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Inventory Model with Different Deterioration Rates under Linear Demand and Time Varying Holding Cost

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ABSTRACT: *An inventory model with different deterioration rates under linear trend in demand with time varying holding cost is developed. Shortages are not allowed. Numerical examples are provided to illustrate the model and sensitivity analysis is also carried out for parameters.*

KEY WORDS: *Inventory model, Varying Deterioration, Linear demand, Time varying holding cost*

I. INTRODUCTION

In recent years much work has been done regarding inventory models for deteriorating items. Research in the area of deteriorating items inventory model started with the work of Within [14] who considered fashion goods deteriorating at the end of prescribed storage period. Ghare and Schrader [4] considered inventory problem under constant demand and constant deterioration. Shah and Jaiswal [11] considered an order level inventory model for items deteriorating at a constant rate. Aggarwal [1] discussed an order level inventory model with constant rate of deterioration. Dave and Patel [3] developed the deteriorating items inventory model with linear trend in demand. They considered demand as linear function of time. Burewell [2] developed economic lot size model for price dependent demand under quantity and freight discounts. Salameh and Jaber [10] developed a model to determine the total profit per unit of time and the economic order quantity for a product purchased from the supplier. Mukhopadhyay et al. [7] developed an inventory model for deteriorating items with a price-dependent demand rate. The rate of deterioration was taken to be time-proportional and a power law form of the price-dependence of demand was considered. Teng and Chang [12] considered the economic production quantity model for deteriorating items with stock level and selling price dependent demand. Other research work related to deteriorating items can be found in, for instance (Raafat [8], Goyal and Giri [5], Ruxian et al. [9]).

Tripathy and Mishra [13] dealt with development of an inventory model when the deterioration rate follows Weibull two parameter distribution, demand rate is a function of selling price and holding cost is time dependent. The model was developed by taking care of with and without shortage both cases. Mathew [6] developed an inventory model for deteriorating items with mixture of Weibull rate of decay and demand as function of both selling price and time.

Inventory models for non-instantaneous deteriorating items have been an object of study for a long time. Generally the products are such that there is no deterioration initially. After certain time deterioration starts and again after certain time the rate of deterioration increases with time. Here we have used such a concept and developed the deteriorating items inventory models.

In this paper we have developed an inventory model with different deterioration rates for the cycle time under time varying holding cost. Shortages are not allowed. Numerical example is provided to illustrate the model and sensitivity analysis of the optimal solutions for major parameters is also carried out.

II. ASSUMPTIONS AND NOTATIONS

The following notations are used for the development of the model:

NOTATIONS:

D(t) : Demand rate is a linear function of time t ($a+bt$, $a>0$, $0<b<1$)
A : Replenishment cost per order
c : Purchasing cost per unit
p : Selling price per unit
T : Length of inventory cycle
I(t) : Inventory level at any instant of time t, $0 \le t \le T$
Q : Order quantity
θ : Deterioration rate during $\mu_1 \le t \le \mu_2$, $0< \theta<1$

θt : Deterioration rate during , $\mu_2 \le t \le T$, $0 < \theta < 1$
π : Total relevant profit per unit time.

ASSUMPTIONS:

The following assumptions are considered for the development of the model.

- The demand of the product is declining as a linear function of time.
- Replenishment rate is infinite and instantaneous.
- Lead time is zero.
- Shortages are not allowed.

III. THE MATHEMATICAL MODEL AND ANALYSIS

Let I(t) be the inventory at time t ($0 \le t \le T$) as shown in figure.

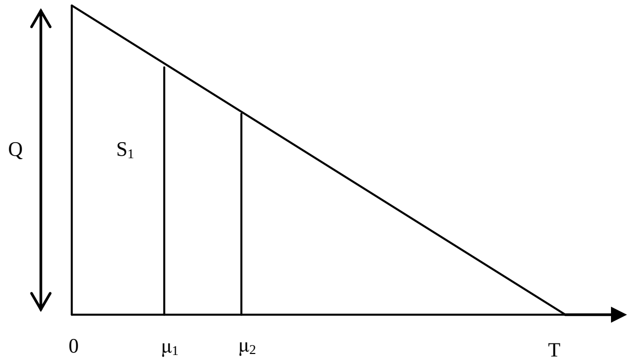


Figure 1

The differential equations which describes the instantaneous states of I(t) over the period (0, T) is given by

