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Assessing Elementary Pre-service Teachers' Knowledge for Teaching Geometry

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ABSTRACT*: Invoking the first three van Hiele levels of geometric thought, pre-service elementary teachers (N = 52) were assessed on their content and pedagogical geometry knowledge as well as their spatial visualization skills. The pre-service teachers' first and second level responses to the geometry test items indicated a severe deficit in their geometric understanding. By not demonstrating geometric thought at the third van Hiele level, these teachers lacked the skills to competently teach students in grades 1 through 8.*

KEYWORDS -*Pre-Service Teachers, Geometric Thought, van Hiele, Spatial Visualization*

I. Introduction

One of the most influential variables that impacts the type of instruction that occurs in mathematics classrooms is the depth of understanding of the teacher [1, 2, 3]. In order to improve student achievement, the level of knowledge possessed by pre-service teachers who enter the workforce each year must be considered. Current reform-based curriculum frameworks for mathematics suggest that students be engaged in challenging activities that require them to use complex reasoning skills and divergent thinking skills to solve multi-step tasks [4]. In order to facilitate this kind of mathematical understanding, pre-service teachers need to possess a thorough conceptual and procedural understanding of mathematics, as well as an understanding of research-based, developmentally-appropriate pedagogical practices [5, 6, 7]. Additionally, teachers must be able to view the content from the multiple perspectives of their students [8].

With regards to geometric thinking, pre-service elementary teachers must possess an understanding of shapes, congruency, transformations, location, and spatial visualization [4, 9, 10] in the context of the first three van Hiele levels of geometric thought: Visualization, Analysis, and Ordering/Informal Deduction [11, 12]. According to Spear [12], "the first three levels identify thinking within the capability of elementary school students (p. 393)". In order to be successful in high school geometry courses, students should enter high school with a geometric level of thought that is at least at the Ordering/Informal Deduction level [10]. Consequently, all pre-service elementary and middle level mathematics teachers should be functioning, at a minimum, at the Ordering/Informal Deduction van Hiele level [3]. Not only must pre-service teachers be functioning at or above the third van Hiele level, but they must also know what each level requires in terms of understanding specific topics within each of the broad Geometry content categories so that they can identify the level(s) of their students' geometric thinking, as well as monitor their students' progression to higher levels.

For over 25 years, the van Hiele theory has been accepted as a way of measuring one's level of geometric thought [13, 14]. According to the theory, learners of geometry progress through sequential levels that are not age dependent, but rather dependent on appropriate geometric experiences. Furthermore, when geometric experiences occur within the context of the classroom, instruction must match the level of thinking of the student if learning is to occur [10]. Typically, students exhibit a dominant level of thinking when responding to geometry content questions; however, many times students will demonstrate thinking at multiple levels, particularly two consecutive levels, suggesting that they are in transition from one level to the next [13, 14, 15]. Thus, a student functioning at Level 1-2 would demonstrate thinking of both Levels 1 and 2 [15]. Gutierrez, Jaime, and Fortuny [16] indicated that the van Hiele levels of geometric thought were not discrete and that attainment of a higher level does not occur at once, but rather over the course of months or years. They concluded that students' geometric thinking exists in varying degrees of each level. Furthermore, students' level of thinking can differ across various geometric concepts; i.e. one's thinking regarding the hierarchical nature of quadrilaterals could be at Level 2, while one's thinking about the properties of three-dimensional shapes could be at Level 1 [13, 15, 16, 17, 18].

The five van Hiele levels of geometric thought are Level 1, Visualization; Level 2, Analysis; Level 3, Ordering/Informal Deduction; Level 4, Deduction, and Level 5, Rigor [10, 19]. Additionally, Clements and Battista [20] have identified the existence of a Level 0, Pre-recognition, which they described as "children initially perceive geometric shapes, but attend to only a subset of a shape's visual characteristics. They are unable to identify many common shapes" (p. 356). At the visual level (Visualization), non-verbal thinking occurs; shapes are judged by the way they look. Two-dimensional shapes are identified based on their appearance and one's mental images of that shape, rather than on the shape's mathematical properties. At the Analysis level, shapes are identified according to the properties they possess; classes of shapes and the properties that define them are considered. Thinking that is indicative of the Ordering/Informal Deduction level includes the formulation of definitions of shapes based on the logical ordering of properties. The Deduction level is characterized by the creation of logic-driven proofs of geometric properties which involve axioms, definitions, theorems, corollaries, and postulates. According to van de Walle et al [10], the highest level, Rigor, involves an understanding of the "distinctions and relationships between different axiomatic systems" (p. 406).

