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Line Search Techniques by Fibonacci Search

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ABSTRACT: *We shall consider useful iterative techniques for solving first unconstrained nonlinear problems. These techniques usually require many iterations of rather tedious computations. As a result such techniques usually require the use of a high speed computer. A large number of these techniques exists and we shall be contended with examining some of the important ones here. A so called best general purpose algorithm suitable for solving all types of nonlinear programmes has not been devised so far. The search for one such algorithm continues. Throughout, emphasis is placed on the computational aspects of the techniques. Less attention is paid to the theoretical development and economy(rate of convergence, stability etc.) of various methods. For highly specialized problems of large systems required careful theoretical considerations. Nevertheless, many problems not possessing such a specialized structure can be satisfactorily solved using iterative techniques.*

KEY WORDS: *Nonlinear Programming, Unconstrained Nonlinear Problems, Line Search, Fibonacci Search, Optimization, Eight-point Fibonacci search.*

I. INTRODUCTION

Line Search:

We began the study of nonlinear programming theory with one-dimensional problem, it is logical to start here again with single variable problems in the study of numerical optimization.

In line search techniques, we begin with finding a maximum or a minimum of the objective function along a straight line. Many of the multidimensional search techniques require a sequence of unidirectional (line) searches. For simplicity we will assume that for all the time f is a real valued function of a real variable x over an interval $[a_1, b_1]$. We shall also assume that f is unimodel i.e. f has a single relative minimum or maximum x*, i.e. f is strictly decreasing (or increasing) when $x < X^*$ and f is strictly increasing (or decreasing) when $x > x^*$.

Fibonacci Search:

We consider again the problem of minimizing a function that is unimodel over the interval [a ,b]. Let $L_1 = b - a$ and suppose that $L_{1/2} < L_2 < L_1$. Then set $x^1_L = b_1 - L_2$ and $x^1_R = a_1 + L_2$, where $a_1 = a$ and $b_1 = b$. We can note that x^1_L and x^1_R are symmetrically placed about the midpoint of the interval [a , b]. Without loss of generality suppose that $f(x^1_L) < f(x^1_R)$. Then the minimum of f cannot occur in the interval $(x^1_R, b_1]$ and hence, $(x^1_R, b_1]$ is discarded leaving $[a_1, x^1_R]$. Let $a_2 = a_1$, $b_2 = x^1_R$ and $L_3 = L_1 - L_2$. Then, set $x^2_L = b_2 - L_3$ and $x^2_R = a_2 + L_3$, where he superscript 2 in the expression x^2_L denotes the second iteration and not the square of x_L. Again, notice that x^2_L and x^2_R are symmetrically placed about the midpoint of $[a_2,b_2]$. We shall require that the points be positioned so that $L_3 = ½ L_2$. After n-2 iterations we could have a situation more compact. Next, let $L_n = L_{n-2} - L_{n-1}$ and select x^{n-1}_L and x^{n-1}_R so that they are symmetrically positioned about the midpoint $[a_{n-2}, x^{n-2}_R]$, requiring that $L_n > L_{n-1/2}$. This would mean that $x^{n-1}_L = b_{n-1} - L_n$ and $x^{n-1}_R = x^{n-2}_R$. If $\varepsilon = x^{n-1}_R - x^{n-1}_L$, then it would follow that

$$L_{n-1} = 2L_n - \varepsilon \qquad \text{……………………… (1)}$$

In general, after the (n-k)th iteration, where k = 2,3, …. n-1, we would have

$$L_{n-(k-2)} = L_{n-k} - L_{n-(k-1)} \qquad \text{………………………….(2)}$$

Thus, using (1) and (2), we would have the following:

$$L_{n-1} = 2L_n - \varepsilon$$
$$L_{n-2} = L_{n-1} + L_n = 2L_n - \varepsilon + L_n = L_n - \varepsilon$$
$$L_{n-3} = L_{n-2} + L_{n-1} = 3L_n - \varepsilon + 2L_n - \varepsilon = 5L_n - 2\varepsilon \qquad \text{…………………(3)}$$
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…….
$$L_{n-k} = L_{n-(k-1)} + L_{n-(k-2)} = F_{k+1}L_n - F_{k-1}\varepsilon$$

Where integral coefficients F_{k+1} and F_{k-1} are generated by recurrence relations:

$$F_0 = F_1 = 1$$
$$F_{k+1} = F_k + F_{k-1},\ k = 1, 2, 3 \ldots\ldots \qquad \text{……………………(4)}$$

The Sequence of integers generated by (4) is called the Fibonacci number sequence.
If we let K = n -1 in equation (3), we get the following relationship between L_1 and L_n

$$L_1 = L_{n-(n-1)} = F_{(n-1)+1} L_n - F_{(n-1)-1} \varepsilon = F_n L_n - F_{n-2} \varepsilon.$$

$$\text{or} \quad L_n = L_1 + F_{n-2} \varepsilon. / F_n$$

Likewise, if we put k = n-2 in (3), then we get

$$L_2 = L_{n-(n-2)} = F_{(n-2)+1} L_n - F_{(n-2)-1} \varepsilon = F_{n-1} L_n - F_{n-3} \varepsilon.$$

$$\text{or} \quad L_2 = F_{n-1} \quad L_1 + F_{n-2} \varepsilon. / F_n - F_{n-3} \varepsilon = F_{n-1} L_1 / F_n + F_{n-1} F_{n-2} - F_n F_{n-3} / F_n \ \varepsilon.$$

on simplification we have

$$F_{n-1} F_{n-2} - F_n F_{n-3} = F^2_{n-1} - F_{n-1} F_{n-3} = (-1)^n. \text{ (by induction)}$$

Hence

$$L_2 = F_{n-1} L_1 / F_n + (-1)^n \varepsilon / F_n \qquad \text{………………..(5)}$$

Thus, for a predetermined number n of search points and a predetermined ε, the Fibonacci search technique for finding the minimum of a unimodel function over an interval [a,b] can be put in the format.

