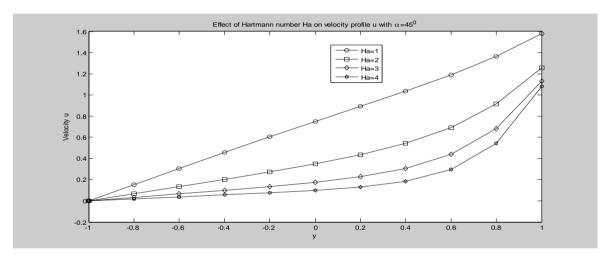
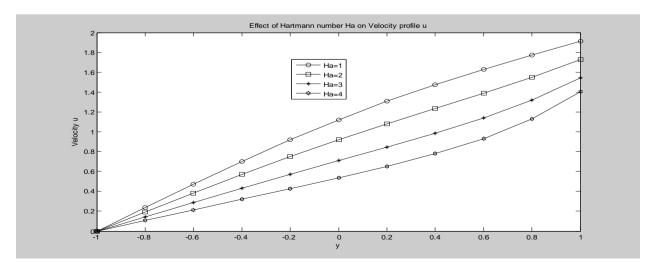


Figure 3: Effect of Hartmann number Ha on velocity profile u with $\alpha = 45^{\circ}, B = 1, t = 0.5, \varepsilon = 0.02, Gr = 1, N = 1 and \omega = 1.$



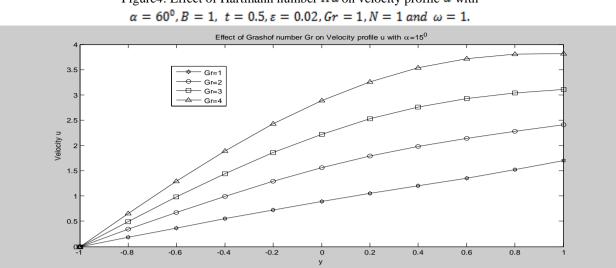


Figure 4: Effect of Hartmann number Ha on velocity profile u with

Figure 5: Effect of Grashof number Gr on velocity profile u with $\alpha = 15^{\circ}, B = 1, t = 0.5, \varepsilon = 0.02, Ha = 1, N = 1 and \omega = 1.$

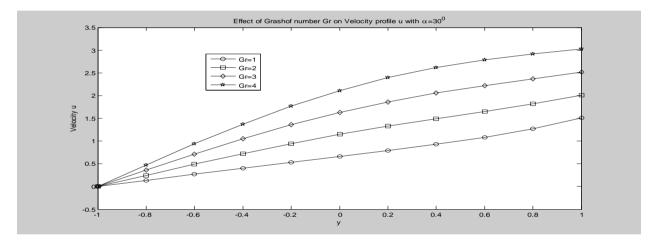
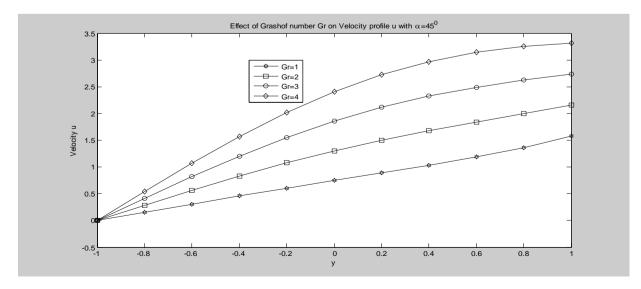


Figure 6: Effect of Grashof number Gr on velocity profile u with $\alpha = 30^{\circ}, B = 1, t = 0.5, \varepsilon = 0.02, Ha = 1, N = 1 and \omega = 1.$



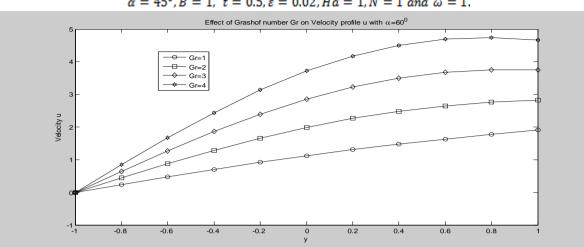


Figure 7: Effect of Grashof number Gr on velocity profile u with $\alpha = 45^{\circ}, B = 1, t = 0.5, \varepsilon = 0.02, Ha = 1, N = 1 and \omega = 1$.

