Two warehouses deteriorating items inventory model under partial backlogging, inflation and permissible delay in payments

Shital S. Patel

Department of Statistics, Veer Narmad South Gujarat University, Surat, INDIA

ABSTRACT: A deteriorating items inventory model with two warehouses under time varying holding cost and linear demand with inflation and permissible delay in payments is developed. Shortages are allowed and partially backlogged. A rented warehouse (RW) is used to store the excess units over the capacity of the own warehouse. Numerical examples are provided to illustrate the model and sensitivity analysis is also carried out for parameters.

KEYWORDS: Deterioration, Two-warehouse, Inventory, partial backlogging, Inflation, Permissible delay in payment

I. INTRODUCTION

In past few decades deteriorating items inventory models were widely studied. An inventory model with constant rate deterioration was developed by Ghare and Schrader [12]. The model was further extended by considering variable rate of deterioration by Covert and Philip [10]. Shah [30] further extended the model by considering shortages. The related work are found in (Nahmias [23], Raffat [25], Goyal and Giri [14], Mandal [21]), Mishra et al. [22]).

The classical inventory model generally deal with single storage facility with the assumption that the available warehouse of the organization has unlimited capacity. But in actual practice, to take advantages of price discount for bulk purchases, a rented warehouse (RW) is used to store the excess units over the capacity of the own warehouse (OW). The cost of storing items at the RW is higher than that at the OW but provides better preserving facility with a lower rate of deterioration.

Hartley [15] was the first one to consider a two warehouse model. A two warehouse deterministic inventory model with infinite rate of replenishment was developed by Sarma [29]. Pakkala and Achary [24] extended the two-warehouse inventory model for deteriorating items with finite rate of replenishment and shortages. Yang [31] considered a two warehouse inventory model for deteriorating items with constant rate of demand under inflation in two alternatives when shortages are completely backordered. Dye et al. [11] considered a two warehouse inventory model with partially backlogging. Jaggi et al. [16] developed an inventory model with linear trend in demand under inflationary conditions and partial backlogging rate in a two warehouse system. Related work is also find in (Benkherouf [3], Bhunia and Maiti [4], Kar et al. [19], Rong et al. [26], Sana et al. [28], Agarwal and Banerjee [1], Bhunia et al. [5]).

Goyal [13] first considered the economic order quantity model under the condition of permissible delay in payments. Aggarwal and Jaggi [2] extended Goyal’s [13] model to consider the deteriorating items. Aggarwal and Jaggi’s [2] model was further extended by Jamal et al. [17] to consider shortages. The related work are found in (Chung and Dye [7], Jamal et al. [18], Salameh et al. [27], Chung et al. [8], Chang et al. [6]). Chung and Huang [9] proposed a two warehouse inventory model for deteriorating items under permissible delay in payments, but they assumed that the deterioration rate of the two warehouse were same. Liao and Huang [20] considered an order level inventory model for deteriorating items in two warehouse and a permissible delay in payments.

In this paper we have developed a two-warehouse inventory model under time varying holding cost and linear demand with inflation and permissible delay in payments. Shortages are allowed and partially backlogged. Numerical examples are provided to illustrate the model and sensitivity analysis of the optimal solutions for major parameters is also carried out.
II. ASSUMPTIONS AND NOTATIONS

NOTATIONS:
The following notations are used for the development of the model:

- \( D(t) \) : Demand rate is a linear function of time \( t \) \((a+bt, a>0, 0<b<1)\)
- \( A \) : Replenishment cost per order for two warehouse system
- \( c \) : Purchasing cost per unit
- \( p \) : Selling price per unit
- \( c_2 \) : Shortage cost per unit
- \( c_3 \) : Cost of lost sales per unit
- \( HC(OW) \): Holding cost per unit time is a linear function of time \( t \) \((x_1+y_1t, x_1>0, 0<y_1<1)\) in OW
- \( HC(RW) \): Holding cost per unit time is a linear function of time \( t \) \((x_2+y_2t, x_2>0, 0<y_2<1)\) in RW
- \( I_e \) : Interest earned per year
- \( I_p \) : Interest charged per year
- \( M \) : Permissible period of delay in settling the accounts with the supplier
- \( T \) : Length of inventory cycle
- \( I(t) \) : Inventory level at any instant of time \( t \), \( 0 \leq t \leq T \)
- \( W \) : Capacity of owned warehouse
- \( I_d(t) \) : Inventory level in OW at time \( t \)
- \( I_r(t) \) : Inventory level in RW at time \( t \)
- \( Q_1 \) : Inventory level initially
- \( Q_2 \) : Shortage of inventory
- \( Q \) : Order quantity
- \( R \) : Inflation rate
- \( t_r \) : Time at which the inventory level reaches zero in RW in two warehouse system
- \( \theta_1t \) : Deterioration rate in OW, \( 0< \theta_1<1 \)
- \( \theta_2t \) : Deterioration rate in RW, \( 0< \theta_2<1 \)
- \( TC_i \) : Total relevant cost per unit time \( (i=1,2,3) \)

