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Inventory Model with Different Deterioration Rates with Stock and Price Dependent Demand under Time Varying Holding Cost and Shortages

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ABSTRACT: *An inventory model for deteriorating items with stock and price dependent demand is developed. Holding cost is considered as function of time. Shortages are allowed and completely backlogged. Numerical example is provided to illustrate the model and sensitivity analysis is also carried out for parameters.*

KEY WORDS: *Inventory model, Deterioration, Price dependent demand, Stock dependent demand, Time varying holding cost, Shortages*

I. INTRODUCTION

In real life, deterioration of items is a general phenomenon for many inventory systems and therefore deterioration effect cannot be ignored. Many researchers have studied EOQ models for deteriorating items in past. Ghare and Schrader [2] considered no-shortage inventory model with constant rate of deterioration. The model was extended by Covert and Philip [1] by considering variable rate of deterioration. By considering shortages, the model was further extended by Shah and Jaiswal [14]. The related work are found in (Nahmias [9], Raffat [12], Goyal and Giri [3], Ouyang et al. [10], Wu et al. [16]).

Hill [4] considered inventory model with ramp type demand rate. Mandal and Pal [6] developed inventory model with ramp type demand with shortages. Hung [5] considered inventory model with arbitrary demand and arbitrary deterioration rate. Salameh and Jaber [13] developed a model to determine the total profit per unit of time and the economic order quantity for a product purchased from the supplier. Mukhopadhyay et al. [8] developed an inventory model for deteriorating items with a price-dependent demand rate. The rate of deterioration was taken to be time-proportional and a power law form of the price-dependence of demand was considered. Teng and Chang [15] considered the economic production quantity model for deteriorating items with stock level and selling price dependent demand. Mathew [7] developed an inventory model for deteriorating items with mixture of Weibull rate of decay and demand as function of both selling price and time. Patel and Parekh [11] developed an inventory model with stock dependent demand under shortages and variable selling price.

Inventory models for non-instantaneous deteriorating items have been an object of study for a long time. Generally the products are such that there is no deterioration initially. After certain time deterioration starts and again after certain time the rate of deterioration increases with time. Here we have used such a concept and developed the deteriorating items inventory models.

In this paper we have developed an inventory model with stock and price dependent demand with different deterioration rates for the cycle time. Shortages are allowed and completely backlogged. To illustrate the model, numerical example is taken and sensitivity analysis for major parameters on the optimal solutions is also carried out.

II. ASSUMPTIONS AND NOTATIONS

The following notations are used for the development of the model:

NOTATIONS:

D(t) : Demand rate is a linear function of price and inventory level ($a + bI(t) - \rho p$, $a>0$, $0<b<1$, $\rho>0$)

A : Replenishment cost per order

c : Purchasing cost per unit

p : Selling price per unit

T : Length of inventory cycle

I(t) : Inventory level at any instant of time t, $0 \le t \le T$

Q_1 : Order quantity intially

Q_2 : Shortages of quantity
Q : Order quantity
θ : Deterioration rate during $\mu_1 \le t \le \mu_2$, $0 < \theta < 1$
θt : Deterioration rate during , $\mu_2 \le t \le T$, $0 < \theta < 1$
c_2 : Shortage cost per unit item
π : Total relevant profit per unit time.

ASSUMPTIONS:

The following assumptions are considered for the development of the model.

- The demand of the product is declining as a function of price and inventory level.
- Replenishment rate is infinite and instantaneous.
- Lead time is zero.
- Shortages are permitted and completely backlogged.
- Deteriorated units neither be repaired nor replaced during the cycle time.

III. THE MATHEMATICAL MODEL AND ANALYSIS

Let I(t) be the inventory at time t ($0 \le t \le T$) as shown in figure.

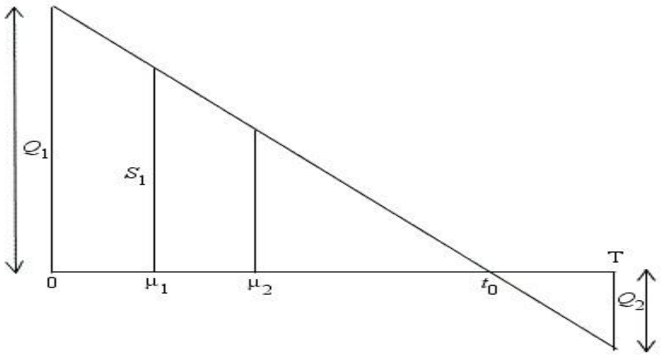


Figure 1

The differential equations which describes the instantaneous states of I(t) over the period (0, T) are given by :

$$\frac{dI(t)}{dt} = -(a + bI(t) - \rho p), \qquad 0 \le t \le \mu_1 \tag{1}$$

$$\frac{dI(t)}{dt} + \theta I(t) = -(a + bI(t) - \rho p), \qquad \mu_1 \le t \le \mu_2 \tag{2}$$

$$\frac{dI(t)}{dt} + \theta t I(t) = -(a + bI(t) - \rho p), \qquad \mu_2 \le t \le t_0 \tag{3}$$

$$\frac{dI(t)}{dt} = -(a + bI(t) - \rho p), \qquad t_0 \le t \le T \tag{4}$$

with initial conditions $I(0) = Q_1$, $I(\mu_1) = S_1$, $I(t_0) = 0$, and $I(T) = -Q_2$.
Solutions of these equations are given by

$$I(t) = Q_1(1 - bt) - (at + \frac{1}{2}bt^2 - \rho pt - \frac{1}{2}\rho bpt^2 - abt^2 + \rho bpt^2), \tag{5}$$

$$I(t) = \begin{bmatrix} a(\mu_1 - t) - \rho p(\mu_1 - t) + \frac{1}{2}a(\theta+b)(\mu_1^2 - t^2) - \frac{1}{2}\rho p(\theta+b)(\mu_1^2 - t^2) \\ - a(\theta+b)t(\mu_1 - t) + \rho p(\theta+b)t(\mu_1 - t) - ab\theta t(\mu_1^2 - t^2) + \rho pb\theta t(\mu_1^2 - t^2) \end{bmatrix} + S_1\left[1 + (\theta+b)(\mu_1 - t)\right] \tag{6}$$

$$I(t) = \begin{bmatrix} a(t_0-t) - \rho p(t_0-t) + \frac{1}{2}ab(t_0^2-t^2) - \frac{1}{2}\rho pb(t_0^2-t^2) + \frac{1}{6}a\theta(t_0^3-t^3) - \frac{1}{6}\rho p\theta(t_0^3-t^3) - abt(t_0-t) + b\rho pt(t_0-t) \\ - \frac{1}{6}ab\theta t(t_0^3-t^3) + \frac{1}{6}\rho bp\theta t(t_0^3-t^3) - \frac{1}{2}a\theta t^2(t_0-t) + \frac{1}{2}\rho p\theta t^2(t_0-t) + \frac{1}{4}\rho bp\theta t^2(t_0^2-t^2) - \frac{1}{4}ab\theta t^2(t_0^2-t^2) \end{bmatrix} \tag{7}$$

$$I(t) = \left[a(t_0 - t) - \rho p(t_0 - t) + \frac{1}{2}ab(t_0^2 - t^2) - \frac{1}{2}\rho pb(t_0^2 - t^2) - abt(t_0 - t) + b\rho pt(t_0 - t)\right]. \tag{8}$$

(by neglecting higher powers of θ)

From equation (5), putting $t = \mu_1$, we have

$$Q_1 = \frac{S_1}{(1-b\mu_1)} + \frac{1}{(1-b\mu_1)}\left(a\mu_1 - \rho p\mu_1 + \frac{1}{2}b\mu_1^2 - \frac{1}{2}b\rho p\mu_1^2 - ab\mu_1^2 + \rho bp\mu_1^2\right). \tag{9}$$

From equations (6) and (7), putting $t = \mu_2$, we have

$$I(\mu_2) = \begin{bmatrix} a(\mu_1-\mu_2) - \rho p(\mu_1-\mu_2) + \frac{1}{2}a(\theta+b)(\mu_1^2-\mu_2^2) - \frac{1}{2}\rho p(\theta+b)(\mu_1^2-\mu_2^2) \\ -a(\theta+b)\mu_2(\mu_1-\mu_2) + \rho p(\theta+b)\mu_2(\mu_1-\mu_2) - ab\theta\mu_2(\mu_1^2-\mu_2^2) + b\rho\theta p\mu_2(\mu_1^2-\mu_2^2) \end{bmatrix} + S_1\left[1+(\theta+b)(\mu_1-\mu_2)\right] \tag{10}$$

$$I(\mu_2) = \begin{bmatrix} a(t_0-\mu_2) - \rho p(t_0-\mu_2) + \frac{1}{2}ab(t_0^2-\mu_2^2) - \frac{1}{2}\rho bp(t_0^2-\mu_2^2) + \frac{1}{6}a\theta(t_0^3-\mu_2^3) - \frac{1}{6}\rho\theta p(t_0^3-\mu_2^3) \\ -ab\mu_2(t_0-\mu_2) + b\rho p\mu_2(t_0-\mu_2) - \frac{1}{6}ab\theta\mu_2(t_0^3-\mu_2^3) + \frac{1}{6}\rho b\theta p\mu_2(t_0^3-\mu_2^3) - \frac{1}{2}a\theta\mu_2^2(t_0-\mu_2) \\ + \frac{1}{2}\rho\theta p\mu_2^2(t_0-\mu_2) + \frac{1}{4}\rho b\theta p\mu_2^2(t_0^2-\mu_2^2) - \frac{1}{4}ab\theta\mu_2^2(t_0^2-\mu_2^2) \end{bmatrix}. \tag{11}$$

