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Graceful Labeling For Open Star of Graphs

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ABSTRACT: *In this paper we have proved that $S(t\cdot K_{m,n})$, $S(t\cdot P_n\times P_m)$, $P_n^t(tn\cdot K_{m,r})$ are graceful graphs. For this we define open star of graphs and one point union for path of graphs.*

KEY WORDS - *Graceful labeling, complete bipartite graph, grid graph, open star of graphs and one point union of path graphs.*

AMS **subject classification number:** 05C78.

I. INTRODUCTION:

The graceful labeling was introduced by A Rosa [7] in 1967. Golomb [3] proved that the complete bipartite graph is graceful. Barrientos [1] proved that union of complete bipartite graphs is also graceful. Vaidya et. al [8] introduced a star of cycle and proved that it is cordial as well as 3-equitable. Kaneria and Makadia [5] proved that star of a cycle $C_n (n \equiv 0 (mod\ 4))$ is graceful. Kaneria et. al [6] proved that join sum of $K_{m,n}$, path union of $K_{m,n}$ and star of $K_{m,n}$ are graceful graphs. For detail survey of graph labeling one can refer Gallian [2].

In this paper we have introduced open star of graphs and one point union for path of graphs. We also proved that $S(t\cdot K_{m,n})$, $S(t\cdot P_n\times P_m)$ and $P_n^t(tn\cdot K_{m,r})$ are graceful graphs.

We begin with a simple undirected, finite graph $G=(V,E)$, with $p=|V|$ vertices and $q=|E|$ edges. In this work $K_{m,n}$ denotes a complete bipartite graph and $P_n\times P_m$ denotes a grid graph on mn vertices. For all terminology and notations we follow Harary [4]. We shall give brief summary of definitions which are useful in this paper.

Definition−1.1 **:** A function f is called graceful labeling of a graph $G=(V,E)$ if $f: V \longrightarrow \{0,1,\dots,q\}$ is injective and induced function $f^*: E \longrightarrow \{1,2,\dots,q\}$ defined as $f^*(e)=|f(u)-f(v)|$ is bijective for every edge $e=(u,v) \in E$. A graph G is called graceful graph if it admits a graceful labeling.

Definition−1.2 **:** Let G be a graph and $G_1, G_2, \dots, G_n$, $n\geq 2$ be n copies of graph G. Then the graph obtained by adding an edge from G_i to $G_{i+1} (i=1,2,\dots,n-1)$ is called path union of graph G.

Definition−1.3 **:** A graph obtained by replacing each vertex of $K_{1,n}$ except the apex vertex by the graphs $G_1, G_2, \dots, G_n$ is known as open star of graphs. We shall denote such graph by $S(G_1, G_2, \dots, G_n)$.

If we replace each vertices of $K_{1,n}$ except the apex vertex by a graph G. i.e. $G_1=G_2=\dots=G_n=G$, such open star of a graph, we shall denote by $S(n\cdot G)$.

Definition−1.4 **:** A graph G is obtained by replacing each edge of $K_{1,t}$ by a path P_n of length n on $n+1$ vertices is called one point union for t copies of path P_n. We shall denote such graph G by P_n^t.

Definition−1.5 **:** A graph G obtained by replacing each vertices of P_n^t except the central vertex by the graphs $G_1, G_2, \dots, G_{tn}$ is known as one point union for path of graphs. We shall denote such graph G by $P_n^t(G_1, G_2, \dots, G_{tn})$, where P_n^t is the one point union of t copies of path P_n.

If we replace each vertices of P_n^t except the central vertex by a graph H, i.e. $G_1=G_2=\dots=G_{tn}=H$, such one point union of path graph, we shall denote it by $P_n^t(tn\cdot H)$.

II. MAIN RESULTS:

Theorem−2.1 : $S(t \cdot K_{m,n})$ is graceful.

Proof: Let G be a graph obtained by replacing each vertices of $K_{1,t}$ except the apex vertex by the graph $K_{m,n}$. Let u_0 is the apex vertex of $K_{1,t}$. i.e. it is central vertex of the graph G. Let $u_{l,i}(1 \leq i \leq m), v_{l,j}(1 \leq j \leq n)$ be the vertices of l^{th} copy of $K_{m,n}^{(l)}$ of $K_{m,n}$ in $G, \forall\, l = 1,2, \ldots, t$.