ASSUMPTIONS:
The following assumptions are considered for the development of two warehouse model.

- The demand of the product is declining as a linear function of time.
- Replenishment rate is infinite and instantaneous.
- Lead time is zero.
- Shortages are allowed and partially backlogged.
- OW has a fixed capacity \( W \) units and the RW has unlimited capacity.
- The goods of OW are consumed only after consuming the goods kept in RW.
- The unit inventory costs per unit in the RW are higher than those in the OW.
- During the time, the account is not settled; generated sales revenue is deposited in an interest bearing account. At the end of the credit period, the account is settled as well as the buyer pays off all units sold and starts paying for the interest charges on the items in stocks.

III. THE MATHEMATICAL MODEL AND ANALYSIS

At time \( t=0 \), a lot size of certain units enter the system. \( W \) units are kept in OW and the rest is stored in RW. The items of OW are consumed only after consuming the goods kept in RW. In the interval \([0,t_r]\), the inventory in RW gradually decreases due to demand and deterioration and it reaches to zero at \( t=t_r \). In OW, however, the inventory \( W \) decreases during the interval \([0,t]\) due to deterioration only, but during \([t, t_0]\), the inventory is depleted due to both demand and deterioration. By the time to \( t_0 \), both warehouses are empty. Shortages occur during \((t_0,T)\) of size \( Q_2 \) units. The figure describes the behaviour of inventory system.
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Hence, the inventory level at time $t$ at RW and OW are governed by the following differential equations:

\[
\frac{dI_r(t)}{dt} + \theta_r t I_r(t) = -(a+bt), \quad 0 \leq t \leq t_r
\]

with boundary conditions $I_r(t_r) = 0$ and

\[
\frac{dI_o(t)}{dt} + \theta_o t I_o(t) = 0, \quad 0 \leq t \leq t_r
\]

with initial condition $I_0(0) = W$, respectively.

While during the interval $(t_r, t_0)$, the inventory in OW reduces to zero due to the combined effect of demand and deterioration both. So the inventory level at time $t$ at OW, $I_o(t)$, is governed by the following differential equation:

\[
\frac{dI_o(t)}{dt} + \theta_o t I_o(t) = -(a+bt), \quad t_r \leq t \leq t_0
\]

with the boundary condition $I_o(t_0) = 0$.

Similarly during $(t_0, T)$ the shortage level at time $t$, $I_s(t)$ is governed by the following differential equation:

\[
\frac{dI_s(t)}{dt} = -e^{\alpha T-t}(a+bt), \quad t_0 \leq t \leq T,
\]

with the boundary condition $I_s(t_0) = 0$.

The solutions to equations (1) to (4) are given by:

\[
I_r(t) = \left[ a(t_r - t) + \frac{1}{2} b(t_r^2 - t^2) + \frac{1}{6} a\theta_2 (t_r^3 - t^3) + \frac{1}{2} b\theta_2 (t_r^2 - t^2) - \frac{1}{2} a\theta_1 t^2 (t_r - t) - \frac{1}{4} b\theta_1 t^2 (t_r^2 - t^2) \right], \quad 0 \leq t \leq t_r
\]

\[
I_o(t) = W \left( 1 - \theta_1 t^2 \right), \quad 0 \leq t \leq t_r
\]

