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Analysis of the discrete time queue length distribution with a bulk Service rule $(\mathbf{L,K})$

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ABSTRACT: *In this paper we consider analysis of the discrete time queue length distribution with a bulk service rule (L,K). The analysis is carried out under the assumption that service times of batches are independent of the number of packets in any batch and there is no simultaneous arrival of packets and departure of batch in a single slot. The packets arrive one by one and their inter arrival times follow geometric distribution. The arriving packets are queued in FIFO order. One server transports packets in batches of minimum number L and maximum K the service times following negative binomial (α, p_1) distribution. The server accesses new arrivals even after service has started on any batch of initial number 'j' $(L \leq j < K)$. This operation continues till random service time of the ongoing batch is completed or the maximum capacity of the batch being served attains "K' whichever occurs first. The distribution of the system occupancy just before and after the departure epochs is obtained using discrete-time analysis. The primary focus is on various performance measures of the steady state distribution of the batch server at departure instants and also on numerical illustrations.*

KEYWORDS*: Discrete-time queue, Slot, Packets, Batch service, Departure epochs, System occupancy and Inter departure time.*

I. INTRODUCTION AND MOTIVATION

This paper considers analysis of the discrete time queue length distribution with a bulk service rule (L,K) and the distribution of system occupancy immediately before and after the departure epoch is obtained using discrete-time analysis . Discrete-time queueing models are suitable for the performance evaluation of Asynchronous Transfer Mode (ATM) switches. In ATM, different types of traffic need different quality of service standards. The delay characteristics of delay-sensitive traffic (e.g., voice) are more stringent than those of delay-insensitive traffic . This discrete time analysis of a queue of packets of information in binary encoded (digital) form has been made in the study of ATM (Asynchronous Transfer Mode) networks will help cut cost the transportation of packets arriving from a manufacturing industry and await transportation in a transit location/warehouse. In these data packets have constant volume/size and their transportation can be modeled even when time is divided into units (slots) of discrete length. For an application of discrete time analysis the reader is referred to Tran-Gia [12] and Hameed and Yasser [2].

Time is assumed to be divided into equal intervals called slots. Packets arrive one by one their inter arrival time following geometric distribution where in the probability of a packet arriving for dispatch in a slot is 'p' and it's not arriving is 'q'. If an arriving packet finds the server busy it joins the queue in first in and first out (FIFO) order. A server transports packets in batches of the minimum number L and maximum K the service time following common Negative Binomial NB(α, p_1) distribution where in the probability of a batch getting success in a slot is 'p_1'and not so is 'q_1'.

The server accesses new arrivals into each ongoing service batch of initial size 'j' $(L \leq j < K)$ even after service has started on that batch. This is operated till the service time of the ongoing service batch is completed or when the batch being served attains the maximum capacity 'K' whichever occurs first. A batch that admits late arrivals as stated above is called the accessible batch . The analysis of accessible and non-accessible batch service queues has been carried out by Sivasamy[9], Sivasamy and Elangovan [10]. For more on continuous time queues with accessible batches studies are available in Kleinrock [4]. Mathias [7] has derived the inter departure time distribution for batches admitting non-accessible batches using discrete-time analysis and correlation between inter departure times and batch sizes. Mathias and Alexander [6] have shown how discrete-time analysis can be adapted to numerical analysis of a queue of packets in a batch server in which additional packets cannot be added to the service facility, though there might be room for them even after a service has been started. Neuts[8] proposed the "general bulk service rule " in which service being only when a certain number of customers in the queue under are available. Sivasamy and Pukazhenthi [11] has analyzed the discrete time bulk service queue with accessible batch with the arrivals and service times as generally and negative binomially distribution. Baburaj and Rekha [1] presented an (a;b) policy bulk queue service times are assumed to be geometrically distributed.

The analysis of the present study is continued under the assumption that service times of batches are independent of the number of packets transported in any batch ruling out the simultaneous occurrence of a packet arriving and a batch departing in a single slot. The primary focus is on deriving various performance measures of the steady state distribution of the batch server at departure instants and on numerical illustrations. The rest of the paper is organized as follows: In section 2, presents the description of the model, i.e. arrival distribution and service distribution. Section 3, deals with the steady state distribution of system occupancy at departure epochs, the joined distribution of inter-departure time and number of pockets in a batch. In section 4 presents numerical study for various performance measures of the steady state distribution of the batch server at departure instants. A brief conclusion is presented in section 5.