II. ALGORITHM

Algorithm for Fibonacci Search:

Step 1: Set $a_1 = a$, $b_1 = b$ and $L_1 = b_1 - a_1$.
Step 2 : Use equation (5) to compute L_2.
Step 3: Let $x^1_L = b_1 - L_2$ and $x^1_R = a_1 + L_2$.
Step 4: Evaluate $f(x^1_L)$ and $f(x^1_R)$.
Step 5: Evaluate $L_3 = L_1 - L_2$.
Step 6: If $f(x^1_L) > f(x^1_R)$, then set $a_2 = x^1_L$, $b_2 = b_1$, and $x^2_L = x^1_R$
$x^2_R = a_2 + L_3$, Go to step 7.
If $f(x^1_L) > f(x^1_R)$, then set $a_2 = a_1$, $b_2 = x^1_R$, $x^2_R = x^1_L$ and
$x^2_L = b_2 - L_3$. Go to step 7.
Step 7: Return to step 4 and continue for n-1 trials.

We can note that when the process ends the search points are ε units apart and the interval of uncertainty has length L_n, i.e., the true minimum occurs at appoint not more than Ln units from the estimated solution. Moreover, the length of $L_n = L_1 + F_{n-2}\varepsilon / F_n$ is determined at the beginning of the calculation once L_1 and ε are specified.

III. EXAMPLE

We can apply the algorithm to an important example:

Using an Eight-point Fibonacci search with ε = 0.0001, determine minimum x Sin(1/x) over [1/2π, 1/π]. First, let $a_1 = 1/2\pi = 0.15915494$ and $1/\pi = 0.31830989$; $L_1 = b_1 - a_1 = 0.15915494$.Using equation (5) let us compute

$L_2 = F_7 L_1/F_8 + (-1)^8 \epsilon/F_8 = 0.9830452$
$x^1_L = b_1 - L_2 = 0.22000537$
and $x^1_R = a_1 + L_2 = 0.25745946$
Hence,
$f(x^1_L) = -0.21694297$
and
$f(x^1_R) = -0.17407923.$
Also,
$L_3 = L_1 - L_2 = 0.06085043$
Since, $f(x^1_L) < f(x^1_R)$, we get
$a_2 = a_1 = 0.15915494$
$b_2 = x^1_R = 0.225745946$
$x^2_R = x^1_L = 0.22000537$
and $x^2_L = b_2 - L_2 = 0.19660903.$
Next computing $f(x^2_L)$, $f(x^2_R)$ and L_4, we get
$f(x^2_L) = -0.18302913$
$f(x^2_R) = -0.21694297$
and $L_4 = L_2 - L_3 = 0.03745409$
Since $f(x^2_L) > f(x^2_R)$, we get
$a_3 = x^2_L = 0.19660903$
$b_3 = b_2 = 0.25745946$
$x^3_L = x^2_R = 0.22000537$
$x^3_R = a_3 + L_4 = 0.23406312.$

The process is continued for a total of seven trials. In Table 1 the results of the seven trials are given. We can see from the table that the two final trial points x^7_L and x^7_R are $\varepsilon = 0.0001$ apart. The optimal solution is estimated to be f(0.22462463) = -0.21704508 and the interval of uncertainty is $L_8 = [a_7, x^7_R]$ which has length 0.00471916.

Table 1: Eight-point Fibonacci search

n	a_i	b_i	x^i_L	x^i_R	$f(x^i_L)$	$f(x^i_R)$	L_{i+1}
1	0.15915494	0.31830989	0.22000537	0.25745946	-0.21694237	-0.17407923	0.09830452
2	0.15915494	0.25745946	0.19660903	0.22000537	-0.18302913	-0.21694297	0.06085043
3	0.19660903	0.25745946	0.22000537	0.23406312	-0.21694297	-0.21176526	0.037455409
4	0.19660903	0.23406312	0.21066684	0.22000537	-0.21054189	-0.21694297	0.02339634
5	0.21066684	0.23406312	0.22000537	0.22472463	-0.21694297	-0.21702661	0.01405775
6	0.22000537	0.23406312	0.22472463	0.22934389	-0.21702661	-0.21527183	0.00933859
7	0.22000537	0.22934389	0.22472463	0.22472463	-0.21704508	-0.21702661	0.00471916

Min x sin(1/x) over [1/2π . 1/π]

IV. CONCLUSION

A useful iterative technique for solving first unconstrained nonlinear problems. These techniques usually require much iteration of rather tedious computations. As a result such techniques usually require the use of a high speed computer. In a line search techniques, we find a maximum or a minimum of the objective functions. We can apply the algorithm for Fibonacci search with an example and summarize results in Table.

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