Previously, researchers [3, 15, 18, 21, 22, 23, 24, 25, 26, 27] have found that pre-service elementary and middle level teachers lack a level of geometric thinking necessary to future success as mathematics teachers. At best, pre-service teachers typically demonstrate a procedural understanding of geometry as evidenced by memorized definitions and properties of shapes, rather than a conceptual understanding based on a synthesis of properties of shapes and recognition of minimal properties which define shapes [21]. This lack of conceptual understanding is also evidenced by pre-service teachers' struggles with the hierarchical relationships among classes of quadrilaterals [22, 23, 25]. Besides attaining the necessary level of geometric thinking with regards to content, pre-service teachers must also possess adequate pedagogical content knowledge (PCK) of geometry, as well as knowledge of particular materials appropriate for geometry instruction [28]. Thus, in order to teach geometry, pre-service teachers must have knowledge of various representations of geometric concepts, appropriate examples and non-examples, and instructional materials that do not misrepresent the concepts being taught.

Furthermore, researchers have documented that K-12 students encounter difficulties when trying to learn geometric content or their understanding is flawed by misconceptions [14, 16, 29, 30, 31]. In particular, students have been found to (1) regard "sides" of polygons as only those that appear in a vertical orientation; (2) refer to "straight lines" rather than "parallel", indicating a lack of understanding of the precise definition of a line; (3) identify parallelograms based on the presence or absence of oblique angles; (4) incorrectly identify right triangles, isosceles triangles, quadrilaterals and altitudes of various types of triangles, particularly when presented with these polygons such that one or more sides are not parallel or perpendicular to the sides of the frame of reference; (5) prefer one name for a given polygon rather than equal acceptance of multiple names (i.e. preference for naming a square "square" rather than equal acceptance of "rectangle", "rhombus" or "square"); and (6) have difficulty identifying characteristics of rhombi, squares, and/or parallelograms [14, 15, 30].

Weaknesses in geometric understanding have also been substantiated through the 2003 Trends in Mathematics and Science Study in which United States eighth graders scored lowest on geometry of the five mathematical areas assessed; United States fourth and eighth graders were reported as scoring in the bottom half of all fourth and eighth graders in geometry [32]. Unal et al [32] speculate that the reason for this could be that teachers who lack sufficient geometric knowledge are unable to provide learning opportunities to their students that facilitate geometric understanding. According to Unal et al [32], "teachers whose geometric knowledge and/or spatial ability is limited may not have the capacity to make adjustments to curriculum to address the needs of students with varying learning needs." Additionally, pre-service teachers have been found to possess several misconceptions; namely, (1) rhombi are not considered to be parallelograms; (2) squares are not considered to be rectangles; (3) inclusion of unnecessary characteristics of polygons when formulating definitions; (4) omission of critical characteristics of polygons when formulating definitions; and (5) polygons must be convex; concave polygons are "shapes" [22, 23, 25, 27]. Cunningham and Roberts [21] reported that while pre-service teachers' geometric content improved after explicit instruction, their understanding lacked the depth needed to supplement the prototypical geometry examples provided by many textbooks; i.e. only presenting triangles with altitudes that are enclosed by the sides of the triangles or only presenting convex polygons in the context of presenting examples of diagonals. Furthermore, pre-service elementary teachers have been found to have significantly weaker spatial visualization skills when compared to other undergraduates, particularly those majoring in engineering fields, architecture, mathematics, and secondary mathematics education [33].

Given that researchers have consistently found inadequacies in elementary pre-service teachers' geometric understanding and spatial ability, the purposes of this study were 1) to assess elementary pre-service teachers' geometry content knowledge (GCK), geometry pedagogical content knowledge (GPCK), and their spatial visualization skills (SVS) in terms of the first three van Hiele levels of geometric thought; and 2) to identify misconceptions held by pre-service teachers with regards to geometric content. Within the context of this study, it was assumed that not all pre-service teachers were functioning at the Ordering/Informal Deduction level (Level 3) of the van Hiele model; and, spatial visualization was defined as one's ability to "mentally rotate, twist, turn, reflect or otherwise move a three-dimensional object presented in two dimensions" [34].