\[
I_o(t) = \left[ a(t_0 - t) + \frac{1}{2} b(t_0^2 - t^2) + \frac{1}{6} a\theta_1 (t_0^3 - t^3) + \frac{1}{2} b\theta_1 (t_0^2 - t^2) - \frac{1}{2} a\theta_0 t^2 (t_0 - t) - \frac{1}{4} b\theta_0 t^2 (t_0^2 - t^2) \right], \quad t_r \leq t \leq t_0
\]

\[
I_s(t) = \left[ -\frac{1}{3} b\delta t^3 - \frac{1}{2} (a\delta + b(1-\delta T)) t^2 - at(1-\delta T) + \frac{1}{3} b\delta t_0^3 + \frac{1}{2} a\delta t_0^2 + \frac{1}{2} bt_0^2 - \frac{1}{2} b\delta T t_0^2 - a\delta t_0 T + at_0 \right], \quad t_0 \leq t \leq T
\]

(by neglecting higher powers of $\theta_1$, $\theta_2$)

Using the condition $I_s(t) = Q_1 - W$ at $t=0$ in equation (5), we have
\[
Q_i = W + \left[ a(t_i - t) + \frac{1}{2} b(t_i^2 - t^2) + \frac{1}{6} a^2 t_i^3 + \frac{1}{8} b^2 t_i^4 \right],
\]

\[
Q_i = W + \left[ a(t_i - t) + \frac{1}{2} b(t_i^2 - t^2) + \frac{1}{6} a^2 t_i^3 + \frac{1}{8} b^2 t_i^4 \right].
\]

(9)

Using the condition \( I(t) = Q - Q_i \) at \( t = T \) in equation (8), we have

\[
Q - Q_i = -a(T - t) + b(T^2 - t^2).
\]

(10)

Using the continuity of \( I(t) \) at \( t = T \) in equations (6) and (7), we have

\[
I_0(t_i) = W(1 - 0 t^2) = a(t_i - t) + \frac{1}{2} b(t_i^2 - t^2) + \frac{1}{6} a^2 t_i^3 + \frac{1}{8} b^2 t_i^4
\]

\[+ \frac{1}{8} b^2 t_i^4 (t_i - t) - \frac{1}{2} a^2 t_i^3 (t_i - t) - \frac{1}{4} b^2 t_i^4 (t_i - t^2)\]

which implies that

\[
t_i = \frac{-a + \sqrt{a^2 + 2bW - bW t_i^2 + b^2 t_i^2 + 2abt_i}}{b}.
\]

(12)

(by neglecting higher powers of \( t_i \))

From equation (12), we note that \( t_i \) is a function of \( t_i \), therefore \( t_i \) is not a decision variable.

Based on the assumptions and descriptions of the model, the total annual relevant costs \( TC \), include the following elements:

(i) Ordering cost (OC) = \( A \)

(ii) \( HC(RW) = \int_0^t (x_i + y_i t) I_0(t) e^{w t} dt = \int_0^t (x_i + y_i t) I_0(t) e^{w t} dt \)

\[
= a(t_i - t) + \frac{1}{2} b(t_i^2 - t^2) + \frac{1}{6} a^2 t_i^3 + \frac{1}{8} b^2 t_i^4
\]

\[+ \frac{1}{8} b^2 t_i^4 (t_i - t) - \frac{1}{2} a^2 t_i^3 (t_i - t) - \frac{1}{4} b^2 t_i^4 (t_i - t^2)\]

(13)

(by neglecting higher powers of \( R \))

(iii) \( HC(OW) = \int_0^t (x_i + y_i t) I_0(t) e^{w t} dt \)

\[
= \int_0^t (x_i + y_i t) W(1 - 0 t^2) e^{w t} dt + \int_0^t (x_i + y_i t) I_0(t) e^{w t} dt
\]