II. MODEL DESCRIPTION

2.1. ARRIVAL DISTRIBUTION

The queuing model under consideration as pointed out in the preceding section consists of a queue of packets with infinite capacity and based on the bulk service rule (L, K). The inter arrival times are independent and identically distributed (i.i.d) random variables and have common distribution $\{ a(k) = Pr\,(A_n = k): k = 0,1,2 \ldots.\}$, i.e.

$$a(k) = p\,(1-p)^{k-1} \quad : k = 0,1,2 \ldots. \qquad \ldots(1)$$

Hence, the mean inter arrival times $(1/p)$ and variance inter times are (q/p_2)

$$i.e\ E(A) = \frac{1}{p}, E(A^2) = (2-p)/p^2 \ \ and \ \ var(A) = q/p^2 \qquad \ldots(2)$$

2.2. SERVICE DISTRIBUTION

The service times of batches are also independent and identically distributed (i.i.d) random variables and have common Pascal Negative-binomial distribution $\{b(m; \alpha, p_1) = Pr\,(B_n = m) : m = \alpha,\ \alpha+1, \alpha+2 \ldots.\}$ so that

$$b(m, \alpha, p_1) = \begin{pmatrix} m-1 \\ \alpha-1 \end{pmatrix} p_1\ \alpha\ (1-p_1)m-\alpha \ : m = \alpha, \alpha+1, \alpha+2 \ldots. \qquad \ldots(3)$$

and B_n is the number of slots required to complete a batch service at the α^{th} success in a sequence of independent Bernoulli trails with probability p_1 for success and $q_1 = 1 - p_1$ is the probability of failure. The mean and $var\ (B)$ of (3) are as follows

Mean service times:$(\alpha\ /p_1)$ and variance, of services times: $(\alpha\ q_1/p_1)$ … (4)

Thus load 'ρ' of the server is

$$\rho = \frac{\mathrm{E(B)}}{\mathrm{K\,E(A)}} = \frac{\alpha\,\mathrm{p}}{\mathrm{K\,p_1}} \qquad \ldots(5)$$

To ensure that the discrete-time queueing system is stable, assumptions needed for subsequent analyses have been summarized. Each slot is exactly equal to the transmission time of a batch of size K. The time interval $(k, k+1)$ will be referred to as slot $k+1$; $k = 0, 1, 2 \ldots$ Arriving packets are of fixed number and queued in a buffer of infinite capacity until they enter the server. A packet cannot enter service on its arrival in the slot. The probability of a packet arriving at the close of the slot is 'p' and it's not arriving at that point is '$1-p$' which implies that the inter-arrival time is geometrically distributed with parameter 'p'. A batch of packets of size $j\ (L \leq j \leq K)$ starts service at the beginning of a slot, and will end service at α^{th} success just before the end of a slot. The probabilities of the batch server completes either a success or a failure at the end of a slot are 'p_1' and '$q_1 = 1 - p_1$' respectively. For more information see Hunter[3]. Service time of a batch is independent of the number of packets in a batch and simultaneous occurrence of both arrival of a packet and departure of a batch in a single slot is ruled out. All the random variables in the analysis are non negative and have integer value. The value of the load 'ρ' of the server is less than unity i.e.,

$$\rho = \frac{E(B)}{K\,E(A)} = \frac{\alpha\, p}{K\, p_1} < 1$$

III. DISTRIBUTION OF SYSTEM OCCUPANCY AT DEPARTURE EPOCHS

Let $X_n \in \{0, 1, 2, \dots, \infty\}$ be the number of packets accumulated in the system (queue + service) just after the server has left with n[th] batch. The steady state distribution $\{x(k) = \lim_{n\to\infty} x_n(k) = \lim_{n\to\infty} \Pr(X_n = k): \; k = 0,1,2,\dots\infty\}$ of system occupancy at departure epochs is derived in this section using the embedded Markov Chain (MC) technique.

Let V_n denoting the number of packets that reach the system during the n^{th} service. Then the distribution $\{v(k): k = 0,1,2\dots\}$ of V_n can be derived as follows:

$$v(k) = \sum_{m=k+\alpha}^{\infty} \binom{m-\alpha}{k} p^k (1-p)^{m-\alpha-k} \binom{m-1}{\alpha-1} \quad p_1^{\alpha} (1-p_1)^{m-\alpha} = 0,1,.. \quad \dots (6)$$

For $k = 0,1,2\dots\infty$

$$v(k) = \binom{k+\alpha-1}{\alpha-1} \beta^{\alpha} (1-\beta)^k \quad \dots (7)$$

Where $\beta = \frac{P_1}{P+P_1-PP_1}$ Also $E(V) = \frac{\alpha(1-\beta)}{\beta}$ and $Var\,(V) = \frac{\alpha(1-\beta)}{\beta^2}$