Figure 8: Effect of Grashof number *Gr* on velocity profile *u* with $\alpha = 60^{\circ}, B = 1, t = 0.5, \varepsilon = 0.02, Ha = 1, N = 1 and \omega = 1$.

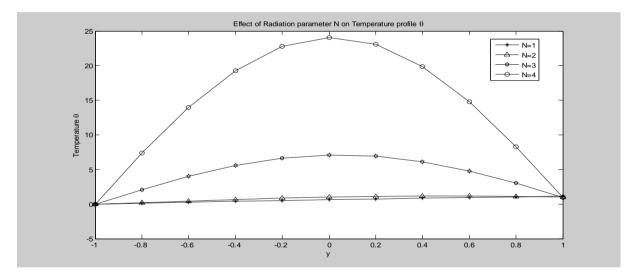


Figure 9: Effect of Radiation parameter N on temperature distribution θ with B = 1, t = 0.5, $\varepsilon = 0.02$, Ha = 1, N = 1, Gr = 1 and $\omega = 1$.

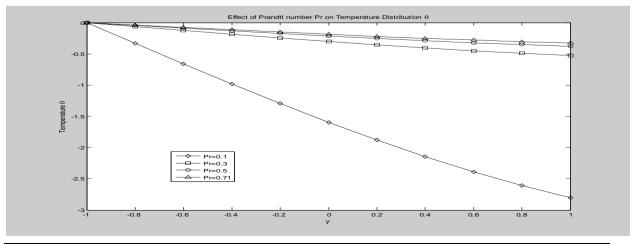


Figure 10: Effect of Prandtl number Pr on temperature distribution θ with B = 1, t = 0.5, $\varepsilon = 0.02$, Ha = 1, N = 1, Gr = 1 and $\omega = 1$.

V. SUMMARY AND CONCLUSION

In this section we studied the effect of inclined Hartmann in an unsteady MHD couette flow between two infinite parallel porous plates in an inclined magnetic field. The momentum and energy equations are written in a dimensionless form using the dimensionless parameters. Perturbation method was employed to solve the velocity profile and temperature distribution. However, at high Hartmann number Ha and high radiation parameter N, the velocity u decreases. When the magnetic field is high, it reduces the energy loss through the plates. But large Nusselt number Nu corresponds to more active convection. Also, when the Prandlt number Princreases, the temperature distribution θ decreases and increase in radiation parameter N increases the temperature distribution θ This work can be applied in electric power generator, extrusion of plastics in the manufacture of Rayon and Nylon etc

Appendix

$$\begin{split} m_{1} &= \frac{B + \sqrt{B^{2} + 4BM^{2}}}{2}, m_{2} = \frac{B - \sqrt{B^{2} + 4BM^{2}}}{2}, m_{3} = \frac{B + \sqrt{B^{2} + 4b}}{2}, m_{4} = \frac{B - \sqrt{B^{2} + 4b}}{2}, C_{1} = C_{2} tanN \\ C_{2} &= \frac{1}{2sinN}, C_{3} = -C_{4}e^{2\sqrt{a_{1}}}, C_{4} = sinn\pi, C_{5} = \frac{L_{2} - C_{6}e^{m_{2}}}{e^{m_{1}}}, C_{6} = \frac{L_{1}e^{m_{1} - L_{2}e^{-m_{2}}}{e^{m_{1} - m_{2} - e^{m_{2}}}, C_{7} = \frac{L_{1}e^{m_{1} - L_{2}e^{-m_{2}}}{e^{m_{3} - m_{4} - e^{m_{4} - m_{3}}} \\ C_{8} &= \frac{L_{4} - C_{8}e^{m_{4}}}{e^{m_{3}}}, K_{1} = \frac{G_{7}C_{1}}{N^{2} + M^{2}}, K_{2} = \frac{G_{7}C_{2}}{N^{2} + M^{2}}, K_{3} = \frac{G_{7}C_{3}}{N^{2} + b}, K_{4} = \frac{G_{7}C_{4}}{N^{2} + b} \\ K_{5} &= \frac{-G_{7}C_{3}}{a - \sqrt{a}B - b}, K_{6} = \frac{-G_{7}C_{4}}{a - \sqrt{a}B - b}, L_{1} = -(K_{1}cosN + K_{2}sinN), L_{2} = 1 - (K_{1}cosN + K_{2}sinN) \\ L_{3} &= -(K_{5}e^{-\sqrt{a}} + K_{6}e^{\sqrt{a}}), L_{3} = -(K_{5}e^{\sqrt{a}} + K_{6}e^{-\sqrt{a}}) \end{split}$$