$$\frac{dI(t)}{dt} = -(a + bt), \qquad 0 \le t \le \mu_1 \tag{1}$$

$$\frac{dI(t)}{dt} + \theta I(t) = -(a + bt), \qquad \mu_1 \le t \le \mu_2 \tag{2}$$

$$\frac{dI(t)}{dt} + \theta t I(t) = -(a + bt), \qquad \mu_2 \le t \le T \tag{3}$$

with initial conditions I(0) = Q, $I(\mu_1) = S_1$ and I(T) = 0.
Solutions of these equations are given by

$$I(t) = Q - (at + \frac{1}{2}bt^2), \tag{4}$$

$$I(t) = \begin{bmatrix} a(\mu_1 - t) + \frac{1}{2}b(\mu_1^2 - t^2) + \frac{1}{2}a\theta(\mu_1^2 - t^2) \\ + \frac{1}{3}b\theta(\mu_1^3 - t^3) - a\theta t(\mu_1 - t) - \frac{1}{2}b\theta t(\mu_1^2 - t^2) \end{bmatrix} + S_1\left[1 + \theta(\mu_1 - t)\right] \tag{5}$$

$$I(t) = \begin{bmatrix} a(T - t) + \frac{1}{2}b(T^2 - t^2) + \frac{1}{6}a\theta(T^3 - t^3) \\ + \frac{1}{8}b\theta(T^4 - t^4) - \frac{1}{2}a\theta t^2(T - t) - \frac{1}{4}b\theta t^2(T^2 - t^2) \end{bmatrix}. \tag{6}$$

(by neglecting higher powers of θ)

From equation (4), putting $t = \mu_1$, we have

$$Q = S_1 + \left(a\mu_1 + \frac{1}{2}b\mu_1^2\right). \tag{7}$$

From equations (5) and (6), putting $t = \mu_2$, we have

$$I(\mu_2) = \begin{bmatrix} a(\mu_1 - \mu_2) + \frac{1}{2}b\left(\mu_1^2 - \mu_2^2\right) + \frac{1}{2}a\theta\left(\mu_1^2 - \mu_2^2\right) \\ + \frac{1}{3}b\theta\left(\mu_1^3 - \mu_2^3\right) - a\theta\mu_2(\mu_1 - \mu_2) - \frac{1}{2}b\theta\mu_2\left(\mu_1^2 - \mu_2^2\right) \end{bmatrix} + S_1\left[1 + \theta(\mu_1 - \mu_2)\right] \tag{8}$$

$$I(\mu_2) = \begin{bmatrix} a(T - \mu_2) + \frac{1}{2}b\left(T^2 - \mu_2^2\right) + \frac{1}{6}a\theta\left(T^3 - \mu_2^3\right) \\ + \frac{1}{8}b\theta\left(T^4 - \mu_2^4\right) - \frac{1}{2}a\theta\mu_2^2(T - \mu_2) - \frac{1}{4}b\theta\mu_2^2\left(T^2 - \mu_2^2\right) \end{bmatrix}. \tag{9}$$

So from equations (8) and (9), we get

$$S_1 = \frac{1}{\left[1+\theta(\mu_1 - \mu_2)\right]}\begin{bmatrix} a(T - \mu_2) + \frac{1}{2}b\left(T^2 - \mu_2^2\right) + \frac{1}{6}a\theta\left(T^3 - \mu_2^3\right) + \frac{1}{8}b\theta\left(T^4 - \mu_2^4\right) - \frac{1}{2}a\theta\mu_2^2(T - \mu_2) \\ - \frac{1}{4}b\theta\mu_2^2\left(T^2 - \mu_2^2\right) - a(\mu_1 - \mu_2) - \frac{1}{2}b\left(\mu_1^2 - \mu_2^2\right) - \frac{1}{2}a\theta\left(\mu_1^2 - \mu_2^2\right) - \frac{1}{3}b\theta\left(\mu_1^3 - \mu_2^3\right) \\ + a\theta\mu_2(\mu_1 - \mu_2) + \frac{1}{2}b\theta\mu_2\left(\mu_1^2 - \mu_2^2\right) \end{bmatrix}. \tag{10}$$