(14)
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\[
W\left(\frac{1}{10}y_{r}Rb_{0}t_{i}^{5} - \frac{1}{8}(y_{r} - x)R\theta_{i}t_{i}^{4} + \frac{1}{3}\left(-\frac{1}{2}x_{i}\theta_{i} - y_{r}R\right) t_{i}^{3} + \frac{1}{2}(y_{r} - x)R\theta_{i}t_{i}^{2} + x_{i}t_{i}\right)
\]

\[
\left[\begin{array}{c}
- \frac{1}{56}y_{r}Rb_{0}t_{i}^{5} + \frac{1}{6}\left(\frac{1}{8}(y_{r} - x)R\theta_{i}b_{0} - \frac{1}{3}y_{r}R\theta_{i}\right) t_{i}^{4} \\
+ \frac{1}{5}\left(\frac{1}{8}x_{i}b_{0} + \frac{1}{3}(y_{r} - x)R\theta_{i}a_{i} - y_{r}R\theta_{i}\left(-\frac{1}{2}t_{i} + \frac{1}{2}bt_{i}^{5} + at_{i}\right) - \frac{1}{2}b\right) t_{i}^{4}
\end{array}\right] + \frac{1}{4}\left(\begin{array}{c}
\frac{1}{2}x_{i}a_{i} + (y_{r} - x)R\theta_{i}\left(-\frac{1}{2}t_{i} + \frac{1}{2}bt_{i}^{5} + at_{i}\right) + y_{r}R\theta_{i}t_{i}^{4} \\
+ \frac{1}{3}\left(\begin{array}{c}
x_{i}\left(-\frac{1}{2}t_{i} + \frac{1}{2}bt_{i}^{5} + at_{i}\right) - \frac{1}{2}b\right)
\end{array}\right) (y_{r} - x)R \theta_{i} + y_{r}R \theta_{i}\left(\begin{array}{c}
\frac{1}{6}a_{i}t_{i}^{4} + \frac{1}{2}bt_{i}^{5} + at_{i}\end{array}\right)
\end{array}\right]
\]

\[
\left[\begin{array}{c}
+ \frac{1}{7}\left(\begin{array}{c}
x_{i}\left(-\frac{1}{2}t_{i} + \frac{1}{2}bt_{i}^{5} + at_{i}\right) - \frac{1}{2}b\right) (y_{r} - x)R \theta_{i} + y_{r}R \theta_{i}\left(\begin{array}{c}
\frac{1}{6}a_{i}t_{i}^{4} + \frac{1}{2}bt_{i}^{5} + at_{i}\end{array}\right)
\end{array}\right]
\]

(iv) Deterioration cost:
The amount of deterioration in both RW and OW during \([0,t_{0}]\) are:

\[
\int_{0}^{t_{0}} t_{i} dt + \int_{0}^{t_{0}} t_{i} dt
\]

So deterioration cost

\[
DC = c\left[\int_{0}^{t_{0}} t_{i} dt + \int_{0}^{t_{0}} t_{i} dt\right]
\]

\[
= c\left[\int_{0}^{t_{0}} t_{i} e^{\theta_{i}t} dt + \int_{0}^{t_{0}} t_{i} e^{\theta_{i}t} dt\right]
\]

\[
= c\left[\int_{0}^{t_{0}} t_{i} e^{\theta_{i}t} dt + \int_{0}^{t_{0}} e^{\theta_{i}t} dt\right] + \int_{0}^{t_{0}} t_{i} e^{\theta_{i}t} dt
\]

\[
\left[\begin{array}{c}
\frac{1}{56}R\theta_{i}b_{0} + \frac{1}{6}R \theta_{i} b_{0} - \frac{1}{3}R \theta_{i} a_{i} + \frac{1}{2}R \theta_{i} b_{0} + \frac{1}{2}R \theta_{i} b_{0} + \frac{1}{2}R \theta_{i} b_{0} + \frac{1}{2}R \theta_{i} b_{0} + \frac{1}{2}R \theta_{i} b_{0}
\end{array}\right] t_{i}^{4}
\]

\[
= c\theta_{i} + \frac{1}{4}\left(\begin{array}{c}
\frac{1}{2}R \theta_{i} b_{0} + \frac{1}{2}R \theta_{i} b_{0} + \frac{1}{2}R \theta_{i} b_{0} + \frac{1}{2}R \theta_{i} b_{0} + \frac{1}{2}R \theta_{i} b_{0} + \frac{1}{2}R \theta_{i} b_{0} + \frac{1}{2}R \theta_{i} b_{0} + \frac{1}{2}R \theta_{i} b_{0}
\end{array}\right) t_{i}^{4}
\]

