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Construction of Balanced Incomplete Block Designs

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ABSTRACT: *Incomplete block designs were introduced to eliminate heterogeneity to a greater extent is possible with randomized block design and latin square design when the number of treatments is large. Some of the incomplete block designs are, balanced incomplete block design, partially balanced incomplete block design, youden square design, lattice design etc. In this paper an attempt is made to propose two new balanced incomplete block design construction methods using two associate class partially balanced incomplete block designs.*

Keywords: *BIBD, Triangular type PBIBD, PBIBD.*

I. INTRODUCTION

The arrangement of ‘v’ treatments in ‘b’ blocks, each of sizes $k_1, k_2, \dots, k_b$, each of the treatment appears $r_1, r_2, \dots, r_v$ blocks such that some pairs of treatments occur in λ_1 blocks, some pairs of treatments occur in λ_2 blocks, so on and the rest of pairs of treatments occur in λ_m blocks then the design is said to be a “General Incomplete Block Design”. The Incomplete block designs were introduced to eliminate heterogeneity to a greater extent is possible with randomized blocks and latin squares when the number of treatments is large. Some of the incomplete block designs are, balanced incomplete block design, partially balanced incomplete block design, youden square design, lattice design etc.

DEFINITION 1.1: The arrangement of ‘v’ treatments in ‘b’ blocks each of size ‘k’, each treatment appears exactly in ‘r’ blocks and every pair of treatments occurs exactly ‘λ’ times, then the design is said to ‘Balanced Incomplete Block Design (BIBD)’. The parameters v, b, r, k and λ satisfies the parametric relations: vr = bk; $\lambda(v-1) = r(k-1)$ and $b \geq v$.

DEFINITION 1.2: The arrangement of ‘v’ treatments in ‘b’ blocks each of size ‘k’ (<v) each treatment appears in ‘r’ blocks is said to be an m-associate class partially balanced incomplete block design (PBIBD) if each pair of treatments appears in λ_1 or λ_2, ... or λ_m blocks. The parameters v, b, r, k, $\lambda_1, \lambda_2, \dots \lambda_m$, $n_1, n_2, \dots n_m$ satisfies the parametric relations: vr = bk; $\Sigma n_i = v-1$; $\Sigma n_i\lambda_i = r(k-1)$; $P_{ij}^k = P_{ji}^k$; $n_i p^i_{jk} = n_j p^j_{ik}$; $\Sigma P_{ij}^k = n_i$ if $i \neq k$ and $\Sigma P_{ij}^k = n_i-1$ if i=k; where i, j, k = 1,2,….m. If the number of associations is two then it is called a two-associate class PBIBD.

DEFINITION 1.3: The arrangement of v= ½ s(s-1) treatments in a square array of order s, such that the positions in the principal diagonal are left blank, and the ½ s(s-1) positions above the principal diagonal are filled up with the ½ s(s-1) treatment symbols and the positions below the principal diagonal are filled up by the ½ s(s-1) treatment symbols in such a manner that the resultant arrangement is symmetrical about the principal diagonal, the resulting design is said to be a triangular type PBIBD. In this design any two treatments are said to be first associates if they belong to same row or same column and are said to be second associates otherwise.

The parameters of triangular type PBIBD are v = ½ s(s-1), b=s, r=2, k=s-1, λ_1=0, λ_2=1, n_1=2(s-2), n_2= ½ (s-2)(s-3) and the two association matrices are

$$P_{ij}^{(1)} = \begin{bmatrix} s-2 & s-3 \\ s-3 & \frac{1}{2}(s-3)(s-4) \end{bmatrix} \text{ and } P_{ij}^{(2)} = \begin{bmatrix} 4 & 2s-8 \\ 2(s-4) & \frac{1}{2}(s-4)(s-5) \end{bmatrix}$$

DEFINITION 1.4: The arrangement of $v=s^2$ treatments in ‘s’ rows, ‘s’ columns such that each treatment occurs once in each row and in each column (Latin square of order s) and, the resulting design is a latin square type PBIBD. In this design any two treatments in the same row or column are first associates and otherwise they are said to be second associate.

II. CONSTRUCTION OF BALANCED INCOMPLETE BLOCK DESIGNS

In this section an attempt is made to propose two new methods for the construction of balanced incomplete block designs using partially balanced incomplete block designs.

THEOREM 2.1: A BIBD with parameters v′ = n, b′= ½ n(n-1), r′= n-1, k′= n-3, λ′=1can be constructed with the existence of a triangular type PBIBD with parameters v= ½ n(n-1), b=n, r=2, k=n-1, λ_0 = 0, λ_1 = 1.

Proof: Consider a two associate class triangular type partially balanced incomplete block design with parameters $v = ½\, n\, (n-1)$, $b=n$, $r=2$, $k = n-1$, $\lambda_1 = 1$, $\lambda_0 = 0$, whose incidence matrix is N. Construct a design with number of blocks as number of treatments ($v'=b$), generate the blocks corresponding to each treatment of PBIBD ($b'=v$) such that the block containing block numbers of each treatment as blocks with block size ($k'=r$) and each treatment replicated 'k' times ($r'= k$). The resulting design is a BIBD with the parameters $v' = n$, $b'= ½\, n(n-1)$, $r'= n-1$, $k'= 2$, $\lambda'=1$.