II. Method

a. Participants

The sample for this study was comprised of N = 52 elementary pre-service teachers from three cohorts with varying levels of teaching and mathematics preparation. Approximately 20% of the students had completed only the foundations of mathematics education course (FO)(19.6%), nearly 30% had finished both the foundations of mathematics education course & the geometry and measurement content course (FG) (29.4%), and over 50% were near completion of the third course in the series of three courses: foundations of mathematics education, geometry and measurement content, and mathematics pedagogy (PG). Because of the small cell syndrome, no other characteristics were obtained to ensure the participants' anonymity.

b. Instrumentation

Each participate completed the Assessment of Geometric Knowledge for Teaching, AGKT [35]. This assessment was developed to evaluate three domains, (a) geometry content knowledge (GCK), (b) geometry pedagogical content knowledge (GPCK) which included knowledge of appropriate geometric materials and manipulatives, and (c) spatial visualization skills (SVS). These domains each contained four-items scored on a binary (correct or incorrect) metric. Additionally, there were 26-items designed to correlate with geometry content knowledge and/or pedagogical content knowledge. These 26-items were likewise scored on a binary (correct or incorrect) metric. Individual items evaluated understanding of composition and decomposition of 2-D shapes, symmetry, congruence, properties of quadrilaterals, properties of 3-D shapes, Euler's formula, location in terms of the coordinate plane, and transformations (slides, rotations, and reflections) within the first three van Hiele levels. Specifically, 4 of the items aligned with geometric thought indicative of van Hiele level 1, Visualization; 16 items aligned with geometric thought indicative of van Hiele level 2, Analysis; and 6 items aligned with geometric thought indicative of van Hiele level 3, Ordering/Informal Deduction. Based on this distribution of items by level, pre-service teachers who scored relatively high, 85% or above, on the assessment were considered to be functioning at van Hiele level 2, the necessary level for appropriate elementary geometry instruction to occur. Likewise, the lower the score, the greater the likelihood that the pre-service teacher was functioning at van Hiele levels 0 and/or 1, depending on the specific geometric content under consideration [13]. Incorrect items were used as indicators of a lack of thinking at a given level of geometric thought. The content of the AGKT items was selected based on recommendations of the Conference Board for the Mathematical Sciences [9] for pre-service teacher preparation programs and the geometry content advocated for elementary and middle level students by the National Council of Teachers of Mathematics in Principals and Standards for School Mathematics [36]. The content was also aligned with the geometry expectations of the Common Core State Standards for Mathematics [4] for grades Kindergarten through sixth grade.

c. Statistical Analyses

Descriptive statistics were conducted to eliminate items lacking sufficient levels of variance (i.e., a minimum ratio of 80/20) [37]. Cronbach's alpha was performed to assess the internal consistency of the items comprising the three domains. A 3 X 3 mixed ANOVA was conducted to determine any differences among the three groups on the three domains. One-way ANOVAs with Bonferroni post-hocs were employed as follow-up tests. An all-possible-subsets regression analysis was conducted to identify the Visualization Items that achieved statistically significant part-correlation coefficients for both the Content and Pedagogy Scores The all-possible-subsets analysis is considered to be more sophisticated and credible than the traditional stepwise method by presenting numerous models that best fit the data [37]. Whereas the stepwise method creates a single model by including or excluding single predictors at each step, the all-possible-subsets technique constructs the number of models for consideration that equals 2 raised to the power of the number of predictors minus 1. In this example, the number of plausible models equals [(25) – 1] = 31 of which the top ten best fitting models were reported.

III. Results

a. Data Screening

The items failing to reach the 80/20 ratio minimum are presented in Fig. 1.