\[
+ \frac{1}{2}\left(\begin{array}{c}
\frac{1}{6}R \theta_{i} b_{0} + \frac{1}{2}R \theta_{i} b_{0} + \frac{1}{2}R \theta_{i} b_{0} + \frac{1}{2}R \theta_{i} b_{0} + \frac{1}{2}R \theta_{i} b_{0} + \frac{1}{2}R \theta_{i} b_{0} + \frac{1}{2}R \theta_{i} b_{0} + \frac{1}{2}R \theta_{i} b_{0}
\end{array}\right) t_{i}^{4}
\]

\[
+ \frac{1}{2}\left(\begin{array}{c}
\frac{1}{6}R \theta_{i} b_{0} + \frac{1}{2}R \theta_{i} b_{0} + \frac{1}{2}R \theta_{i} b_{0} + \frac{1}{2}R \theta_{i} b_{0} + \frac{1}{2}R \theta_{i} b_{0} + \frac{1}{2}R \theta_{i} b_{0} + \frac{1}{2}R \theta_{i} b_{0} + \frac{1}{2}R \theta_{i} b_{0}
\end{array}\right) t_{i}^{4}
\]
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\[
\begin{align*}
\theta_t &+ \frac{1}{6} (\theta_t - \frac{1}{2} \theta_t + a \theta_t - R \theta_t - \frac{1}{2} \theta_t - b \theta_t + at) t^t + \frac{1}{5} \left( \frac{1}{3} a_0 - R \left( \frac{1}{2} \theta_t + \frac{1}{2} a \theta_t - b \theta_t + at \right) \right) t^t \\
- \epsilon_0 &+ \frac{1}{4} \left( \frac{1}{2} \theta_t + a \theta_t + b \theta_t + R \theta_t + \frac{1}{2} \theta_t + b \theta_t - b \theta_t - a \theta_t \right) \right) t^t + \frac{1}{3} \left( a - R \left( \frac{1}{2} \theta_t + \frac{1}{2} a \theta_t + b \theta_t + at \right) \right) t^t \\
&+ \frac{1}{2} \left( \frac{1}{8} a_0, t_0^t + a \theta_t + b \theta_t + at \right) t^t
\end{align*}
\]  

(16)

(v) Shortage cost:

\[
SC = - c_2 \int_0^T (a + b t) e^{R t} dt = - c_2 \int_0^T \left[ - \frac{1}{2} b \theta_t^t + \frac{1}{3} (a \theta_t + b (1-\delta T)) t^t - R (1-\delta T) \right] e^{R t} dt
\]

(17)

(vi) Cost due to lost sales:

\[
LS = c_3 \int_0^T (a + b t) (1 - (1 - \delta (T - t))) e^{R t} dt = c_3 \int_0^T \left[ a T (T - t) + \frac{1}{2} (b T - a \delta T) (T^t - t^t) \right] e^{R t} dt
\]

(18)

(vii) Interest Earned: There are two cases:

**Case I : (M ≤ t ≤ T):**

In this case interest earned is:

\[
IE_1 = pI_0 \int_0^M (a + b t) e^{R t} dt = pI_0 \left[ \frac{1}{4} b R M^3 + \frac{1}{3} (b R + a) M^3 + \frac{1}{2} a M^3 \right]
\]

(19)

**Case II : (t ≤ M ≤ T):**

In this case interest earned is:

\[
IE_2 = pI_0 \int_0^t (a + b t) e^{R t} dt + (a + b t_0) (M - t_0)
\]

\[
= pI_0 \left[ \frac{1}{2} b R t_0^t + \frac{1}{3} (b R + a) t_0^t + \frac{1}{2} a t_0^t + (a + b t_0) (M - t_0) \right]
\]

(20)