Putting value of S_1 from equation (10) into equation (7), we have

$$Q = \frac{1}{\left[1+\theta(\mu_1-\mu_2)\right]}\begin{bmatrix} a(T-\mu_2)+\frac{1}{2}b\left(T^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(T^3-\mu_2^3\right)+\frac{1}{8}b\theta\left(T^4-\mu_2^4\right)-\frac{1}{2}a\theta\mu_2^2(T-\mu_2)-\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right) \\ -a(\mu_1-\mu_2)-\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right)-\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right)-\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2(\mu_1-\mu_2)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right) \end{bmatrix} \tag{11}$$

$$+\left(a\mu_1+\frac{1}{2}b\mu_1^2\right).$$

Using (11) in (4), we have

$$I(t)=\frac{1}{\left[1+\theta(\mu_1-\mu_2)\right]}\begin{bmatrix} a(T-\mu_2)+\frac{1}{2}b\left(T^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(T^3-\mu_2^3\right)+\frac{1}{8}b\theta\left(T^4-\mu_2^4\right)-\frac{1}{2}a\theta\mu_2^2(T-\mu_2)-\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right) \\ -a(\mu_1-\mu_2)-\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right)-\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right)-\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2(\mu_1-\mu_2)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right) \end{bmatrix} \tag{12}$$

$$+ a(\mu_1 - t) \quad \frac{1}{2}b\left(\mu_1^2 - t^2\right).$$

Based on the assumptions and descriptions of the model, the total annual relevant profit (μ), include the following elements:

(i) Ordering cost (OC) = A (13)

(ii) $HC = \int_0^T (x+yt)I(t)dt = \int_0^{\mu_1}(x+yt)I(t)dt + \int_{\mu_1}^{\mu_2}(x+yt)I(t)dt + \int_{\mu_2}^{T}(x+yt)I(t)dt$

$$= \frac{1}{5}\left(\frac{1}{8}xb\theta + \frac{1}{3}ya\theta\right)T^5 + x\left(aT+\frac{1}{2}bT^2+\frac{1}{6}a\theta T^3+\frac{1}{8}b\theta T^4\right)T+\frac{1}{3}\left(x\left(-\frac{1}{2}b - \frac{1}{2}a\theta T - \frac{1}{4}b\theta T^2\right)-ya\right)T^3$$

$$+\frac{1}{4}\left(\frac{1}{3}xa\theta + y\left(-\frac{1}{2}b - \frac{1}{2}a\theta T - \frac{1}{4}b\theta T^2\right)\right)T^4 + \frac{1}{2}\left(-xa + y\left(aT + \frac{1}{2}bT^2+\frac{1}{6}a\theta T^3+\frac{1}{8}b\theta T^4\right)\right)T^2$$

$$+\frac{1}{30}yb\theta\mu_2^5 + \frac{1}{48}yb\theta T^6 - \frac{1}{30}yb\theta\mu_1^5 - \frac{1}{4}\left(\frac{1}{3}xa\theta + y\left(-\frac{1}{2}b - \frac{1}{2}a\theta T - \frac{1}{4}b\theta T^2\right)\right)\mu_2^4$$

$$-\frac{1}{3}\left(x\left(-\frac{1}{2}b - \frac{1}{2}a\theta T - \frac{1}{4}b\theta T^2\right) - ya\right)\mu_2^3 - \frac{1}{2}\left(-xa + y\left(aT + \frac{1}{2}bT^2+\frac{1}{6}a\theta T^3+\frac{1}{8}b\theta T^4\right)\right)\mu_2^2$$

$$-\frac{1}{48}yb\theta\mu_2^6 - x\left(aT + \frac{1}{2}bT^2+\frac{1}{6}a\theta T^3+\frac{1}{8}b\theta T^4\right)\mu_2 - \frac{1}{5}\left(\frac{1}{8}xb\theta + \frac{1}{3}ya\theta\right)\mu_2^5$$

$$+x\left(\begin{array}{l} a\mu_1+\frac{1}{2}b\mu_1^2+\frac{1}{2}a\theta\mu_1^2+\frac{1}{3}b\theta\mu_1^3 \\ +\frac{1}{\left[1+\theta\left(\mu_1-\mu_2\right)\right]} \\ \left(\left(\begin{array}{l} a\left(T-\mu_2\right)+\frac{1}{2}b\left(T^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(T^3-\mu_2^3\right)+\frac{1}{8}b\theta\left(T^4-\mu_2^4\right)-\frac{1}{2}a\theta\mu_2^2\left(T-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right) \\ -a\left(\mu_1-\mu_2\right)-\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right)-\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right)-\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2\left(\mu_1-\mu_2\right)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right) \end{array}\right)\left(1+\theta\mu_1\right)\right) \end{array}\right)\mu_2$$