The step by step procedure for the construction of BIBD is presented below.

Step 1: Consider a triangular type PBIBD with parameters $v = n\, (n-1)/2$, $b=n$, $r=2$, $k = n-1$, $\lambda_1 = 1$, $\lambda_0 = 0$ whose incidence matrix is N.

Step 2: Consider each block number of PBIBD as treatment, corresponding to each treatment list the block number of its occurrence.

Step 3: The resulting is a BIBD is with parameters $v' = n$, $b'= ½\, n(n-1)$, $r'= n-1$, $k'= 2$, $\lambda'=1$.

The procedure is illustrated in the example 2.1.

EXAMPLE 2.1: Consider a triangular type PBIBD with parameters $v= 10$, $b=5$, $r=2$, $k=4$, $\lambda_0=0$, $\lambda_1=1$ whose incidence matrix is N. The resultant BIBD with parameters $v'=5$, $b'=10$, $r'=4$, $k'=2$, $\lambda'=1$ is presented in Table 2.1. The efficiency of the BIBD is 0.625.

Triangular type PBIBD	Incidence Matrix of PBIBD	Incidence Matrix of BIBD	BIBD
$\begin{array}{c\|cccc} & \multicolumn{4}{c}{\text{Blocks}} \\ \hline \text{I} & 1 & 2 & 3 & 4 \\ \text{II} & 1 & 5 & 6 & 7 \\ \text{III} & 2 & 5 & 8 & 9 \\ \text{IV} & 3 & 6 & 8 & 10 \\ \text{V} & 4 & 7 & 9 & 10 \end{array}$	$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \\ 2 & 3 \\ 2 & 4 \\ 2 & 5 \\ 3 & 4 \\ 3 & 5 \\ 4 & 5 \end{bmatrix}$

Table 2.1

THEOREM 2.2: A BIBD with parameters $v'=v= s^2$, $b'= 2s^2$, $k'= ½(s^2-1)$, $r'= s^2-1$ and $\lambda'= 3$ can be constructed with the existence of a two associate class partially balanced incomplete block design with parameters $v=s^2$, $b=2s$, $r=2$, $k=s$, $\lambda_0 = 0$, $\lambda_1 = 1$, $n_1=n_2$.

Proof: Consider a partially balanced incomplete block design with parameters $v=s^2$, $b=2s$, $r=2$, $k=s$, $\lambda_0 = 0$, $\lambda_1 = 1$, $n_1 = n_2$ (= ½ (s^2-1)). Corresponding to each treatment 1st associate group of treatments as one block and 2nd associate group of treatments as another block, generate $2s^2$ blocks with the s^2 treatments such that each treatment is repeated s^2-1 times and each block contains ½ (s^2-1) treatments. The resulting design is a BIBD with parameters $v'=v$, $b'= 2s^2$, $k'= ½(s^2-1)$, $r'= s^2-1$ and $\lambda'= 3$. □

The detailed step by step procedure of construction of BIBD is presented below.

Step 1: Consider a simple lattice design with parameters $v=s^2$, $b=2s$, $r=2$, $k=s$, $\lambda_0 = 0$, $\lambda_1 = 1$, $n_1=n_2$, whose incidence matrix is N.

Step 2: Identify the treatments belong to 1st and 2nd association schemes of each treatment.

Step 3: Constructing the $2s^2$ blocks with the s^2 treatments such that corresponding to each treatment 1st association treatments as one block and 2nd associate treatments as another block.

Step 4: The resulting BIBD is with parameters $v'=v$, $b'= 2s^2$, $k'= ½(s^2-1)$, $r'= s^2-1$ and $\lambda'= 3$.

It can be noted that the design is existing when s=3 and the procedure is illustrated in the example 2.2.

EXAMPLE 2.2: Consider a PBIBD with parameters v = 9, b= 6, k=3, r=2, λ_0= 0, λ_1=1. The resulting BIBD with parameters v=9, b=18, r=8, k=4, n_1=4, n_2=4, λ=3 is presented below. The efficiency of the BIBD is

PBIBD	Incidence Matrix of PBIBD	BIBD	Incidence Matrix of BIBD
$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 2 & 3 & 4 & 7 \\ 1 & 3 & 5 & 8 \\ 1 & 2 & 6 & 9 \\ 5 & 6 & 1 & 7 \\ 4 & 6 & 2 & 8 \\ 4 & 5 & 3 & 9 \\ 1 & 4 & 8 & 9 \\ 7 & 9 & 2 & 5 \\ 7 & 8 & 3 & 6 \\ 5 & 6 & 8 & 9 \\ 4 & 6 & 7 & 9 \\ 4 & 5 & 7 & 8 \\ 2 & 3 & 8 & 9 \\ 1 & 3 & 5 & 9 \\ 1 & 2 & 7 & 8 \\ 2 & 3 & 5 & 6 \\ 1 & 3 & 4 & 6 \\ 1 & 2 & 4 & 5 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$

Table 2.2

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