(viii) Interest Payable: There are three cases described as in figure:
Case I: \((M \leq t_r \leq T)\):
In this case, annual interest payable is:

\[
IP_1 = c_{l_p} \left\{ \int_0^{T} I_1(t)e^{-\delta t}dt + \frac{1}{R} \int_0^{t_r} I_1(t)e^{-\delta t}dt + \int_0^{t_r} I_0(t)e^{-\delta t}dt \right\}
\]

\[
= c_{l_p} \left[ -\frac{1}{48} R_0 b t_i^6 + \frac{1}{5} \left( \frac{1}{8} b_0 - \frac{1}{3} R_0 \right) a t_i^5 + \frac{1}{4} \left( \frac{1}{3} a - R \left( \frac{1}{2} a_0 + \frac{1}{2} b \right) \right) t_i^4 \right. \\
+ \left. \frac{1}{8} \left( \frac{1}{2} a_0 + \frac{1}{2} b \right) t_i^3 + \frac{1}{2} \left( a - R \left( \frac{1}{8} b_0 t_i^4 + \frac{1}{6} a_0 t_i^4 + \frac{1}{2} b t_i^2 + a t_i \right) \right) t_i^2 \right]
\]

\[
= c_{l_p} \left[ \frac{1}{8} R_0 b t_i^6 + \frac{1}{5} \left( \frac{1}{8} b_0 - \frac{1}{3} R_0 \right) a t_i^5 + \frac{1}{4} \left( \frac{1}{3} a - R \left( \frac{1}{2} a_0 + \frac{1}{2} b \right) \right) t_i^4 \right. \\
+ \left. \frac{1}{8} \left( \frac{1}{2} a_0 + \frac{1}{2} b \right) t_i^3 + \frac{1}{2} \left( a - R \left( \frac{1}{8} b_0 t_i^4 + \frac{1}{6} a_0 t_i^4 + \frac{1}{2} b t_i^2 + a t_i \right) \right) t_i^2 \right]
\]

\[
+ c_{l_p} a t_i + \frac{1}{8} R_0 a t_i^4 - \frac{1}{6} a_0 t_i^4 - \frac{1}{2} R t_i^2 \right\} c_{l_p} \left[ M + \frac{1}{8} R_0 M^4 - \frac{1}{6} R M^4 - \frac{1}{2} R M^2 \right]
\]

\[
= c_{l_p} \left[ \frac{1}{8} R_0 b t_i^6 + \frac{1}{5} \left( \frac{1}{8} b_0 - \frac{1}{3} R_0 \right) a t_i^5 + \frac{1}{4} \left( \frac{1}{3} a - R \left( \frac{1}{2} a_0 + \frac{1}{2} b \right) \right) t_i^4 \right. \\
+ \left. \frac{1}{8} \left( \frac{1}{2} a_0 + \frac{1}{2} b \right) t_i^3 + \frac{1}{2} \left( a - R \left( \frac{1}{8} b_0 t_i^4 + \frac{1}{6} a_0 t_i^4 + \frac{1}{2} b t_i^2 + a t_i \right) \right) t_i^2 \right]
\]

\[
- c_{l_p} \left[ \frac{1}{8} R_0 b t_i^6 + \frac{1}{5} \left( \frac{1}{8} b_0 - \frac{1}{3} R_0 \right) a t_i^5 + \frac{1}{4} \left( \frac{1}{3} a - R \left( \frac{1}{2} a_0 + \frac{1}{2} b \right) \right) t_i^4 \right. \\
+ \left. \frac{1}{8} \left( \frac{1}{2} a_0 + \frac{1}{2} b \right) t_i^3 + \frac{1}{2} \left( a - R \left( \frac{1}{8} b_0 t_i^4 + \frac{1}{6} a_0 t_i^4 + \frac{1}{2} b t_i^2 + a t_i \right) \right) t_i^2 \right]
\]

\[
+ c_{l_p} a t_i + \frac{1}{8} R_0 a t_i^4 - \frac{1}{6} a_0 t_i^4 - \frac{1}{2} R t_i^2 \right\] 

(21)

Case II: \((t_r \leq M \leq T)\):
In this case interest payable is:

\[
IP_2 = c_{l_p} \int_0^{T} I_1(t)e^{-\delta t}dt
\]