$$+x\left(\begin{array}{l} \frac{1}{\left[1+\theta\left(\mu_1-\mu_2\right)\right]} \\ \left(\left(\begin{array}{l} a\left(T-\mu_2\right)+\frac{1}{2}b\left(T^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(T^3-\mu_2^3\right)+\frac{1}{8}b\theta\left(T^4-\mu_2^4\right)-\frac{1}{2}a\theta\mu_2^2\left(T-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right) \\ -a\left(\mu_1-\mu_2\right)-\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right)-\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right)-\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2\left(\mu_1-\mu_2\right)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right) \end{array}\right)+a\mu_1+\frac{1}{2}b\mu_1^2\right)\mu_1 \end{array}\right)$$

$$+x\left(\begin{array}{l} \frac{1}{3}\left(-\frac{1}{2}xb-ya\right)\mu_1^3+\frac{1}{4}\left(\frac{1}{6}xb\theta+y\left(-\frac{1}{2}b+\frac{1}{2}a\theta\right)\right)\mu_2^4 \\ +\frac{1}{3}\left(\begin{array}{l} x\left(-\frac{1}{2}b+\frac{1}{2}a\theta\right) \\ +y\left(\begin{array}{l} -a-\frac{1}{\left[1+\theta\left(\mu_1-\mu_2\right)\right]} \\ \left(\left(\begin{array}{l} a\left(T-\mu_2\right)+\frac{1}{2}b\left(T^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(T^3-\mu_2^3\right)+\frac{1}{8}b\theta\left(T^4-\mu_2^4\right)-\frac{1}{2}a\theta\mu_2^2\left(T-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right) \\ -a\left(\mu_1-\mu_2\right)-\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right)-\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right)-\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2\left(\mu_1-\mu_2\right)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right) \end{array}\right)\theta\right) \\ -\frac{1}{2}b\theta\mu_1^2-a\theta\mu_1 \end{array}\right) \end{array}\right)\mu_1^3 \end{array}\right)$$

$$+\left(\frac{1}{2}\left(x\left(\begin{array}{l} -a-\frac{1}{\left[1+\theta\left(\mu_1-\mu_2\right)\right]} \\ \left(\left(\begin{array}{l} a\left(T-\mu_2\right)+\frac{1}{2}b\left(T^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(T^3-\mu_2^3\right)+\frac{1}{8}b\theta\left(T^4-\mu_2^4\right)-\frac{1}{2}a\theta\mu_2^2\left(T-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right) \\ -a\left(\mu_1-\mu_2\right)-\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right)-\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right)-\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2\left(\mu_1-\mu_2\right)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right) \end{array}\right)\theta\right)-\frac{1}{2}b\theta\mu_1^2-a\theta\mu_1 \end{array}\right)\right)\mu_2^2\right)$$

$$+\left(\frac{1}{2}\left(y\left(\begin{array}{l} a\mu_1+\frac{1}{2}b\mu_1^2+\frac{1}{2}a\theta\mu_1^2+\frac{1}{3}b\theta\mu_1^3 \\ +\frac{1}{\left[1+\theta\left(\mu_1-\mu_2\right)\right]} \\ \left(\left(\begin{array}{l} a\left(T-\mu_2\right)+\frac{1}{2}b\left(T^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(T^3-\mu_2^3\right)+\frac{1}{8}b\theta\left(T^4-\mu_2^4\right)-\frac{1}{2}a\theta\mu_2^2\left(T-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right) \\ -a\left(\mu_1-\mu_2\right)-\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right)-\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right)-\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2\left(\mu_1-\mu_2\right)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right) \end{array}\right)\left(1+\theta\mu_1\right)\right) \end{array}\right)\right)\mu_2^2\right)$$

$$+\frac{1}{2}\left(-xa+y\left(\begin{aligned}&\frac{1}{\left[1+\theta\left(\mu_1-\mu_2\right)\right]}\\&\left(\left(\begin{aligned}&a\left(T-\mu_2\right)+\frac{1}{2}b\left(T^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(T^3-\mu_2^3\right)+\frac{1}{8}b\theta\left(T^4-\mu_2^4\right)-\frac{1}{2}a\theta\mu_2^2\left(T-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right)\\&-a\left(\mu_1-\mu_2\right)-\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right)-\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right)-\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2\left(\mu_1-\mu_2\right)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right)\end{aligned}\right)+a\mu_1+\frac{1}{2}b\mu_1^2\right)\end{aligned}\right)\right)\mu_1^2$$