So from equations (10) and (11), we get

$$S_1 = \frac{1}{\left[1+(\theta+b)(\mu_1-\mu_2)\right]} \begin{bmatrix} a(t_0-\mu_2) - \rho p(t_0-\mu_2) + \frac{1}{2}ab(t_0^2-\mu_2^2) - \frac{1}{2}\rho bp(t_0^2-\mu_2^2) + \frac{1}{6}a\theta(t_0^3-\mu_2^3) - \frac{1}{6}\rho\theta p(t_0^3-\mu_2^3) \\ -ab\mu_2(t_0-\mu_2) + b\rho p\mu_2(t_0-\mu_2) - \frac{1}{6}ab\theta\mu_2(t_0^3-\mu_2^3) + \frac{1}{6}\rho b\theta p\mu_2(t_0^3-\mu_2^3) - \frac{1}{2}a\theta\mu_2^2(t_0-\mu_2) \\ + \frac{1}{2}\rho\theta p\mu_2^2(t_0-\mu_2) + \frac{1}{4}\rho b\theta p\mu_2^2(t_0^2-\mu_2^2) - \frac{1}{4}ab\theta\mu_2^2(t_0^2-\mu_2^2) - a(\mu_1-\mu_2) + \rho p(\mu_1-\mu_2) - \frac{1}{2}a(\theta+b)(\mu_1^2-\mu_2^2) \\ + \frac{1}{2}\rho p(\theta+b)(\mu_1^2-\mu_2^2) + a(\theta+b)\mu_2(\mu_1-\mu_2) - \rho p(\theta+b)\mu_2(\mu_1-\mu_2) + ab\theta\mu_2(\mu_1^2-\mu_2^2) - b\rho\theta p\mu_2(\mu_1^2-\mu_2^2) \end{bmatrix}. \tag{12}$$

Putting value of S_1 from equation (12) into equation (9), we have

$$Q_1 = \frac{1}{(1-b\mu_1)\left[1+(\theta+b)(\mu_1-\mu_2)\right]} \begin{bmatrix} a(t_0-\mu_2) - \rho p(t_0-\mu_2) + \frac{1}{2}ab(t_0^2-\mu_2^2) - \frac{1}{2}\rho bp(t_0^2-\mu_2^2) + \frac{1}{6}a\theta(t_0^3-\mu_2^3) - \frac{1}{6}\rho\theta p(t_0^3-\mu_2^3) - ab\mu_2(t_0-\mu_2) \\ + b\rho p\mu_2(t_0-\mu_2) - \frac{1}{6}ab\theta\mu_2(t_0^3-\mu_2^3) + \frac{1}{6}\rho b\theta p\mu_2(t_0^3-\mu_2^3) - \frac{1}{2}a\theta\mu_2^2(t_0-\mu_2) + \frac{1}{2}\rho\theta p\mu_2^2(t_0-\mu_2) \\ + \frac{1}{4}\rho b\theta p\mu_2^2(t_0^2-\mu_2^2) - \frac{1}{4}ab\theta\mu_2^2(t_0^2-\mu_2^2) - a(\mu_1-\mu_2) + \rho p(\mu_1-\mu_2) - \frac{1}{2}a(\theta+b)(\mu_1^2-\mu_2^2) + \frac{1}{2}\rho p(\theta+b)(\mu_1^2-\mu_2^2) \\ + a(\theta+b)\mu_2(\mu_1-\mu_2) - \rho p(\theta+b)\mu_2(\mu_1-\mu_2) + ab\theta\mu_2(\mu_1^2-\mu_2^2) - b\rho\theta p\mu_2(\mu_1^2-\mu_2^2) \end{bmatrix} \tag{13}$$

$$+ \frac{\left(a\mu_1 - \rho p\mu_1 + \frac{1}{2}b\mu_1^2 + \frac{1}{2}b\rho p\mu_1^2 - ab\mu_1^2\right)}{(1-b\mu_1)}.$$

Putting $t = T$ in equation (8), we have

$$Q_2 = \left[a(T-t_0) - \rho p(T-t_0) + \frac{1}{2}ab(T^2-t_0^2) - \frac{1}{2}\rho pb(T^2-t_0^2) - abT(T-t_0) + b\rho pT(T-t_0)\right].. \tag{14}$$

Using (13) in (5), we have

$$
\begin{aligned}
I(t) = {} & \frac{(1-bt)}{(1-b\mu_1)\left[1+(\theta+b)(\mu_1-\mu_2)\right]} \\
& \left[\begin{aligned}
& a(t_0-\mu_2)-\rho p(t_0-\mu_2)+\frac{1}{2}ab\left(t_0^2-\mu_2^2\right)-\frac{1}{2}\rho bp\left(t_0^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{6}\rho\theta p\left(t_0^3-\mu_2^3\right)-ab\mu_2(t_0-\mu_2) \\
& +b\rho p\mu_2(t_0-\mu_2)-\frac{1}{6}ab\theta\mu_2\left(t_0^3-\mu_2^3\right)+\frac{1}{6}\rho b\theta p\mu_2\left(t_0^3-\mu_2^3\right)-\frac{1}{2}a\theta\mu_2^2(t_0-\mu_2)+\frac{1}{2}\rho\theta p\mu_2^2(t_0-\mu_2) \\
& +\frac{1}{4}\rho b\theta p\mu_2^2\left(t_0^2-\mu_2^2\right)-\frac{1}{4}ab\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-a(\mu_1-\mu_2)+\rho p(\mu_1-\mu_2)-\frac{1}{2}a(\theta+b)\left(\mu_1^2-\mu_2^2\right)+\frac{1}{2}\rho p(\theta+b)\left(\mu_1^2-\mu_2^2\right) \\
& +a(\theta+b)\mu_2(\mu_1-\mu_2)-\rho p(\theta+b)\mu_2(\mu_1-\mu_2)+ab\theta\mu_2\left(\mu_1^2-\mu_2^2\right)-b\rho\theta p\mu_2\left(\mu_1^2-\mu_2^2\right)
\end{aligned}\right] \\
& +\frac{(1-bt)\left(a\mu_1-\rho p\mu_1+\frac{1}{2}b\mu_1^2+\frac{1}{2}b\rho p\mu_1^2-ab\mu_1^2\right)}{(1-b\mu_1)}-\left(at+\frac{1}{2}bt^2-\rho pt+\frac{1}{2}\rho bpt^2+abt^2\right)
\end{aligned}
\tag{15}
$$

Based on the assumptions and descriptions of the model, the total relevant profit (π), include the following elements:

(i) Ordering cost (OC) = A (16)

(ii) $HC = \int_0^{t_0}(x+yt)I(t)dt = \int_0^{\mu_1}(x+yt)I(t)dt + \int_{\mu_1}^{\mu_2}(x+yt)I(t)dt + \int_{\mu_2}^{t_0}(x+yt)I(t)dt$

$$
\begin{aligned}
= {} & x\left(at_0-\rho pt_0+\frac{1}{2}abt_0^2-\frac{1}{2}\rho bpt_0^2+\frac{1}{6}a\theta t_0^3-\frac{1}{6}\rho\theta pt_0^3\right)t_0+\frac{1}{6}y\left(\frac{5}{12}ab\theta-\frac{5}{12}\rho bp\right)t_0^6 \\
& +x\left(\begin{aligned}
& a\mu_1-\rho p\mu_1 \\
& +\left(\begin{aligned}
& \frac{1}{\left[1+(\theta+b)(\mu_1-\mu_2)\right]} \\
& \left[\begin{aligned}
& a(t_0-\mu_2)-\rho p(t_0-\mu_2)+\frac{1}{2}ab\left(t_0^2-\mu_2^2\right)-\frac{1}{2}\rho bp\left(t_0^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{6}\rho\theta p\left(t_0^3-\mu_2^3\right)-ab\mu_2(t_0-\mu_2) \\
& +b\rho p\mu_2(t_0-\mu_2)-\frac{1}{6}ab\theta\mu_2\left(t_0^3-\mu_2^3\right)+\frac{1}{6}\rho b\theta p\mu_2\left(t_0^3-\mu_2^3\right)-\frac{1}{2}a\theta\mu_2^2(t_0-\mu_2)+\frac{1}{2}\rho\theta p\mu_2^2(t_0-\mu_2) \\
& +\frac{1}{4}\rho b\theta p\mu_2^2\left(t_0^2-\mu_2^2\right)-\frac{1}{4}ab\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-a(\mu_1-\mu_2)+\rho p(\mu_1-\mu_2)-\frac{1}{2}a(\theta+b)\left(\mu_1^2-\mu_2^2\right) \\
& +\frac{1}{2}\rho p(\theta+b)\left(\mu_1^2-\mu_2^2\right)+a(\theta+b)\mu_2(\mu_1-\mu_2)-\rho p(\theta+b)\mu_2(\mu_1-\mu_2)+ab\theta\mu_2\left(\mu_1^2-\mu_2^2\right)-b\rho\theta p\mu_2\left(\mu_1^2-\mu_2^2\right)
\end{aligned}\right] \\
& \left(1+(\theta+b)\mu_1\right)+\frac{1}{2}a(\theta+b)\mu_1^2-\frac{1}{2}\rho p(\theta+b)\mu_1^2
\end{aligned}\right)
\end{aligned}\right)\mu_2
\end{aligned}
$$