\[
= c_{l_p} \left[ \frac{1}{8} R_0 b t_i^6 + \frac{1}{5} \left( \frac{1}{8} b_0 - \frac{1}{3} R_0 \right) a t_i^5 + \frac{1}{4} \left( \frac{1}{3} a - R \left( \frac{1}{2} a_0 + \frac{1}{2} b \right) \right) t_i^4 \right. \\
+ \left. \frac{1}{8} \left( \frac{1}{2} a_0 + \frac{1}{2} b \right) t_i^3 + \frac{1}{2} \left( a - R \left( \frac{1}{8} b_0 t_i^4 + \frac{1}{6} a_0 t_i^4 + \frac{1}{2} b t_i^2 + a t_i \right) \right) t_i^2 \right]
\]
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\[
\begin{align*}
\frac{1}{48} R_0 b M_t' + \frac{1}{5} \left( \frac{1}{8} R_0 a - \frac{1}{3} R_0 a \right) M_t' + \frac{1}{4} \left( \frac{1}{3} a - R_0 \right) \left( \frac{1}{2} b + a M_t' \right) - \frac{1}{2} b \right) M_t' \\
- c_t = \frac{1}{3} \left( - \frac{1}{2} \left( \frac{1}{2} b + a M_t' \right) + \frac{1}{6} b + R_0 \right) M_t' + \frac{1}{2} \left( - a - R_0 \right) \left( \frac{1}{8} b_0 t_i' + \frac{1}{6} a_0 t_i' + \frac{1}{2} b t_i' \right) M_t' \\
+ \frac{1}{8} \left( b_0 t_i' + a_0 t_i' \right) M_t' + \frac{1}{2} b t_i' M_t + a_0 t_i' M_t
\end{align*}
\]

(22)

Case III: \((t_0 \leq M \leq T)\):

In this case, no interest charges are paid for the item. So, \(I_{P_1} = 0\).

The retailer’s total cost during a cycle, \(T(t_i, T), i=1,2,3\) consisted of the following:

\[
T_{C_i} = \frac{1}{T} \left[ A + HC(OW) + HC(RW) + DC + SC + LS + I_{P_1} - IE_i \right]
\]

(24)

and \(t_i\) is approximately related to \(t_i\) through equation (12).

Substituting values from equations (13) to (18) and equations (19) to (23) in equation (24), total costs for the three cases will be as under:

\[
T_{C_1} = \frac{1}{T} \left[ A + HC(OW) + HC(RW) + DC + SC + LS + I_{P_1} - IE_1 \right]
\]

(25)

\[
T_{C_2} = \frac{1}{T} \left[ A + HC(OW) + HC(RW) + DC + SC + LS + I_{P_1} - IE_2 \right]
\]

(26)

\[
T_{C_3} = \frac{1}{T} \left[ A + HC(OW) + HC(RW) + DC + SC + LS + I_{P_1} - IE_3 \right]
\]

(27)

The optimal value of \(t_r = t_r^*\), \(T = T^*\) (say), which minimizes \(T_{C_i}\) can be obtained by solving equation (25), (26) and (27) by differentiating it with respect to \(t_i\) and \(T\) and equate it to zero i.e.

\[
\frac{\partial T_{C_i}(t_i, T)}{\partial t_i} = 0, \quad \frac{\partial T_{C_i}(t_i, T)}{\partial T} = 0, \quad i=1,2,3,
\]

(28)

provided it satisfies the condition

\[
\frac{\partial^2 T_{C_i}(t_i, T)}{\partial t_i^2} > 0, \quad \frac{\partial^2 T_{C_i}(t_i, T)}{\partial T^2} > 0, \quad \frac{\partial^2 T_{C_i}(t_i, T)}{\partial t_i \partial T} > 0, \quad i=1,2,3.
\]

(29)

IV. NUMERICAL EXAMPLES

Case I: Considering \(A= Rs.150, W = 100, a = 200, b=0.05, c=Rs. 10, p= Rs. 15, 0_1=0.1, 0_2 =0.06, x_i = Rs. 1, y_1=0.05, x_2= Rs. 3, y_2=0.06, I_p= Rs. 0.15, I_e= Rs. 0.12, R = 0.06, c_2 = Rs. 8, c_3 = Rs. 2, 0 = 0.8, M=0.01 year, in appropriate units. The optimal value of \(t_i^* =0.0831, T^*=0.8081\) and \(T^*_1 = Rs. 349.8128\).