REFERENCES

- A.K. Al-Hadhrami, L. Elliot, M.D. Ingham, X. Wen, (2003): Flow through horizontal channels of porous materials, Inter. J. of Energy Research 27, 875-889.
- [2] A. Raptis, C. Perkidis, (1999): Radiation and Free Convention flow past a moving plate, Int, J. of Applied Mechanics and Engineering, 4(4): 817-821.
- [3] A.S Idowu and J.O Olabode (2014): Unsteady MHD Poiseuille Flow between Two Infinite Parallel Plates in an Inclined Magnetic Field with Heat Transfer. IOSR Journal of Mathematics 10(3): 47 – 53.
- C.B. Singh, (1993): MHD Steady flow of liquid between two parallel plates, In: Proc. of First Conference of Kenya Mathematical Society, 24-26.
- [5] C.B. Singh, P.c. Ram, (1978): Unsteady MHD Fluid Flow through a channel: Journal of Scientific Research, 28(2).
- [6] D.G. Drake, (1965): Flow in a channel due to a periodic pressure gradient, Quart. Journal of Mech. And Appl. Maths., 18(1).
 [7] Dileep Singh Chauchan and Priyanka Rastogi (2012): Heat transfer effects on rotating MHD couette flow in a channel partially
- field by a porous medium with hall current. Journal of Appl. Sci. Eng. 15 (3): 281 290.
 [8] E.M. Sparrow, R.D. Cess, (1961): The effect of a magnetic field on free convention heat transfer, Int. J. heat Mass Tran., 3(4):
- [8] E.M. Sparlow, R.D. Cess, (1961): The effect of a magnetic field on free convention near transfer, int. J. near Mass Train., $\mathbf{3}(4)$: 267-274.
- [9] H. Alfven, (1942): Existence of electromagnetic-hydrodynamics waves, Nature, **150**(3805), 405-406.
- [10] J.A. Shercliff, (1956): Entry of conducting and non-conducting fluids in pipes, Journal of Mathematical Proc. of the Cambridge Philosophical Soc., 52, 573-583.
- [11] John Mooney, Nick Stokes, (1997): Time-varying MHD flows with free surfaces, Inter. Conf. on CFD in Mineral Metal Processing and Power Generation CSIRO.
- [12] Joseph K.M, Daniel S and Joseph G.M (2014): Unsteady MHD Couette Flow between Two Infinite Parallel Plates in an Inclined Magnetic Field with Heat Transfer, Inter. J. Math. Stat. Inv. 2 (3): 103 110.
- [13] N. Sandeep and V. Sugunamma (2013): Effect of an inclined magnetic field on unsteady free convection flow of a dusty viscous fluid between two infinite flat plates filled by a porous medium. Int. Journal of Appl. Maths. Modelling, **1** (1): 16 33.
- [14] P.A. Davidson, (2001): An introduction to MHD, First Edition, Cambridge University Press, UK.
- [15] P.C. Ram, C.B. Singh, U. Singh, (1984): Hall effects on Heat and Mass Transfer flow through porous medium, Astrophysics Space Science, 100: 45-51.
- [16] S. Ganesh, S. Krishnambal, (2007): Unsteady MHD Stokes flow of viscous fluid between two parallel porous plates, Journal of Applied Sciences, 7, 374-379.
- [17] Stamenkovic M. Zivojin, Dragiza D. Nikodijevic, Bratislav D. Blagojevic, Slobodan R. Savic, (2010): MHD flow and Heat Transfer of two immiscible fluids between moving plates, Transaction of the Canadian Society for Mechanical Engineering, 4(34): 351-372.
- [18] U.S. Rajput, P.K. Sahu, (2011): Transient free convection MHD flow between two long vertical parallel plates with constant Temperature and variable mass diffusion, Int. Journal of Math. Analysis, 34(5): 1665-1671.
- [19] V.M. Soundalgekar, Mohammed Abdulla-Ali, (1986): Free Convention effects on MHD flow past an impulsively started infinite vertical isothermal plate, Reg. J. Energy Heat and Mass Transfer 8(2): 119-125.
- [20] W.A. Manyonge, D.W. Kiema, C.C.W. Iyaya, (2012): Steady MHD Poiseuille Flow between two infinite parallel porous plates in an inclined magnetic field, Inter. Journal of Pure and Applied Maths., 76(5): 661-668.