We shall join $u_{l,1}$ with the vertex u_0 by an edge to form the open star of graphs $G, \forall\, l = 1,2, \ldots, t$. We define labeling function $f: V(G) \longrightarrow \{0,1, \ldots, q\}$, where $q = t(mn + 1)$ as follows:

$$f(u_0) = 0;$$

$$f(v_{1,j}) = j, \qquad \forall\, j = 1,2, \ldots, n;$$

$$f(u_{1,i}) = q - (i-1)n, \qquad \forall\, i = 1,2, \ldots, m;$$

$$f(v_{2,j}) = f(v_{1,j}) + [q - (mn+1)], \quad \forall\, j = 1,2, \ldots, n;$$

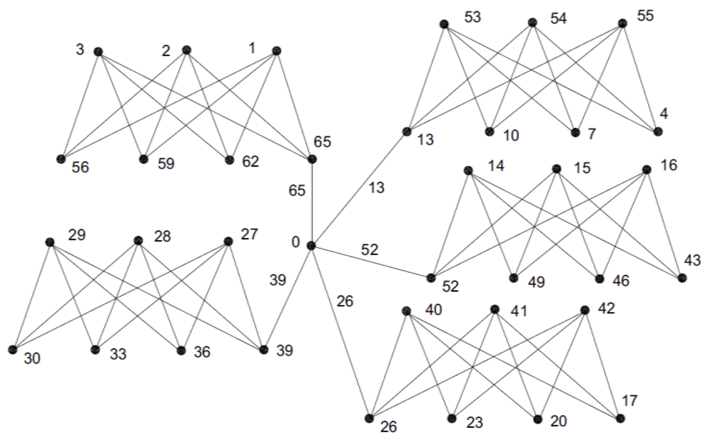
$$f(u_{2,i}) = f(u_{1,i}) - [q - (mn+1)], \quad \forall\, i = 1,2, \ldots, m;$$

$$f(v_{l,j}) = f(v_{l-2,j}) - (-1)^l(mn+1), \quad \forall\, j = 1,2, \ldots, n, \ \forall\, l = 3,4, \ldots, t;$$

$$f(u_{l,i}) = f(u_{l-2,i}) + (-1)^l(mn+1), \quad \forall i = 1,2, \ldots, m, \ \forall\, l = 3,4, \ldots, t;$$

Above labeling pattern give rise graceful labeling to the graph G and so it is a graceful graph.

Illustration−2.2 : open star of 5 copies of $K_{4,3}$ and its graceful labeling shown in figure−1.



Figure−1 A graph obtained by open star of $K_{4,3}$ and its graceful labeling .

Theorem−2.3 : $S(t \cdot P_n \times P_m)$ is graceful.

Proof : Let G be a graph obtained by replacing each vertices of $K_{1,t}$ except the apex vertex by the grid graph $P_n \times P_m$. Let u_0 is the central vertex for the graph G. i.e. it is apex vertex of the original graph $K_{1,t}$. Let $u_{s,i,j}(1 \leq i \leq n, 1 \leq j \leq m)$ be the vertices of s^{th} copy of $(P_n \times P_m)^{(s)}$ of $P_n \times P_m$ in $G, \forall\, s = 1,2, \ldots, t$.

We shall join $u_{s,1,1}$ with the vertex u_0 by an edge to form the open star of graphs $G, \forall\, s = 1,2, \ldots, t$.

We know that the grid graph $P_n \times P_m$ is a graceful graph on $p = mn$ vertices and $q = 2mn - (m+n)$ edges. Let $f: V(P_n \times P_m) \longrightarrow \{0,1,\dots,q\}$ be a graceful labeling with two sequences of labels, among one is increasing and another one is decreasing, which start by $f(v_{1,1}) = q$ and end with $(v_{n,m}) = \left\lfloor \frac{q}{2} \right\rfloor$, where $V(P_n \times P_m) = \{v_{i,j}/1 \le i \le n, 1 \le j \le m\}$.