Case II: Considering \(A= Rs.150, W = 100, a = 200, b=0.05, c= Rs. 10, p= Rs. 15, 0_1=0.1, 0_2 =0.06, x_i = Rs. 1, y_1=0.05, x_2= Rs. 3, y_2=0.06, I_p= Rs. 0.15, I_e= Rs. 0.12, c_2 = Rs. 8, c_3 = Rs. 2, 0 = 0.8, M=0.55 year, in appropriate units. The optimal value of \(t_i^* =0.0867, T^*=0.7151\) and \(T^*_2 = Rs. 215.9081\).

Case III: Considering \(A= Rs.150, W = 100, a = 200, b=0.05, c= Rs. 10, p= Rs. 15, 0_1=0.1, 0_2 =0.06, x_i = Rs. 1, y_1=0.05, x_2= Rs. 3, y_2=0.06, I_p= Rs. 0.15, I_e= Rs. 0.12, c_2 = Rs. 8, c_3 = Rs. 2, 0 = 0.8, M = 0.65 year, in appropriate units. The optimal value of \(t_i^* =0.1014, T^*=0.7101\) and \(T^*_3 = Rs. 185.5066\).

The second order conditions given in equation (29) are also satisfied. The graphical representation of the convexity of the cost functions for the three cases are also given.

<table>
<thead>
<tr>
<th>Case</th>
<th>(t_i) and cost</th>
<th>(T) and cost</th>
<th>(t_i, T) and cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case II</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case III</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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V. SENSITIVITY ANALYSIS

On the basis of the data given in example above we have studied the sensitivity analysis by changing the following parameters one at a time and keeping the rest fixed.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case I ((M \leq t_r \leq T))</th>
<th>Case II ((t_r \leq M \leq T))</th>
<th>Case III ((t_0 \leq M \leq T))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+10%</td>
<td>0.0951 0.7620</td>
<td>367.8316</td>
<td>0.1068 0.6783</td>
</tr>
<tr>
<td>+5%</td>
<td>0.0895 0.7841</td>
<td>358.9034</td>
<td>0.0973 0.6959</td>
</tr>
<tr>
<td>-5%</td>
<td>0.0758 0.8344</td>
<td>340.5512</td>
<td>0.0749 0.7362</td>
</tr>
<tr>
<td>-10%</td>
<td>0.0674 0.8633</td>
<td>331.1092</td>
<td>0.0616 0.7596</td>
</tr>
<tr>
<td>(x_1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+10%</td>
<td>0.0774 0.8055</td>
<td>353.8748</td>
<td>0.0830 0.7144</td>
</tr>
<tr>
<td>+5%</td>
<td>0.0802 0.8068</td>
<td>351.8509</td>
<td>0.0848 0.7148</td>
</tr>
<tr>
<td>-5%</td>
<td>0.0860 0.8094</td>
<td>347.7605</td>
<td>0.1085 0.7154</td>
</tr>
<tr>
<td>-10%</td>
<td>0.0888 0.8107</td>
<td>345.6942</td>
<td>0.0904 0.7157</td>
</tr>
<tr>
<td>(x_2)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+10%</td>
<td>0.0786 0.8037</td>
<td>350.0558</td>
<td>0.0833 0.7119</td>
</tr>
</tbody>
</table>
From the table we observe that as parameter a increases/ decreases average total cost increases/ decreases in case I and case II and there is very slight increase/ decrease in case III with respective increase/ decrease in parameter a.

From the table we observe that with increase/ decrease in parameters x₁ and θ₁, there is corresponding increase/ decrease in total cost for case I, case II and case III respectively.

Also, we observe that with increase and decrease in the value of x₂, θ₂, δ and R, there is corresponding increase/ decrease in total cost for case I, case II and case III. From the table we observe that with increase/ decrease in parameter, there is corresponding increase/ decrease in total cost for case I, case II and case III respectively. Moreover, we observe that with increase/ decrease in the value of A, there is corresponding increase/ decrease in total cost in cases I, II and III.

VI. CONCLUSION

We have developed a two warehouse inventory model for deteriorating items with linear demand and partial backlogging under inflationary conditions and permissible delay in payments in this model. It is assumed that rented warehouse holding cost is greater than own warehouse holding cost but provides a better storage facility and the deterioration rate is low in rented warehouse. Sensitivity with respect to parameters have been carried out. The results show that there is increase/ decrease in cost when there is increase/ decrease in the parameter values.

REFERENCES


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