Fig. 1. Items deleted with inadequate variance

	Content	Pedagogy	Visualization
Items	Q 5[b]		
	Q 11[a]		
		Q2 [a]	
		Q8 [a]	
			Q5 Cylinder [b]

	Q1 Rectangle [b]
	Q5 Pyramid [b]
	Q7 Square [b]
	Q4 Low Viz [b]
	Q4 Med Viz [b]
	Q3 Pattern [b]
	Q6 Triangle [b]
	Q2 Tri [b]

[a] < 0.20
[b] > 0.80

b. Score Reliability

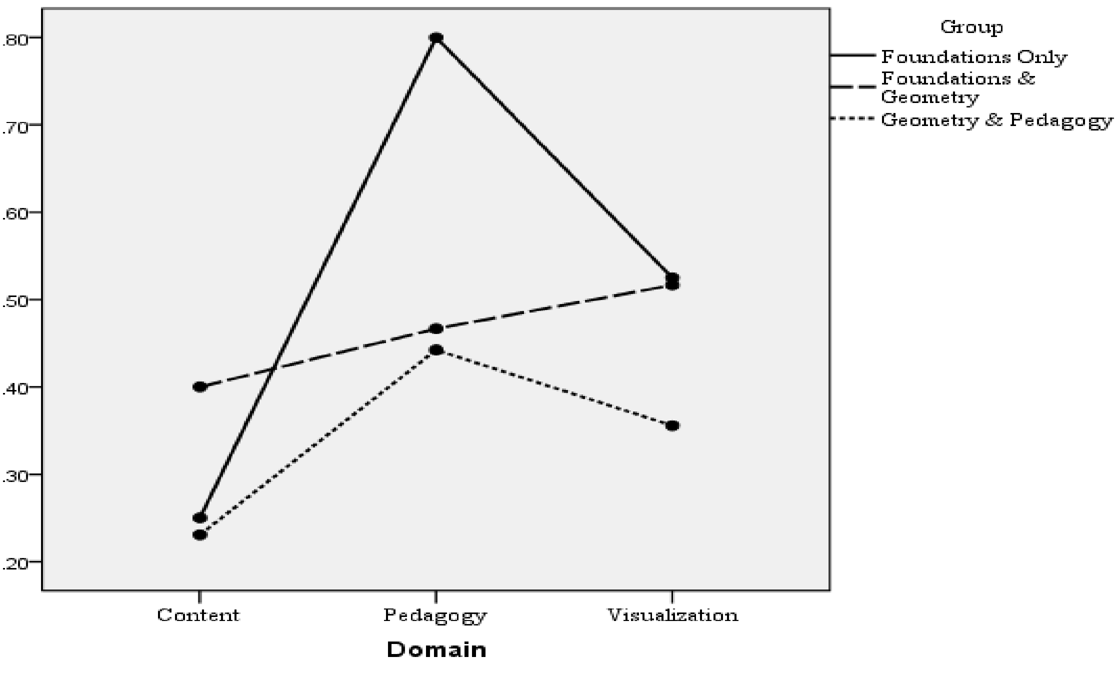
Results of the Cronbach's alpha determined that all the coefficients were below acceptable standards for research purposes (i.e., < 0.50) [38]. Because reliability is a property of the scores and not a characteristic of the instrument [39], these findings strongly suggest that the pre-service teachers in this sample lacked an integrated understanding of the following, (a) geometry, (b) pedagogy of geometry, and (c) related visualization skills.

c. Group by Domain Scores

Results indicated a significant interaction effect, $p < .05$. Follow-up tests detected that the Foundations Only Group scored significantly greater than the other two groups in Pedagogy. There were no other significant differences (See Fig. 2.).

Fig. 2

	Group	*M*	*SD*	*N*
Content	Foundations Only	.25	.35	10
	Foundations & Geometry	.40	.39	15
	Geometry & Pedagogy	.23	.35	26
	Total	.28	.36	51
Pedagogy	Foundations Only	.80	.35	10
	Foundations & Geometry	.47	.40	15
	Geometry & Pedagogy	.44	.41	26
	Total	.52	.41	51
Visual	Foundations Only	.52	.30	10
	Foundations & Geometry	.52	.32	15
	Geometry & Pedagogy	.36	.28	26
	Total	.44	.30	51



Results of the all-possible-subsets regression analysis identified a linear combination of four content items that accounted for 74% of the Content variance score. The items with their b-weight, Beta, Sig., Zero Order, and Part Coefficient are presented in Fig. 3.