$$
-x\left(\begin{aligned}
& a\mu_1-\rho p\mu_1 \\
& +\frac{1}{\left[1+(\theta+b)(\mu_1-\mu_2)\right]} \\
& \left[\begin{aligned}
& a(t_0-\mu_2)-\rho p(t_0-\mu_2)+\frac{1}{2}ab\left(t_0^2-\mu_2^2\right)-\frac{1}{2}\rho bp\left(t_0^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{6}\rho\theta p\left(t_0^3-\mu_2^3\right)-ab\mu_2(t_0-\mu_2) \\
& +b\rho p\mu_2(t_0-\mu_2)-\frac{1}{6}ab\theta\mu_2\left(t_0^3-\mu_2^3\right)+\frac{1}{6}\rho b\theta p\mu_2\left(t_0^3-\mu_2^3\right)-\frac{1}{2}a\theta\mu_2^2(t_0-\mu_2)+\frac{1}{2}\rho\theta p\mu_2^2(t_0-\mu_2) \\
& +\frac{1}{4}\rho b\theta p\mu_2^2\left(t_0^2-\mu_2^2\right)-\frac{1}{4}ab\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-a(\mu_1-\mu_2)+\rho p(\mu_1-\mu_2)-\frac{1}{2}a(\theta+b)\left(\mu_1^2-\mu_2^2\right)+\frac{1}{2}\rho p(\theta+b)\left(\mu_1^2-\mu_2^2\right) \\
& +a(\theta+b)\mu_2(\mu_1-\mu_2)-\rho p(\theta+b)\mu_2(\mu_1-\mu_2)+ab\theta\mu_2\left(\mu_1^2-\mu_2^2\right)-b\rho\theta p\mu_2\left(\mu_1^2-\mu_2^2\right)
\end{aligned}\right] \\
& \left(\left(1+(\theta+b)\mu_1+\frac{1}{2}a(\theta+b)\mu_1^2-\frac{1}{2}\rho p(\theta+b)\mu_1^2\right)\mu_1\right)
\end{aligned}\right)
$$

$$\left(\begin{array}{l} -\frac{1}{3}\left(\begin{array}{l} x\left(-\frac{1}{2}\rho bp+\frac{1}{2}\rho\theta pt_0+\frac{1}{4}\rho b\theta pt_0^2+\frac{1}{2}ab-\frac{1}{2}a\theta t_0-\frac{1}{4}ab\theta t_0^2\right)\\ +y\left(-a+\rho p+\frac{1}{6}\rho b\theta pt_0^3-abt_0+\rho bpt_0-\frac{1}{6}ab\theta t_0^3\right)\end{array}\right)\mu_2^3\\ -\frac{1}{4}\left(x\left(\frac{1}{3}a\theta-\frac{1}{3}\rho\theta p\right)+y\left(-\frac{1}{2}\rho bp+\frac{1}{2}\rho\theta pt_0+\frac{1}{4}\rho b\theta pt_0^2+\frac{1}{2}ab-\frac{1}{2}a\theta t_0-\frac{1}{4}ab\theta t_0^2\right)\right)\mu_2^4\\ -\frac{1}{5}\left(x\left(\frac{5}{12}ab\theta-\frac{5}{12}\rho b\theta p\right)+y\left(\frac{1}{3}a\theta-\frac{1}{3}\rho\theta p\right)\right)\mu_2^5\\ +\frac{1}{2}\left(x\left(-a+\rho p+\frac{1}{6}\rho b\theta pt_0^3-abt_0+\rho bpt_0-\frac{1}{6}ab\theta t_0^3\right)+y\left(at_0-\rho pt_0+\frac{1}{2}abt_0^2-\frac{1}{2}\rho bpt_0^2+\frac{1}{6}a\theta t_0^3-\frac{1}{6}\rho\theta t_0^3\right)\right)t_0^2\\ -\frac{1}{4}\left(x\left(ab\theta-\rho b\theta p\right)+y\left(\frac{1}{2}a(\theta+b)-\frac{1}{2}\rho p(\theta+b)\right)\right)\mu_1^4 \end{array}\right)$$

$$-\frac{1}{3}\left(\begin{array}{l} x\left(\frac{1}{2}a(\theta+b)-\frac{1}{2}\rho p(\theta+b)\right)\\ +y\left(\begin{array}{l} -a+\frac{1}{\left[1+(\theta+b)(\mu_1-\mu_2)\right]}\\ \left[\begin{array}{l} a(t_0-\mu_2)-\rho p(t_0-\mu_2)+\frac{1}{2}ab(t_0^2-\mu_2^2)-\frac{1}{2}\rho bp(t_0^2-\mu_2^2)+\frac{1}{6}a\theta(t_0^3-\mu_2^3)-\frac{1}{6}\rho\theta p(t_0^3-\mu_2^3)-ab\mu_2(t_0-\mu_2)\\ +b\rho p\mu_2(t_0-\mu_2)-\frac{1}{6}ab\theta\mu_2(t_0^3-\mu_2^3)+\frac{1}{6}\rho b\theta p\mu_2(t_0^3-\mu_2^3)-\frac{1}{2}a\theta\mu_2^2(t_0-\mu_2)+\frac{1}{2}\rho\theta p\mu_2^2(t_0-\mu_2)\\ +\frac{1}{4}\rho b\theta p\mu_2^2(t_0^2-\mu_2^2)-\frac{1}{4}ab\theta\mu_2^2(t_0^2-\mu_2^2)-a(\mu_1-\mu_2)+\rho p(\mu_1-\mu_2)-\frac{1}{2}a(\theta+b)(\mu_1^2-\mu_2^2)+\frac{1}{2}\rho p(\theta+b)(\mu_1^2-\mu_2^2)\\ +a(\theta+b)\mu_2(\mu_1-\mu_2)-\rho p(\theta+b)\mu_2(\mu_1-\mu_2)+ab\theta\mu_2(\mu_1^2-\mu_2^2)-b\rho\theta p\mu_2(\mu_1^2-\mu_2^2) \end{array}\right]\\ (-\theta-b)-a(\theta+b)\mu_1+\rho p(\theta+b)\mu_1-ab\theta\mu_1^2+\rho b\theta p\mu_1^2+\rho p \end{array}\right) \end{array}\right)\mu_1^3$$

$$/-\frac{1}{2}\left(x\left(\begin{array}{l} -a+\frac{1}{\left[1+(\theta+b)(\mu_1-\mu_2)\right]}\\ \left[\begin{array}{l} a(t_0-\mu_2)-\rho p(t_0-\mu_2)+\frac{1}{2}ab(t_0^2-\mu_2^2)-\frac{1}{2}\rho bp(t_0^2-\mu_2^2)+\frac{1}{6}a\theta(t_0^3-\mu_2^3)-\frac{1}{6}\rho\theta p(t_0^3-\mu_2^3)-ab\mu_2(t_0-\mu_2)\\ +b\rho p\mu_2(t_0-\mu_2)-\frac{1}{6}ab\theta\mu_2(t_0^3-\mu_2^3)+\frac{1}{6}\rho b\theta p\mu_2(t_0^3-\mu_2^3)-\frac{1}{2}a\theta\mu_2^2(t_0-\mu_2)+\frac{1}{2}\rho\theta p\mu_2^2(t_0-\mu_2)\\ +\frac{1}{4}\rho b\theta p\mu_2^2(t_0^2-\mu_2^2)-\frac{1}{4}ab\theta\mu_2^2(t_0^2-\mu_2^2)-a(\mu_1-\mu_2)+\rho p(\mu_1-\mu_2)-\frac{1}{2}a(\theta+b)(\mu_1^2-\mu_2^2)+\frac{1}{2}\rho p(\theta+b)(\mu_1^2-\mu_2^2)\\ +a(\theta+b)\mu_2(\mu_1-\mu_2)-\rho p(\theta+b)\mu_2(\mu_1-\mu_2)+ab\theta\mu_2(\mu_1^2-\mu_2^2)-b\rho\theta p\mu_2(\mu_1^2-\mu_2^2) \end{array}\right]\\ (-\theta-b)-a(\theta+b)\mu_1+\rho p(\theta+b)\mu_1-ab\theta\mu_1^2+\rho b\theta p\mu_1^2+\rho p \end{array}\right)\right)$$