We have $G = S(t \cdot P_n \times P_m)$ with $P = tmn + 1$ vertices and $Q = t(q+1) = t(2mn - (m+n) + 1)$ edges. We define labeling function $g: V(G) \longrightarrow \{0,1,\dots,Q\}$ as follows:

$$g(u_0) = 0;$$

$$g(u_{1,i,j}) = f(v_{i,j}) + 1, \qquad \text{When } f(v_{i,j}) < \tfrac{q}{2},$$

$$= f(v_{i,j}) + (Q - q), \qquad \text{When } f(v_{i,j}) \ge \tfrac{q}{2},$$

$$\forall\, i = 1,2,\dots,n,\ \forall\, j = 1,2,\dots,m;$$

$$g(u_{2,i,j}) = g(u_{1,i,j}) + (Q - (q+1)), \quad \text{When } g(u_{1,i,j}) < \tfrac{Q}{2},$$

$$= g(u_{1,i,j}) - (Q - (q+1)), \quad \text{When } g(u_{1,i,j}) > \tfrac{Q}{2},$$

$$\forall\, i = 1,2,\dots,n,\ \forall\, j = 1,2,\dots,m;$$

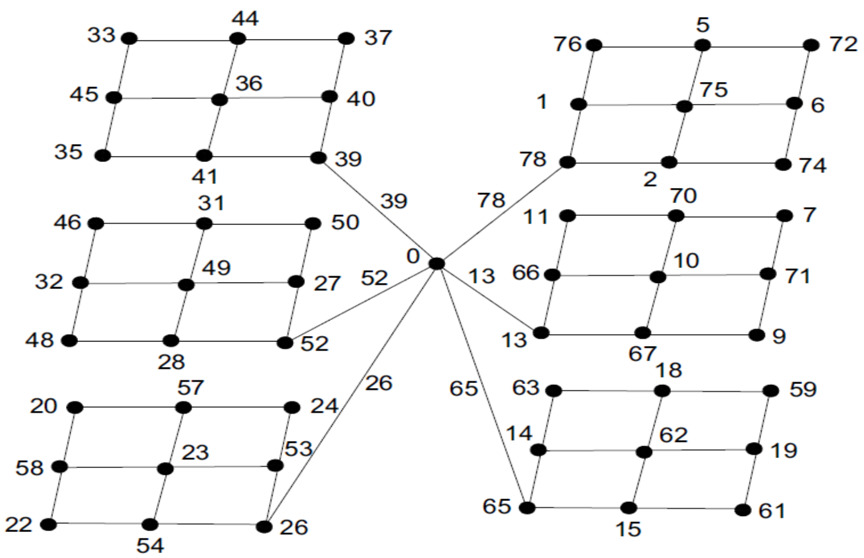
$$g(u_{l,i,j}) = g(u_{l-2,i,j}) + (q+1), \qquad \text{When } g(u_{l-2,i,j}) < \tfrac{Q}{2},$$

$$= g(u_{l-2,i,j}) - (q+1), \qquad \text{When } g(u_{l-2,i,j}) > \tfrac{Q}{2},$$

$$\forall\, i = 1,2,\dots,n,\ \forall\, j = 1,2,\dots,m,\ \forall\, l = 3,4,\dots,t.$$

Above labeling pattern give rise graceful labeling to the graph G and so it is a graceful graph.

Illustration−2.4 **:** open star of 6 copies of $P_3 \times P_3$ and its graceful labeling shown in figure−2.



Figure−2 A n open star of $P_3 \times P_3$ and its graceful labeling.

Theorem−2.5 **:** $P_n^t(tn \cdot K_{m,r})$ is graceful.

Proof: Let G be a graph obtained by replacing each vertices of P_n^t except the central vertex by the graph $K_{m,r}$. i.e. G is the graph obtained by replacing each vertices of $K_{1,t}$ except the apex vertex by the path union of n copies of the graph $K_{m,r}$. Let u_0 be the central vertex for the graph G. Let $u_{s,l,i}(1 \le i \le m), v_{s,l,j}(1 \le j \le r)$ be the vertices of $K_{m,r}$ which is l^{th} copy of the path union of n copies of $K_{m,r}$ lies in s^{th} branch of the graph $G, \forall\, l = 1,2, \dots, n$ and $\forall\, s = 1,2, \dots, t$.

First we shall join $u_{s,l,m}$ with the vertex $v_{s,l+1,1}$ by an edge to form the path union of n copies of $K_{m,r}$ for s^{th} branch of $G, \forall\, l = 1,2, \dots, n-1$ and $\forall\, s = 1,2, \dots, t$. Now we shall join $u_{s,1,1}$ with the vertex u_0 by an edge to form the one point union for path of graphs $G, \forall\, s = 1,2, \dots, t$. We shall define labeling function f for the first copy(branch) of the path union of n copies of $K_{m,r}$ as follows:

$f : V(\textit{path union of } n \textit{ copies of } K_{m,r}) \longrightarrow \{0,1, \dots, q\}$, Where $q = n(mr+1) - 1$ defined by,