$$-\frac{1}{8}yb\mu_1^4-x\left(\begin{aligned}&a\mu_1+\frac{1}{2}b\mu_1^2+\frac{1}{2}a\theta\mu_1^2+\frac{1}{3}b\mu_1^3\\&+\frac{1}{\left[1+\theta\left(\mu_1-\mu_2\right)\right]}\\&\left(\left(\begin{aligned}&a\left(T-\mu_2\right)+\frac{1}{2}b\left(T^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(T^3-\mu_2^3\right)+\frac{1}{8}b\theta\left(T^4-\mu_2^4\right)-\frac{1}{2}a\theta\mu_2^2\left(T-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right)\\&-a\left(\mu_1-\mu_2\right)-\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right)-\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right)-\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2\left(\mu_1-\mu_2\right)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right)\end{aligned}\right)\left(1+\theta\mu_1\right)\right)\end{aligned}\right)\mu_1$$

(14)

(by neglecting higher powers of θ)

(iii) $DC = c\left(\int_{\mu_1}^{\mu_2}\theta I(t)dt+\int_{\mu_2}^{T}\theta tI(t)dt\right) = c\left(\int_{\mu_1}^{\mu_2}\theta I(t)dt+\int_{\mu_2}^{T}\theta tI(t)dt\right)$

$$=c\theta\left(\begin{aligned}&a\left(\mu_1\mu_2-\frac{1}{2}\mu_2^2\right)+\frac{1}{2}b\left(\mu_1^2\mu_2-\frac{1}{3}\mu_2^3\right)+\frac{1}{2}a\theta\left(\mu_1^2\mu_2-\frac{1}{3}\mu_2^3\right)+\frac{1}{3}b\theta\left(\mu_1^3\mu_2-\frac{1}{4}\mu_2^4\right)\\&-a\theta\left(\frac{1}{2}\mu_1\mu_2^2-\frac{1}{3}\mu_2^3\right)-\frac{1}{2}b\theta\left(\frac{1}{2}\mu_1^2\mu_2^2-\frac{1}{4}\mu_2^4\right)\\&+\frac{1}{1+\theta(\mu_1-\mu_2)}\left(\left(\begin{aligned}&a\left(T-\mu_2\right)+\frac{1}{2}b\left(T^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(T^3-\mu_2^3\right)+\frac{1}{8}b\theta\left(T^4-\mu_2^4\right)-\frac{1}{2}a\theta\mu_2^2\left(T-\mu_2\right)\\&-\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right)-a(\mu_1-\mu_2)-\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right)-\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right)-\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)\\&+a\theta\mu_2(\mu_1-\mu_2)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right)\end{aligned}\right)\left(\mu_2+\theta\left(\mu_1\mu_2-\frac{1}{2}\mu_2^2\right)\right)\right)\end{aligned}\right)$$

$$-c\theta\left(\begin{aligned}&\frac{1}{2}\mu_1^2+\frac{1}{3}b\mu_1^3+\frac{1}{6}a\theta\mu_1^3+\frac{1}{8}b\theta\mu_1^4\\&+\frac{1}{1+\theta(\mu_1-\mu_2)}\left(\left(\begin{aligned}&a\left(T-\mu_2\right)+\frac{1}{2}b\left(T^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(T^3-\mu_2^3\right)+\frac{1}{8}b\theta\left(T^4-\mu_2^4\right)-\frac{1}{2}a\theta\mu_2^2\left(T-\mu_2\right)\\&-\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right)-a(\mu_1-\mu_2)-\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right)-\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right)-\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)\\&+a\theta\mu_2(\mu_1-\mu_2)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right)\end{aligned}\right)\left(\mu_1+\frac{1}{2}\theta\mu_1^2\right)\right)\end{aligned}\right)$$