$$-\frac{1}{2}\left(y \left(\begin{array}{l} a\mu_1 - \rho p\mu_1 + \frac{1}{\left[1+(\theta+b)(\mu_1-\mu_2)\right]} \\ \left[\begin{array}{l} a(t_0-\mu_2) - \rho p(t_0-\mu_2) + \frac{1}{2} ab(t_0^2-\mu_2^2) - \frac{1}{2} \rho bp(t_0^2-\mu_2^2) + \frac{1}{6} a\theta(t_0^3-\mu_2^3) - \frac{1}{6}\rho\theta p(t_0^3-\mu_2^3) - ab\mu_2(t_0-\mu_2) \\ + b\rho p\mu_2(t_0-\mu_2) - \frac{1}{6} ab\theta\mu_2(t_0^3-\mu_2^3) + \frac{1}{6}\rho b\theta p\mu_2(t_0^3-\mu_2^3) - \frac{1}{2} a\theta\mu_2^2(t_0-\mu_2) + \frac{1}{2}\rho\theta p\mu_2^2(t_0-\mu_2) \\ + \frac{1}{4}\rho b\theta p\mu_2^2(t_0^2-\mu_2^2) - \frac{1}{4} ab\theta\mu_2^2(t_0^2-\mu_2^2) - a(\mu_1-\mu_2) + \rho p(\mu_1-\mu_2) - \frac{1}{2} a(\theta+b)(\mu_1^2-\mu_2^2) + \frac{1}{2}\rho p(\theta+b)(\mu_1^2-\mu_2^2) \\ + a(\theta+b)\mu_2(\mu_1-\mu_2) - \rho p(\theta+b)\mu_2(\mu_1-\mu_2) + ab\theta\mu_2(\mu_1^2-\mu_2^2) - b\rho\theta p\mu_2(\mu_1^2-\mu_2^2) \end{array}\right] \\ \left(1+(\theta+b)\mu_1 + \frac{1}{2} a(\theta+b)\mu_1^2 - \frac{1}{2}\rho p(\theta+b)\mu_1^2\right) \end{array}\right) \mu_1^2 \right)$$

$$+\frac{1}{3}\left(\begin{array}{l} x\left(-\frac{1}{2}\rho bp + \frac{1}{2}\rho\theta pt_0 + \frac{1}{4}\rho b\theta pt_0^2 + \frac{1}{2} ab - \frac{1}{2} a\theta t_0 - \frac{1}{4} ab\theta t_0^2\right) \\ + y\left(-a+\rho p + \frac{1}{6}\rho b\theta pT^3 - abt_0 + \rho bpt_0 - \frac{1}{6} ab\theta T^3\right)\end{array}\right)t_0^3 + \frac{1}{5}\left(x\left(\frac{5}{12}ab\theta - \frac{5}{12}\rho b\theta p\right) + y\left(\frac{1}{3} a\theta - \frac{1}{3}\rho\theta p\right)\right)t_0^5$$

$$+\frac{1}{4}\left(x\left(\frac{1}{3} a\theta - \frac{1}{3}\rho\theta p\right) + y\left(-\frac{1}{2}\rho bp + \frac{1}{2}\rho\theta pt_0 + \frac{1}{4}\rho b\theta pt_0^2 + \frac{1}{2} ab - \frac{1}{2} a\theta t_0 - \frac{1}{4} ab\theta t_0^2\right)\right)t_0^4$$

$$+ x\left(\begin{array}{l} \frac{1}{(1-b\mu_1)\left[1+(\theta+b)(\mu_1-\mu_2)\right]} \\ \left[\begin{array}{l} a(t_0-\mu_2) - \rho p(t_0-\mu_2) + \frac{1}{2} ab(t_0^2-\mu_2^2) - \frac{1}{2}\rho bp(t_0^2-\mu_2^2) + \frac{1}{6} a\theta(t_0^3-\mu_2^3) - \frac{1}{6}\rho\theta p(t_0^3-\mu_2^3) - ab\mu_2(t_0-\mu_2) \\ + b\rho p\mu_2(t_0-\mu_2) - \frac{1}{6} ab\theta\mu_2(t_0^3-\mu_2^3) + \frac{1}{6}\rho b\theta p\mu_2(t_0^3-\mu_2^3) - \frac{1}{2} a\theta\mu_2^2(t_0-\mu_2) + \frac{1}{2}\rho\theta p\mu_2^2(t_0-\mu_2) \\ + \frac{1}{4}\rho b\theta p\mu_2^2(t_0^2-\mu_2^2) - \frac{1}{4} ab\theta\mu_2^2(t_0^2-\mu_2^2) - a(\mu_1-\mu_2) + \rho p(\mu_1-\mu_2) - \frac{1}{2} a(\theta+b)(\mu_1^2-\mu_2^2) \\ + \frac{1}{2}\rho p(\theta+b)(\mu_1^2-\mu_2^2) + a(\theta+b)\mu_2(\mu_1-\mu_2) - \rho p(\theta+b)\mu_2(\mu_1-\mu_2) + ab\theta\mu_2(\mu_1^2-\mu_2^2) - b\rho\theta p\mu_2(\mu_1^2-\mu_2^2) \end{array}\right] \\ + \frac{a\mu_1 - \rho p\mu_1 + \frac{1}{2} b\mu_1^2 + \frac{1}{2}\rho bp\mu_1^2 - ab\mu_1^2}{(1-b\mu_1)} \end{array}\right)\mu_1$$

$$\left(\begin{array}{l} + \frac{1}{4} y\left(-\frac{1}{2} b - \frac{1}{2}\rho bp + ab\right)\mu_1^4 + \frac{1}{5} y(ab\theta - \rho b\theta p)\mu_2^5 \\ + \frac{1}{4}\left(x(ab\theta - \rho b\theta p) + y\left(\frac{1}{2} a(\theta+b) - \frac{1}{2}\rho p(\theta+b)\right)\right)\mu_2^4 + \frac{1}{3}\left(x\left(\frac{1}{2} a(\theta+b) - \frac{1}{2}\rho p(\theta+b)\right)\right) \\ + y\left(\begin{array}{l} -a + \frac{1}{\left[1+(\theta+b)(\mu_1-\mu_2)\right]} \\ \left[\begin{array}{l} a(t_0-\mu_2) - \rho p(t_0-\mu_2) + \frac{1}{2} ab(t_0^2-\mu_2^2) - \frac{1}{2}\rho bp(t_0^2-\mu_2^2) + \frac{1}{6} a\theta(t_0^3-\mu_2^3) - \frac{1}{6}\rho\theta p(t_0^3-\mu_2^3) - ab\mu_2(t_0-\mu_2) + b\rho p\mu_2(t_0-\mu_2) \\ - \frac{1}{6} ab\theta\mu_2(t_0^3-\mu_2^3) + \frac{1}{6}\rho b\theta p\mu_2(t_0^3-\mu_2^3) - \frac{1}{2} a\theta\mu_2^2(t_0-\mu_2) + \frac{1}{2}\rho\theta p\mu_2^2(t_0-\mu_2) + \frac{1}{4}\rho b\theta p\mu_2^2(t_0^2-\mu_2^2) - \frac{1}{4} ab\theta\mu_2^2(t_0^2-\mu_2^2) \\ - a(\mu_1-\mu_2) + \rho p(\mu_1-\mu_2) - \frac{1}{2} a(\theta+b)(\mu_1^2-\mu_2^2) + \frac{1}{2}\rho p(\theta+b)(\mu_1^2-\mu_2^2) + a(\theta+b)\mu_2(\mu_1-\mu_2) - \rho p(\theta+b)\mu_2(\mu_1-\mu_2) \\ + ab\theta\mu_2(\mu_1^2-\mu_2^2) - b\rho\theta p\mu_2(\mu_1^2-\mu_2^2) \end{array}\right] \\ (-\theta-b) - a(\theta+b)\mu_1 + \rho p(\theta+b)\mu_1 - ab\theta\mu_1^2 + \rho b\theta p\mu_1^2 + \rho p \end{array}\right)\mu_2^3 \end{array}\right)$$

$$
+\frac{1}{2}\left(x \begin{pmatrix}
-a+\dfrac{1}{\left[1+(\theta+b)(\mu_1-\mu_2)\right]} \\
\begin{bmatrix}
a(t_0-\mu_2)-\rho p(t_0-\mu_2)+\frac{1}{2}ab(t_0^2-\mu_2^2)-\frac{1}{2}\rho bp(t_0^2-\mu_2^2)+\frac{1}{6}a\theta(t_0^3-\mu_2^3)-\frac{1}{6}\rho\theta p(t_0^3-\mu_2^3)-ab\mu_2(t_0-\mu_2) \\
+b\rho p\mu_2(t_0-\mu_2)-\frac{1}{6}ab\theta\mu_2(t_0^3-\mu_2^3)+\frac{1}{6}\rho b\theta p\mu_2(t_0^3-\mu_2^3)-\frac{1}{2}a\theta\mu_2^2(t_0-\mu_2)+\frac{1}{2}\rho\theta p\mu_2^2(t_0-\mu_2) \\
+\frac{1}{4}\rho b\theta p\mu_2^2(t_0^2-\mu_2^2)-\frac{1}{4}ab\theta\mu_2^2(t_0^2-\mu_2^2)-a(\mu_1-\mu_2)+\rho p(\mu_1-\mu_2)-\frac{1}{2}a(\theta+b)(\mu_1^2-\mu_2^2) \\
+\frac{1}{2}\rho p(\theta+b)(\mu_1^2-\mu_2^2)+a(\theta+b)\mu_2(\mu_1-\mu_2)-\rho p(\theta+b)\mu_2(\mu_1-\mu_2)+ab\theta\mu_2(\mu_1^2-\mu_2^2)-b\rho\theta p\mu_2(\mu_1^2-\mu_2^2)
\end{bmatrix} \\
(-\theta-b)-a(\theta+b)\mu_1+\rho p(\theta+b)\mu_1-ab\theta\mu_1^2+\rho b\theta p\mu_1^2+\rho p
\end{pmatrix}\right)\mu_2^2
$$