$f(u_{1,1,i}) = q - (i-1), \quad \forall\, i = 1,2, \dots, m;$

$f(v_{1,1,j}) = (j-1)m, \quad \forall\, j = 1,2, \dots, r;$

$f(u_{1,l,i}) = f(u_{1,l-1,i}) - m, \quad \forall\, i = 1,2, \dots, m$ & $\forall\, l = 2,3, \dots, n;$

$f(v_{1,l,j}) = f(v_{1,l-1,j}) + (m+1), \quad \forall\, j = 1,2, \dots, r$ & $\forall\, l = 2,3, \dots, n.$

Above labeling pattern give rise graceful labeling to the path union of n copies of $K_{m,r}$, which lies in first branch of G. Now we shall define labeling $g : V(G) \longrightarrow \{0,1, \dots, Q\}$, where $Q = tn(mr+1)$ as follows:

$g(v_0) = 0;$

$g(v_{1,l,j}) = f(v_{1,l,j}) + 1, \quad \forall\, j = 1,2, \dots, r, \ \forall\, l = 1,2, \dots, n;$

$g(u_{1,l,i}) = (Q - q) + f(u_{1,l,i}), \quad \forall\, i = 1,2, \dots, m, \ \forall\, l = 1,2, \dots, n;$

$g(v_{2,l,j}) = g(v_{1,l,j}) + (Q - (q+1)), \quad \forall\, j = 1,2, \dots, r, \ \forall\, l = 1,2, \dots, n;$

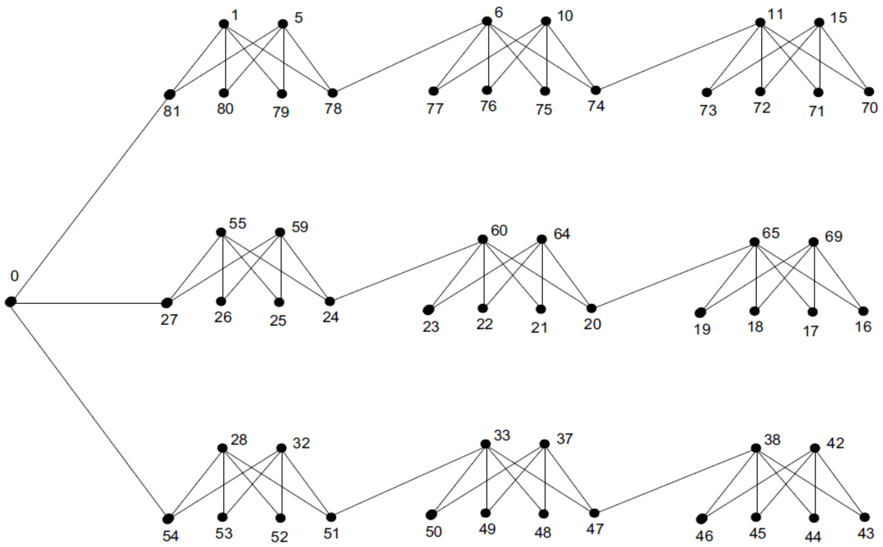
$g(u_{2,l,i}) = g(u_{1,l,i}) - (Q - (q+1)), \quad \forall\, i = 1,2, \dots, m, \ \forall\, l = 1,2, \dots, n;$

$g(v_{s,l,j}) = g(v_{s-2,l,j}) - (-1)^s(q+1), \quad \forall\, j = 1,2, \dots, r, \ \forall\, l = 1,2, \dots, n, \ \forall\, s = 3,4, \dots, t;$

$g(u_{s,l,i}) = g(u_{s-2,l,i}) + (-1)^s(q+1), \quad \forall\, i = 1,2, \dots, m, \ \forall\, l = 1,2, \dots, n; \ \forall\, s = 3,4, \dots, t.$

Above labeling pattern give rise graceful labeling to the graph G and so it is a graceful graph.

Illustration−2.6 **:** One point union for path of $K_{4,2}$ and its graceful labeling shown in figure−3.



Figure−3 A graph obtained by one point union for path of $K_{4,2}$ and its graceful labeling .

III. CONCLUDING REMARKS:

Here we have introduced some definitions like open star of graphs and one point union for path of graphs. We also have given graceful labeling to $S(t \cdot K_{m,n})$, $S(t \cdot P_n \times P_m)$ and $P_n^t(tn \cdot K_{m,r})$. The result obtained here are new and of very general nature. This work contributes three results to the families of graceful labeling. The labeling pattern is demonstrated by means of illustrations.

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