$$+c\theta\left(\begin{aligned}&\frac{1}{48}b\theta T^6+\frac{1}{15}a\theta T^5+\frac{1}{4}\left(-\frac{1}{2}b-\frac{1}{2}a\theta T-\frac{1}{4}b\theta T^2\right)T^4\\&-\frac{1}{3}aT^3+\frac{1}{2}\left(aT+\frac{1}{2}bT^2+\frac{1}{6}a\theta T^3+\frac{1}{8}b\theta T^4\right)T^2\end{aligned}\right)-c\theta\left(\begin{aligned}&\frac{1}{48}b\theta\mu_2^6+\frac{1}{15}a\theta\mu_2^5+\frac{1}{4}\left(-\frac{1}{2}b-\frac{1}{2}a\theta T-\frac{1}{4}b\theta T^2\right)\mu_2^4\\&-\frac{1}{3}a\mu_2^3+\frac{1}{2}\left(aT+\frac{1}{2}bT^2+\frac{1}{6}a\theta T^3+\frac{1}{8}b\theta T^4\right)\mu_2^2\end{aligned}\right)\quad(15)$$

(iv) $SR = p\left(\int_0^T (a+bt)dt\right) = p\left(aT + \frac{1}{2}bT^2\right)$ (16)

The total profit (π) during a cycle consisted of the following:

$$\pi = \frac{1}{T}[SR - OC - HC - DC] \quad (17)$$

Substituting values from equations (13) to (16) in equation (17), we get total profit per unit. Putting $\mu_1 = v_1T$ and $\mu_2 = v_2T$ in equation (17), we get profit in terms of T. Differentiating equation (17) with respect to T and equate it to zero, we have

i.e. $\frac{d\pi}{dT} = 0$ (18)

provided it satisfies the condition

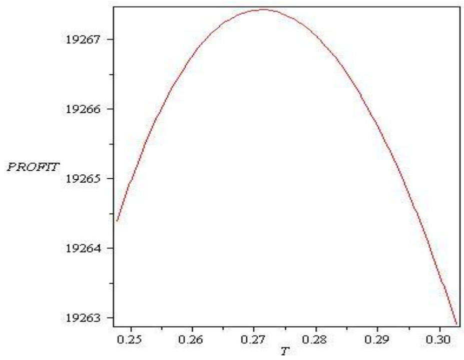
$$\frac{d^2\pi}{dT^2} < 0. \quad (19)$$

IV. NUMERICAL EXAMPLE

Considering A= Rs.100, a = 500, b=0.05, c=Rs. 25, p= Rs. 40, θ=0.05, x = Rs. 5, y=0.05, v_1=0.30, v_2 = 0.50, in appropriate units. The optimal value of T* =0.2712, Profit*= Rs. 19267.4378 and optimum order quantity Q* = 135.8647.

The second order conditions given in equation (19) are also satisfied. The graphical representation of the concavity of the profit function is also given.

T and Profit



Graph 1

V. SENSITIVITY ANALYSIS

On the basis of the data given in example above we have studied the sensitivity analysis by changing the following parameters one at a time and keeping the rest fixed.

Table 1
Sensitivity Analysis

Parameter	%	T	Profit	Q
a	+10%	0.2478	23197.9387	148.9413
	+5%	0.2587	21231.8970	142.5479
	-5%	0.2856	17304.8056	128.7867
	-10%	0.3027	15344.3148	121.3492
θ	+10%	0.2691	19262.4937	134.8622
	+5%	0.2701	19264.9604	135.3386
	-5%	0.2722	19269.9261	136.3403
	-10%	0.2733	19272.4253	136.8656
x	+10%	0.2494	19202.3688	124.9210
	+5%	0.2596	19234.2216	130.0409
	-5%	0.2844	19302.2070	142.4934
	-10%	0.2998	19338.7663	150.2290
A	+10%	0.2967	19197.0030	148.6716
	+5%	0.2842	19231.4302	142.3930
	-5%	0.2574	19305.2698	128.9365
	-10%	0.2429	19345.2402	121.6588

From the table we observe that as parameter a increases/ decreases average total profit and optimum order quantity also increases/ decreases.
Also, we observe that with increase and decrease in the value of θ and x, there is corresponding decrease/ increase in total profit and optimum order quantity.
From the table we observe that as parameter A increases/ decreases average total profit decreases/ increases and optimum order quantity increases/ decreases.

VI. CONCLUSION

In this paper, we have developed an inventory model for deteriorating items with linear demand with different deterioration rates. Sensitivity with respect to parameters have been carried out. The results show that with the increase/ decrease in the parameter values there is corresponding increase/ decrease in the value of profit.

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