$$
+\frac{1}{2}\left(y \begin{pmatrix}
a\mu_1-\rho p\mu_1+\dfrac{1}{\left[1+(\theta+b)(\mu_1-\mu_2)\right]} \\
\begin{bmatrix}
a(t_0-\mu_2)-\rho p(t_0-\mu_2)+\frac{1}{2}ab(t_0^2-\mu_2^2)-\frac{1}{2}\rho bp(t_0^2-\mu_2^2)+\frac{1}{6}a\theta(t_0^3-\mu_2^3)-\frac{1}{6}\rho\theta p(t_0^3-\mu_2^3)-ab\mu_2(t_0-\mu_2) \\
+b\rho p\mu_2(t_0-\mu_2)-\frac{1}{6}ab\theta\mu_2(t_0^3-\mu_2^3)+\frac{1}{6}\rho b\theta p\mu_2(t_0^3-\mu_2^3)-\frac{1}{2}a\theta\mu_2^2(t_0-\mu_2)+\frac{1}{2}\rho\theta p\mu_2^2(t_0-\mu_2) \\
+\frac{1}{4}\rho b\theta p\mu_2^2(t_0^2-\mu_2^2)-\frac{1}{4}ab\theta\mu_2^2(t_0^2-\mu_2^2)-a(\mu_1-\mu_2)+\rho p(\mu_1-\mu_2)-\frac{1}{2}a(\theta+b)(\mu_1^2-\mu_2^2)+\frac{1}{2}\rho p(\theta+b)(\mu_1^2-\mu_2^2) \\
+a(\theta+b)\mu_2(\mu_1-\mu_2)-\rho p(\theta+b)\mu_2(\mu_1-\mu_2)+ab\theta\mu_2(\mu_1^2-\mu_2^2)-b\rho\theta p\mu_2(\mu_1^2-\mu_2^2)
\end{bmatrix} \\
\left(1+a(\theta+b)\mu_1\right)+\frac{1}{2}a(\theta+b)\mu_1^2-\frac{1}{2}\rho p(\theta+b)\mu_1^2
\end{pmatrix}\right)\mu_2^2
$$

$$
+\frac{1}{3}\begin{pmatrix}
x\left(-\frac{1}{2}b-\frac{1}{2}\rho bp+ab\right) \\
+y\begin{pmatrix}
-\dfrac{1}{(1-b\mu_1)\left[1+(\theta+b)(\mu_1-\mu_2)\right]} \\
\left(b\begin{bmatrix}
a(t_0-\mu_2)-\rho p(t_0-\mu_2)+\frac{1}{2}ab(t_0^2-\mu_2^2)-\frac{1}{2}\rho bp(t_0^2-\mu_2^2)+\frac{1}{6}a\theta(t_0^3-\mu_2^3)-\frac{1}{6}\rho\theta p(t_0^3-\mu_2^3)-ab\mu_2(t_0-\mu_2) \\
+b\rho p\mu_2(t_0-\mu_2)-\frac{1}{6}ab\theta\mu_2(t_0^3-\mu_2^3)+\frac{1}{6}\rho b\theta p\mu_2(t_0^3-\mu_2^3)-\frac{1}{2}a\theta\mu_2^2(t_0-\mu_2)+\frac{1}{2}\rho\theta p\mu_2^2(t_0-\mu_2) \\
+\frac{1}{4}\rho b\theta p\mu_2^2(t_0^2-\mu_2^2)-\frac{1}{4}ab\theta\mu_2^2(t_0^2-\mu_2^2)-a(\mu_1-\mu_2)+\rho p(\mu_1-\mu_2)-\frac{1}{2}a(\theta+b)(\mu_1^2-\mu_2^2)+\frac{1}{2}\rho p(\theta+b)(\mu_1^2-\mu_2^2) \\
+a(\theta+b)\mu_2(\mu_1-\mu_2)-\rho p(\theta+b)\mu_2(\mu_1-\mu_2)+ab\theta\mu_2(\mu_1^2-\mu_2^2)-b\rho\theta p\mu_2(\mu_1^2-\mu_2^2)
\end{bmatrix}\right) \\
-\dfrac{b\left(a\mu_1-\rho p\mu_1+\frac{1}{2}b\mu_1^2+\frac{1}{2}\rho bp\mu_1^2-ab\mu_1^2\right)}{(1-b\mu_1)}-a+\rho p
\end{pmatrix}
\end{pmatrix}\mu_1^3
$$

$$
+\frac{1}{2}\left(x \begin{pmatrix}
-\dfrac{1}{(1-b\mu_1)\left[1+(\theta+b)(\mu_1-\mu_2)\right]} \\
\left(b\begin{bmatrix}
a(t_0-\mu_2)-\rho p(t_0-\mu_2)+\frac{1}{2}ab(t_0^2-\mu_2^2)-\frac{1}{2}\rho bp(t_0^2-\mu_2^2)+\frac{1}{6}a\theta(t_0^3-\mu_2^3)-\frac{1}{6}\rho\theta p(t_0^3-\mu_2^3)-ab\mu_2(t_0-\mu_2) \\
+b\rho p\mu_2(t_0-\mu_2)-\frac{1}{6}ab\theta\mu_2(t_0^3-\mu_2^3)+\frac{1}{6}\rho b\theta p\mu_2(t_0^3-\mu_2^3)-\frac{1}{2}a\theta\mu_2^2(t_0-\mu_2)+\frac{1}{2}\rho\theta p\mu_2^2(t_0-\mu_2) \\
+\frac{1}{4}\rho b\theta p\mu_2^2(t_0^2-\mu_2^2)-\frac{1}{4}ab\theta\mu_2^2(t_0^2-\mu_2^2)-a(\mu_1-\mu_2)+\rho p(\mu_1-\mu_2)-\frac{1}{2}a(\theta+b)(\mu_1^2-\mu_2^2)+\frac{1}{2}\rho p(\theta+b)(\mu_1^2-\mu_2^2) \\
+a(\theta+b)\mu_2(\mu_1-\mu_2)-\rho p(\theta+b)\mu_2(\mu_1-\mu_2)+ab\theta\mu_2(\mu_1^2-\mu_2^2)-b\rho\theta p\mu_2(\mu_1^2-\mu_2^2)
\end{bmatrix}\right) \\
-\dfrac{b\left(a\mu_1-\rho p\mu_1+\frac{1}{2}b\mu_1^2+\frac{1}{2}\rho bp\mu_1^2-ab\mu_1^2\right)}{(1-b\mu_1)}-a+\rho p
\end{pmatrix}\right)
$$

$$+\frac{1}{2}y\left(\begin{array}{l}\dfrac{1}{(1-b\mu_1)\left[1+(\theta+b)(\mu_1-\mu_2)\right]}\\ \left(a\left[\begin{array}{l}a(t_0-\mu_2)-\rho p(t_0-\mu_2)+\frac{1}{2}ab\left(t_0^2-\mu_2^2\right)-\frac{1}{2}\rho bp\left(t_0^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{6}\rho\theta p\left(t_0^3-\mu_2^3\right)-ab\mu_2(t_0-\mu_2)\\ +b\rho p\mu_2(t_0-\mu_2)-\frac{1}{6}ab\theta\mu_2\left(t_0^3-\mu_2^3\right)+\frac{1}{6}\rho b\theta p\mu_2\left(t_0^3-\mu_2^3\right)-\frac{1}{2}a\theta\mu_2^2(t_0-\mu_2)+\frac{1}{2}\rho\theta p\mu_2^2(t_0-\mu_2)\\ +\frac{1}{4}\rho b\theta p\mu_2^2\left(t_0^2-\mu_2^2\right)-\frac{1}{4}ab\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-a(\mu_1-\mu_2)+\rho p(\mu_1-\mu_2)-\frac{1}{2}a(\theta+b)\left(\mu_1^2-\mu_2^2\right)+\frac{1}{2}\rho p(\theta+b)\left(\mu_1^2-\mu_2^2\right)\\ +a(\theta+b)\mu_2(\mu_1-\mu_2)-\rho p(\theta+b)\mu_2(\mu_1-\mu_2)+ab\theta\mu_2\left(\mu_1^2-\mu_2^2\right)-b\rho\theta p\mu_2\left(\mu_1^2-\mu_2^2\right)\end{array}\right]\right.\\ \left.+\dfrac{a\mu_1-\rho p\mu_1+\frac{1}{2}b\mu_1^2+\frac{1}{2}\rho bp\mu_1^2-ab\mu_1^2}{(1-b\mu_1)}\right)\end{array}\right)\mu_1^2$$

$$-\frac{1}{6}y\left(\frac{5}{12}ab\theta-\frac{5}{12}\rho b\theta p\right)\mu_2^6-x\left(at_0-\rho pt_0+\frac{1}{2}abt_0^2-\frac{1}{2}\rho bpt_0^2+\frac{1}{6}a\theta t_0^3-\frac{1}{6}\rho\theta t_0^3\right)\mu_2 \qquad (17)$$

$$-\frac{1}{2}\left(x\left(-a+\rho p+\frac{1}{6}\rho b\theta pt_0^3-abt_0+\rho bpt_0-\frac{1}{6}ab\theta t_0^3\right)+y\left(at_0-\rho pt_0+\frac{1}{2}abt_0^2-\frac{1}{2}\rho bpt_0^2+\frac{1}{6}a\theta t_0^3-\frac{1}{6}\rho\theta t_0^3\right)\right)\mu_2^2-\frac{1}{5}y(ab\theta-\rho b\theta p)\mu_1^5$$