The sequence $\{X_n\}$ of random variables can be shown to form a MC on the discrete state space $\{0,1,2,\dots,\infty\}$ with the following one step transition probability matrix $P = (p_{ij})$ where

$$P_{ij} = \begin{cases} \sum_{r=0}^{K-L} v(r) & ; 0 \le i \le L-1 \text{ and } j = 0 \\ v(K-L+j) & ; 0 \le i \le L-1 \text{ and } j \ge 1 \\ \sum_{r=0}^{K-i} v(r) & ; L \le i \le K \text{ and } j = 0 \\ v(K-i+j) & ; L \le i \le K \text{ and } j \ge 1 \\ v(j+K-i) & ; i \ge (K+1) \text{ and } j \ge (i-K) \\ 0 & ; otherwise \end{cases} \quad \dots (8)$$

The unknown probability (row) vector $X_n = (x_0, x_1, x_2, \dots)$ an now be obtained solving the following system of equations:

$$X_n\, P = X_n, X_n\; e = 1 \quad \dots (9)$$

where $'e'$ denotes the row vector of unities. A number of numerical methods could be suggested to solve the system of equations (9). For example an algorithm for solving the system (9) of equations is given by Latouche and Ramaswami [5]. Here an upper bound N and 'i' and 'j' of unit step probability function p_{ij} have been selected so that the p_{ij} values are very small for all $i, j \ge N$ and they could be ignored. Thus $\sum_{j=0}^{N} p_{ij} = 1$ and $p_{iN} = 1 - \sum_{j=0}^{N-1} p_{ij}$ for all $0 \le i \le N$ and $P = (p_{ij})$ is a square matrix of order $(N+1)$. The average system length L_s^+ and average queue length L_q^+ applying equation (10) have obtained.

$$L_s^+ = \sum_{n=0}^{\infty} n\; x(n) \quad and \quad L_q^+ = \sum_{n=0}^{L-1} n \;\; x(n) \; + \sum_{n-K}^{\infty} (n-K)\, x(n) \quad \dots \quad (10)$$

IV. NUMERICAL RESULTS

Some numerical results have been obtained and presented in the form of a table which is self-explanatory. The probability distribution of system occupancy at service completion epochs has been reported in Table 1 for a set of values of $P, P_1, \alpha,$ L and K.

The probability and cumulative probability of the model for the values $L = 3, K = 12$, $P = 0.95$ and $P_1 = 0.10$, $\alpha = 1$ are given in the following table. The average system length and average queue length are also given in the table.

Table 1: Numerical values of {x(n)}

L = 3 K = 12 P = 0.95 P_1= 0.10 α = 1			
Probability	Cum.Probability	Probability	Cum.Probability
x(0)=.10471200	.10471200	x(47)=.00057842	.99505440
x(1)= .09374740	.19845950	x(50)=.00041508	.99645090
x(2)= .08393095	.28239040	x(57)=.00019136	.99836370
x(4)= .06022965	.42480690	x(60)=.00013732	.99882570
x(8)= .04322137	.63045720	x(65)=.00007899	.99932440
x(10)=.03464368	.70379650	x(70)=.00004543	.99961130
x(13)=.02486064	.78744150	x(75)=.00002613	.99965190
x(15)=.01992681	.82962570	x(80)=.00001503	.99987120
x 20)=.01146176	.90200180	x(90)=.00000497	.99995710
x(26)=.00590239	.94953450	x (100)=.00000164	.99998560
x(30)=.00379209	.96757750	x(110)=.00000054	.99999510
x(35)=.00218118	.98135080	x(120)=.00000018	.99999820
x(40)=.00125460	.98927310	x(125)=.00000010	.99999880
System length L_s^+ = 10.7910		**Queue length L_q^+ = 5.66462200**	

It could be observed that $\sum_{k=0}^{\infty} x(k)$ may not be equal to one in the above case. This is because TPM **P** has been truncated from its infinite dimensions to the form of a square matrix of the finite order N. Since the distribution $\{x(k)\}$ is now known, it will be easier to obtain other possible probability distributions of queue length related to various types of transition epochs.

V. CONCLUSION

In this paper, we have analyzed a discrete -time queue of packets with the bulk service rule (L,K). We have obtained the queue length distribution of the system occupancy just before and after the departure epochs. Utilizing these distribution, We have derived some important performance measure . Numerical illustrations in the form of table are reported to demonstrated now the various parameters of the model influence the behavior of the system. This technique used in this paper can be applied to analyzing are complex model such as general distribution $\{\Pr(B = m) = b(m);\ m = 0,1.2,\ldots,\infty\}$, which is either uniform or hyper geometric distribution left for future investigations.

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