(by neglecting higher powers of θ)

(iii) $DC = c\left(\int_{\mu_1}^{\mu_2}\theta I(t)dt+\int_{\mu_2}^{T}\theta tI(t)dt\right)$

$$=c\theta\left(\begin{array}{l}a\left(\mu_1\mu_2-\frac{1}{2}\mu_2^2\right)-\rho p\left(\mu_1\mu_2-\frac{1}{2}\mu_2^2\right)+\frac{1}{2}a(\theta+b)\left(\mu_1^2\mu_2-\frac{1}{3}\mu_2^3\right)-\frac{1}{2}\rho p(\theta+b)\left(\mu_1^2\mu_2-\frac{1}{3}\mu_2^3\right)\\ -a(\theta+b)\left(\frac{1}{2}\mu_1\mu_2^2-\frac{1}{3}\mu_2^3\right)+\rho p(\theta+b)\left(\frac{1}{2}\mu_1\mu_2^2-\frac{1}{3}\mu_2^3\right)-ab\theta\left(\frac{1}{2}\mu_1^2\mu_2^2-\frac{1}{4}\mu_2^4\right)+\rho pb\theta\left(\frac{1}{2}\mu_1^2\mu_2^2-\frac{1}{4}\mu_2^4\right)\\ +\dfrac{1}{1+(\theta+b)(\mu_1-\mu_2)}\\ \left(\begin{array}{l}a(t_0-\mu_2)-\rho p(t_0-\mu_2)+\frac{1}{2}ab\left(t_0^2-\mu_2^2\right)-\frac{1}{2}\rho bp\left(t_0^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{6}\rho\theta p\left(t_0^3-\mu_2^3\right)\\ -ab\mu_2(t_0-\mu_2)+b\rho p\mu_2(t_0-\mu_2)-\frac{1}{6}ab\theta\mu_2\left(t_0^3-\mu_2^2\right)+\frac{1}{6}\rho b\theta p\mu_2\left(t_0^3-\mu_2^2\right)-\frac{1}{2}a\theta\mu_2^2(t_0-\mu_2)\\ +\frac{1}{2}\rho\theta p\mu_2^2(t_0-\mu_2)+\frac{1}{4}\rho b\theta p\mu_2^2\left(t_0^2-\mu_2^2\right)-\frac{1}{4}ab\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-a(t_0-\mu_2)+\rho p(t_0-\mu_2)\\ -\frac{1}{2}a(\theta+b)\left(\mu_1^2-\mu_2^2\right)+\frac{1}{2}\rho p(\theta+b)\left(\mu_1^2-\mu_2^2\right)+a(\theta+b)\mu_2(\mu_1-\mu_2)-\rho p(\theta+b)\mu_2(\mu_1-\mu_2)\\ +ab\theta\mu_2\left(\mu_1^2-\mu_2^2\right)-\rho b\theta p\mu_2\left(\mu_1^2-\mu_2^2\right)\end{array}\right)\\ \left(\mu_2+(\theta+b)\left(\mu_1\mu_2-\frac{1}{2}\mu_2^2\right)\right)\end{array}\right)$$

$$-c\theta\left(\begin{array}{l}\frac{1}{2}a\mu_1^2-\frac{1}{2}\rho p\mu_1^2+\frac{1}{6}a(\theta+b)\mu_1^3-\frac{1}{6}\rho p(\theta+b)\mu_1^3-\frac{1}{4}ab\theta\mu_1^4+\frac{1}{4}\rho b\theta p\mu_1^4\\ +\left(\begin{array}{l}\frac{1}{1+(\theta+b)(\mu_1-\mu_2)}\\ \left(\begin{array}{l}a(t_0-\mu_2)-\rho p(t_0-\mu_2)+\frac{1}{2}ab(t_0^2-\mu_2^2)-\frac{1}{2}\rho bp(t_0^2-\mu_2^2)+\frac{1}{6}a\theta(t_0^3-\mu_2^3)-\frac{1}{6}\rho\theta p(t_0^3-\mu_2^3)\\ -ab\mu_2(\mu_1-\mu_2)+b\rho p\mu_2(\mu_1-\mu_2)-\frac{1}{6}ab\theta\mu_2(t_0^3-\mu_2^3)+\frac{1}{6}\rho b\theta p\mu_2(t_0^3-\mu_2^3)-\frac{1}{2}a\theta\mu_2^2(t_0-\mu_2)\\ +\frac{1}{2}\rho\theta p\mu_2^2(t_0-\mu_2)+\frac{1}{4}\rho b\theta p\mu_2^2(t_0^2-\mu_2^2)-\frac{1}{4}ab\theta\mu_2^2(t_0^2-\mu_2^2)-a(\mu_1-\mu_2)+\rho p(\mu_1-\mu_2)\\ -\frac{1}{2}a(\theta+b)(\mu_1^2-\mu_2^2)+\frac{1}{2}\rho p(\theta+b)(\mu_1^2-\mu_2^2)+a(\theta+b)\mu_2(\mu_1-\mu_2)-\rho p(\theta+b)\mu_2(\mu_1-\mu_2)\\ +ab\theta\mu_2(\mu_1^2-\mu_2^2)-\rho b\theta p\mu_2(\mu_1^2-\mu_2^2)\end{array}\right)\end{array}\right)\\ \left(\mu_1+\frac{1}{2}(\theta+b)\mu_1^2\right)\end{array}\right)$$

$$\begin{aligned}&+c\theta\left(\begin{array}{l}\frac{1}{6}\left(\frac{5}{12}ab\theta-\frac{5}{12}\rho b\theta p\right)t_0^6+\frac{1}{5}\left(\frac{1}{3}a\theta-\frac{1}{3}\rho\theta p\right)t_0^5+\frac{1}{4}\left(-\frac{1}{2}\rho bp+\frac{1}{2}\rho\theta pt_0+\frac{1}{4}\rho b\theta pt_0^2+\frac{1}{2}ab-\frac{1}{2}a\theta t_0-\frac{1}{4}ab\theta t_0^2\right)t_0^4\\ +\frac{1}{3}\left(-a+\rho p+\frac{1}{6}\rho b\theta pt_0^3-abt_0+\rho bpt_0-\frac{1}{6}ab\theta t_0^3\right)t_0^3+\frac{1}{2}\left(at_0-\rho pt_0+\frac{1}{2}abt_0^2-\frac{1}{2}\rho bpt_0^2+\frac{1}{6}a\theta t_0^2-\frac{1}{6}\rho\theta pt_0^3\right)t_0^2\end{array}\right)\\ &-c\theta\left(\begin{array}{l}\frac{1}{6}\left(\frac{5}{12}ab\theta-\frac{5}{12}\rho b\theta p\right)\mu_2^6+\frac{1}{5}\left(\frac{1}{3}a\theta-\frac{1}{3}\rho\theta p\right)\mu_2^5+\frac{1}{4}\left(-\frac{1}{2}\rho bp+\frac{1}{2}\rho\theta pt_0+\frac{1}{4}\rho b\theta pt_0^2+\frac{1}{2}ab-\frac{1}{2}a\theta t_0-\frac{1}{4}ab\theta t_0^2\right)\mu_2^4\\ +\frac{1}{3}\left(-a+\rho p+\frac{1}{6}\rho b\theta pt_0^3-abt_0+\rho bpt_0-\frac{1}{6}ab\theta t_0^3\right)\mu_2^3+\frac{1}{2}\left(at_0-\rho pt_0+\frac{1}{2}abt_0^2-\frac{1}{2}\rho bpt_0^2+\frac{1}{6}a\theta t_0^2-\frac{1}{6}\rho\theta pt_0^3\right)\mu_2^2\end{array}\right)\end{aligned} \tag{18}$$

(iv) Shortage cost (SC) is given by

$$SC=-c_2\left(\int_{t_0}^{T}I(t)dt\right)=-c_2\left(\begin{array}{l}\frac{1}{3}\left(\frac{1}{2}ab-\frac{1}{2}\rho bp\right)(T^3-t_0^3)+\frac{1}{2}(-a+\rho p-abt_0+\rho bpt_0)(T^2-t_0^2)\\ +at_0(T-t_0)-\rho pt_0(T-t_0)+\frac{1}{2}abt_0^2(T-t_0)-\frac{1}{2}\rho bpt_0^2(T-t_0)\end{array}\right) \tag{19}$$

(v) $SR=p\left(\int_0^T(a+bI(t)-\rho p)dt\right)$

$$=p\left(\int_0^{\mu_1}(a+bI(t)-\rho p)dt+\int_{\mu_1}^{\mu_2}(a+bI(t)-\rho p)dt+\int_{\mu_2}^{t_0}(a+bI(t)-\rho p)dt+\int_{t_0}^{T}(a+bI(t)-\rho p)dt\right)$$

$$
\begin{aligned}
= p\Bigg(& a\mu_1 + b\Bigg(\frac{1}{(1-b\mu_1)\left(1+(\theta+b)(\mu_1-\mu_2)\right)} \\
& \Bigg(\Bigg(a(t_0-\mu_2) - \rho p(t_0-\mu_2) + \frac{1}{2}ab\left(t_0^2-\mu_2^2\right) - \frac{1}{2}\rho bp\left(t_0^2-\mu_2^2\right) \\
& + \frac{1}{6}a\theta\left(t_0^3-\mu_2^3\right) - \frac{1}{6}\rho\theta p\left(t_0^3-\mu_2^3\right) - ab\mu_2(t_0-\mu_2) + b\rho p\mu_2(t_0-\mu_2) \\
& - \frac{1}{6}ab\theta\mu_2\left(t_0^3-\mu_2^3\right) + \frac{1}{6}\rho b\theta p\mu_2\left(t_0^3-\mu_2^3\right) - \frac{1}{2}a\theta\mu_2^2(t_0-\mu_2) + \frac{1}{2}\rho\theta p\mu_2^2(t_0-\mu_2) \\
& + \frac{1}{4}\rho b\theta p\mu_2^2\left(t_0^2-\mu_2^2\right) - \frac{1}{4}ab\theta\mu_2^2\left(t_0^2-\mu_2^2\right) - a(\mu_1-\mu_2) + \rho p(\mu_1-\mu_2) \\
& - \frac{1}{2}a(\theta+b)\left(\mu_1^2-\mu_2^2\right) + \frac{1}{2}\rho p(\theta+b)\left(\mu_1^2-\mu_2^2\right) + a(\theta+b)\mu_2(\mu_1-\mu_2) \\
& - \rho p(\theta+b)\mu_2(\mu_1-\mu_2) + ab\theta\mu_2\left(\mu_1^2-\mu_2^2\right) - \rho b\theta p\mu_2\left(\mu_1^2-\mu_2^2\right) \Bigg) \\
& \left(\mu_1 - \frac{1}{2}b\mu_1^2\right)\Bigg) \\
& + \frac{1}{(1-b\mu_1)}\left(a\mu_1 - \rho p\mu_1 + \frac{1}{2}b\mu_1^2 + \frac{1}{2}\rho bp\mu_1^2 - ab\mu_1^2\right)\left(\mu_1 - \frac{1}{2}b\mu_1^2\right) \\
& - \frac{1}{2}a\mu_1^2 - \frac{1}{6}b\mu_1^3 + \frac{1}{2}\rho p\mu_1^2 - \frac{1}{6}\rho bp\mu_1^3 + \frac{1}{3}ab\mu_1^3 \Bigg) \\
& -\rho p\mu_1 \Bigg)
\end{aligned}
$$

$$
\begin{aligned}
+p\Bigg(& a\mu_2 + b\Bigg(a\left(\mu_1\mu_2 - \frac{1}{2}\mu_2^2\right) - \rho p\left(\mu_1\mu_2 - \frac{1}{2}\mu_2^2\right) + \frac{1}{2}a(\theta+b)\left(\mu_1^2\mu_2 - \frac{1}{3}\mu_2^3\right) \\
& - \frac{1}{2}\rho p(\theta+b)\left(\mu_1^2\mu_2 - \frac{1}{3}\mu_2^3\right) - a(\theta+b)\left(\frac{1}{2}\mu_1\mu_2^2 - \frac{1}{3}\mu_2^3\right) \\
& + \rho p(\theta+b)\left(\frac{1}{2}\mu_1\mu_2^2 - \frac{1}{3}\mu_2^3\right) - ab\theta\left(\frac{1}{2}\mu_1^2\mu_2^2 - \frac{1}{4}\mu_2^4\right) + \rho pb\theta\left(\frac{1}{2}\mu_1^2\mu_2^2 - \frac{1}{4}\mu_2^4\right) \\
& + \frac{1}{1+(\theta+b)(\mu_1-\mu_2)} \\
& \Bigg(\Bigg(a(t_0-\mu_2) - \rho p(t_0-\mu_2) + \frac{1}{2}ab\left(t_0^2-\mu_2^2\right) - \frac{1}{2}\rho bp\left(t_0^2-\mu_2^2\right) \\
& + \frac{1}{6}a\theta\left(t_0^3-\mu_2^3\right) - \frac{1}{6}\rho\theta p\left(t_0^3-\mu_2^3\right) - ab\mu_2(t_0-\mu_2) + b\rho p\mu_2(t_0-\mu_2) \\
& - \frac{1}{6}ab\theta\mu_2\left(t_0^3-\mu_2^2\right) + \frac{1}{6}\rho b\theta p\mu_2\left(t_0^3-\mu_2^2\right) - \frac{1}{2}a\theta\mu_2^2(t_0-\mu_2) + \frac{1}{2}\rho\theta p\mu_2^2(t_0-\mu_2) \\
& + \frac{1}{4}\rho b\theta p\mu_2^2\left(t_0^2-\mu_2^2\right) - \frac{1}{4}ab\theta\mu_2^2\left(t_0^2-\mu_2^2\right) - a(t_0-\mu_2) + \rho p(t_0-\mu_2) \\
& - \frac{1}{2}a(\theta+b)\left(\mu_1^2-\mu_2^2\right) + \frac{1}{2}\rho p(\theta+b)\left(\mu_1^2-\mu_2^2\right) + a(\theta+b)\mu_2(\mu_1-\mu_2) \\
& - \rho p(\theta+b)\mu_2(\mu_1-\mu_2) + ab\theta\mu_2\left(\mu_1^2-\mu_2^2\right) - \rho b\theta p\mu_2\left(\mu_1^2-\mu_2^2\right) \Bigg) \\
& \left(\mu_2 + (\theta+b)\left(\mu_1\mu_2 - \frac{1}{2}\mu_2^2\right)\right)\Bigg)\Bigg) \\
& - \rho p\mu_2 \Bigg)
\end{aligned}
$$

$$
\begin{aligned}
&-p\Bigg(a\mu_1 + b\Bigg(\frac{1}{2}a\mu_1^2 - \frac{1}{2}\rho p\mu_1^2 + \frac{1}{6}a(\theta+b)\mu_1^3 - \frac{1}{6}\rho p(\theta+b)\mu_1^3 - \frac{1}{4}ab\theta\mu_1^4 + \frac{1}{4}\rho b\theta p\mu_1^4 \\
&+ \frac{1}{\left(1+(\theta+b)(\mu_1-\mu_2)\right)}\Bigg(\Bigg(a(t_0-\mu_2) - \rho p(t_0-\mu_2) + \frac{1}{2}ab\left(t_0^2-\mu_2^2\right) - \frac{1}{2}\rho bp\left(t_0^2-\mu_2^2\right) \\
&+ \frac{1}{6}a\theta\left(t_0^3-\mu_2^3\right) - \frac{1}{6}\rho\theta p\left(t_0^3-\mu_2^3\right) - ab\mu_2(t_0-\mu_2) + b\rho p\mu_2(t_0-\mu_2) \\
&- \frac{1}{6}ab\theta\mu_2\left(t_0^3-\mu_2^2\right) + \frac{1}{6}\rho b\theta p\mu_2\left(t_0^3-\mu_2^2\right) - \frac{1}{2}a\theta\mu_2^2(t_0-\mu_2) + \frac{1}{2}\rho\theta p\mu_2^2(t_0-\mu_2) \\
&+ \frac{1}{4}\rho b\theta p\mu_2^2\left(t_0^2-\mu_2^2\right) - \frac{1}{4}ab\theta\mu_2^2\left(t_0^2-\mu_2^2\right) - a(t_0-\mu_2) + \rho p(t_0-\mu_2) \\
&- \frac{1}{2}a(\theta+b)\left(\mu_1^2-\mu_2^2\right) + \frac{1}{2}\rho p(\theta+b)\left(\mu_1^2-\mu_2^2\right) + a(\theta+b)\mu_2(\mu_1-\mu_2) \\
&- \rho p(\theta+b)\mu_2(\mu_1-\mu_2) + ab\theta\mu_2\left(\mu_1^2-\mu_2^2\right) - \rho b\theta p\mu_2\left(\mu_1^2-\mu_2^2\right)\Bigg) \\
&\left(\mu_1 + \frac{1}{2}(\theta+b)\mu_1^2\right)\Bigg)\Bigg) \\
&- \rho p\mu_1\Bigg)
\end{aligned}
$$

$$
\begin{aligned}
&+p\left(at_0 + b\left(\frac{1}{2}at_0^2 - \frac{1}{2}\rho pt_0^2 + \frac{1}{6}abt_0^3 - \frac{1}{6}\rho bpt_0^3 + \frac{1}{12}a\theta t_0^4 - \frac{1}{12}\rho\theta pt_0^4 - \frac{1}{12}ab\theta t_0^5 + \frac{1}{12}\rho b\theta pt_0^5\right) - \rho pt_0\right) \\
&-p\Bigg(a\mu_2 + b\Bigg(a\left(t_0\mu_2 - \frac{1}{2}\mu_2^2\right) - \rho p\left(t_0\mu_2 - \frac{1}{2}\mu_2^2\right) + \frac{1}{2}ab\left(t_0^2\mu_2 - \frac{1}{3}\mu_2^3\right) \\
&- \frac{1}{2}\rho bp\left(t_0^2\mu_2 - \frac{1}{3}\mu_2^3\right) + \frac{1}{6}a\theta\left(t_0^3\mu_2 - \frac{1}{4}\mu_2^4\right) - \frac{1}{6}\rho\theta p\left(t_0^3\mu_2 - \frac{1}{4}\mu_2^4\right) \\
&- ab\left(\frac{1}{2}t_0\mu_2^2 - \frac{1}{3}\mu_2^3\right) + b\rho p\left(\frac{1}{2}t_0\mu_2^2 - \frac{1}{3}\mu_2^3\right) - \frac{1}{6}ab\theta\left(\frac{1}{2}t_0^3\mu_2^2 - \frac{1}{5}\mu_2^5\right) \\
&+ \frac{1}{6}\rho b\theta p\left(\frac{1}{2}t_0^3\mu_2^2 - \frac{1}{5}\mu_2^5\right) - \frac{1}{2}a\theta\left(\frac{1}{3}t_0\mu_2^3 - \frac{1}{4}\mu_2^4\right) + \frac{1}{2}\rho\theta p\left(\frac{1}{3}t_0\mu_2^3 - \frac{1}{4}\mu_2^4\right) \\
&+ \frac{1}{4}\rho b\theta p\left(\frac{1}{3}t_0^2\mu_2^3 - \frac{1}{5}\mu_2^5\right) - \frac{1}{4}ab\theta\left(\frac{1}{3}t_0^2\mu_2^3 - \frac{1}{5}\mu_2^5\right)\Bigg) - \rho p\mu_2\Bigg) \\
&+p\Bigg(aT + b\Bigg(\left(Tt_0 - \frac{1}{2}T^2\right) - \rho p\left(Tt_0 - \frac{1}{2}T^2\right) + \frac{1}{2}ab\left(t_0^2T - \frac{1}{3}T^3\right) \\
&- \frac{1}{2}\rho bp\left(t_0^2T - \frac{1}{3}T^3\right) - ab\left(\frac{1}{2}t_0T^2 - \frac{1}{3}T^3\right) + \rho bp\left(\frac{1}{2}t_0T^2 - \frac{1}{3}T^3\right)\Bigg) - \rho pT\Bigg) \\
&-p\left(at_0 + b\left(\frac{1}{2}at_0^2 - \frac{1}{2}\rho pt_0^2 + \frac{1}{6}abt_0^3 - \frac{1}{6}\rho bpt_0^3\right) - \rho pt_0\right)
\end{aligned}
\qquad (20)
$$

The total profit during a cycle, $\pi(t_0,T,p)$ consisted of the following:

$$\pi(t_0,T,p) = \frac{1}{T}\left[SR - OC - HC - DC - SC\right] \qquad (21)$$

Substituting values from equations (16) to (20) in equation (21), we get total profit per unit. Putting $\mu_1 = v_1t_0$ and $\mu_2 = v_2t_0$ in equation (21), we get profit in terms of t_0, T and p. Differentiating equation (21) with respect to t_0, T and p and equate it to zero, we have i.e. $\frac{\partial \pi(t_0,T,p)}{\partial t_0} = 0,\ \frac{\partial \pi(t_0,T,p)}{\partial T} = 0,\ \frac{\partial \pi(t_0,T,p)}{\partial p} = 0$ (22)

provided it satisfies the condition

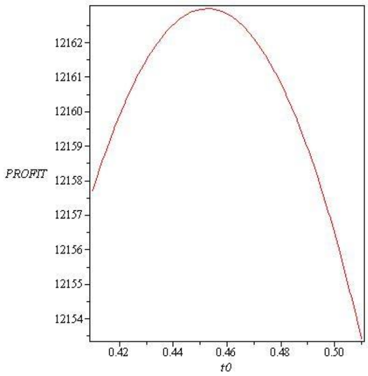
$$\begin{vmatrix} \frac{\partial \pi^2(t_0,T,p)}{\partial t_0^2} & \frac{\partial \pi^2(t_0,T,p)}{\partial t_0 T} & \frac{\partial \pi^2(t_0,T,p)}{\partial t_0 p} \\ \frac{\partial \pi^2(t_0,T,p)}{\partial T t_0} & \frac{\partial \pi^2(t_0,T,p)}{\partial T^2} & \frac{\partial \pi^2(t_0,T,p)}{\partial T p} \\ \frac{\partial \pi^2(t_0,T,p)}{\partial p t_0} & \frac{\partial \pi^2(t_0,T,p)}{\partial p T} & \frac{\partial \pi^2(t_0,T,p)}{\partial p^2} \end{vmatrix} > 0 \tag{23}$$

IV. NUMERICAL EXAMPLE

Considering A= Rs.100, a = 500, b=0.05, c=Rs. 25, ρ= 5, θ=0.05, x = Rs. 5, y=0.05, v_1 = 0.30, v_2 = 0.50, c_2 = Rs. 8, in appropriate units. The optimal values of t_0* = 0.4505, T* =0.5808, p* = 50.5313, and Profit*= Rs. 12162.9820.

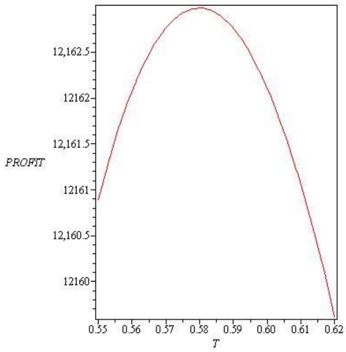
The second order conditions given in equation (23) are also satisfied. The graphical representation of the concavity of the profit function is also given.

t_0 and Profit



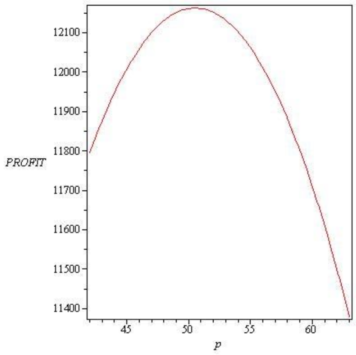
Graph 1

T and Profit



Graph 2

p and Profit



Graph 3

V. SENSITIVITY ANALYSIS

On the basis of the data given in example above we have studied the sensitivity analysis by changing the following parameters one at a time and keeping the rest fixed.

Table 1
Sensitivity Analysis

Parameter	%	t_0	T	p	Profit
a	+20%	0.4569	0.5629	60.5423	17655.9523
	+10%	0.4526	0.5700	55.5345	14782.9921
	-10%	0.4516	0.5965	45.5333	9795.8726
	-20%	0.4567	0.6184	40.5414	7681.6778
x	+20%	0.3794	0.5246	50.5201	12123.7579
	+10%	0.4115	0.5496	50.5246	12142.1646
	-10%	0.4991	0.6206	50.5412	12186.8211
	-20%	0.5628	0.6735	50.5564	12214.5794
θ	+20%	0.4397	0.5707	50.5240	12159.4220
	+10%	0.4465	0.5776	50.5297	12161.2015
	-10%	0.4546	0.5841	50.5330	12164.7889
	-20%	0.4589	0.5875	50.5348	12133.6245

A	+20%	0.4913	0.6344	50.5805	12130.3498
	+10%	0.4714	0.6082	50.5565	12146.3045
	-10%	0.4285	0.5519	50.5048	12180.4949
	-20%	0.4050	0.5212	50.4767	12198.9846
ρ	+20%	0.4165	0.5584	42.1586	10063.9856
	+10%	0.4313	0.5679	45.9632	11017.7569
	-10%	0.4748	0.5967	56.1179	13563.7951
	-20%	0.5122	0.6245	63.1107	15316.7190
c_2	+20%	0.4565	0.5709	50.5498	12197.9785
	+10%	0.4537	0.5756	50.5411	12160.3310
	-10%	0.4455	0.5849	50.5195	12165.9827
	-20%	0.4404	0.5907	50.5074	12169.4242

From the table we observe that as parameter a increases/ decreases average total profit and optimum order quantity also increases/ decreases.

Also, we observe that with increase and decrease in the value of θ, x and ρ, there is corresponding decrease/ increase in total profit and optimum order quantity.

From the table we observe that as parameter A increases/ decreases average total profit decreases/ increases and optimum order quantity increases/ decreases.

VI. CONCLUSION

In this paper, we have developed an inventory model for deteriorating items with price and inventory dependent demand with different deterioration rates. Sensitivity with respect to parameters have been carried out. The results show that with the increase/ decrease in the parameter values there is corresponding increase/ decrease in the value of profit.

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