Fig. 3

	B	Beta	Sig.	Zero-order	Part
(Constant)	-.287		.000		
Q1 Parallelogram	.313	.434	.000	.556	.388
Q7 Parallel	.316	.434	.000	.412	.424
Q1 Triangle	.235	.325	.000	.526	.309
Q7 Triangle	.196	.271	.002	.551	.245

There was a linear combination of two pedagogy items that accounted for 43% of the Pedagogy variance score. The items with their b-weight, Beta, Sig., Zero Order, and Part Coefficient are presented in Fig. 4.

Fig. 4

	Unstandardized Coefficients		Standardized Coefficients			Correlations		
Model	B	Std. Error	Beta	t	Sig.	Zero-order	Partial	Part
(Constant)	.040	.091		.441	.661			
Q3 Tangram	.471	.096	.536	4.932	.000	.551	.580	.536
Q12 All	.295	.089	.361	3.322	.002	.383	.432	.361

Dependent Variable: Pedagogy
$R = 0.66$, $R^2 = .43$, $p < .001$

The descriptive statistics for each variable are presented in Fig. 5.

Fig. 5

Variable	N	Minimum	Maximum	Mean	Std. Deviation
Q1_Content	51	0	1	.25	.44
Q7_Content	51	0	1	.31	.47
Q10_Ped	51	0	1	.59	.50
Q3_Ped	51	0	1	.45	.50
Q12_Viz	51	0	1	.27	.45
Q4_Viz	51	0	1	.53	.50
Q6_Viz	51	0	1	.27	.45
Q9_Viz	51	0	1	.67	.48
Q12All	51	0	1	.53	.50
Q12Reflect	51	0	1	.37	.49
Q12Slide	51	.0	1	.43	.50
Q1Parallelogram	51	0	1	.51	.50
Q7Rectangle	51	0	1	.78	.42
Q1Triangle	51	0	1	.53	.50
Q2Mix	51	0	1	.24	.43
Q2Not3	51	0	1	.43	.50
Q2Right	51	0	1	.78	.42
Q3Tangram	51	0	1	.69	.47
Q4highviz	51	0	1	.65	.48
Q6Square	51	0	1	.51	.50
Q7Parallel	51	0	1	.57	.50
Q7Triangle	51	0	1	.55	.50
Q8Parallel	51	0	1	.75	.44

Q8SqRec	51	0	1	.41	.50
Q8SqRhom	51	0	1	.47	.50
Gender	51	0	1	.06	.24
GeoCompl	51	0	1	.80	.40
Class	51	0	1	.51	.50
Q5_Content	51	0	1	.90	.30
Q2_Ped	51	0	1	.12	.33
Q8_Ped	51	0	1	.12	.33
Q11_Content	51	0	1	.18	.39
Valid N (listwise)	51				

d. General Level of Geometric Thought and Misconceptions Identified

Based on the individual item descriptive statistics, the general van Hiele level of geometric thought of the participating pre-service teachers was determined to be predominantly Level 2. Of the 20 individual items with mean scores of at least 50%, 4 revealed geometric thinking at van Hiele level 1, Visualization; 14 revealed geometric thinking at van Hiele level 2, Analysis; and only 2 revealed geometric thinking at van Hiele level 3, Ordering/Informal Deduction. Moreover, a lack of thinking at van Hiele level 3 was indicated by a majority of the pre-service teachers incorrectly responding to 4 of the 6 van Hiele level 3 items. The individual item descriptive statistics were also used to identify misconceptions held by the pre-service teacher. Items with mean scores of less than 50% led to misconceptions which were categorized as pedagogical, content, or visualization based on the original intent of the item.

i. Pedagogical Misconceptions

Four pedagogical misconceptions were identified:

- When assessing students' ability to decompose polygons, decompositions are equally correct regardless of the description of the required decomposition.
- Regardless of the number of correct solutions, one correct solution is expected to demonstrate understanding.
- Regardless of the number of appropriate manipulative models, a single mathematical manipulative is used to represent concepts.
- When assessing students' understanding of the hierarchical relationships among quadrilaterals, all that students should understand is that squares, rhombi, and rectangles are all parallelograms.

ii. Content Misconceptions

The pre-service teachers' geometry content knowledge was compromised by six identified misconceptions:

- Parallelograms have two lines of symmetry;
- Equilateral triangles have one line of symmetry.
- A square is not a rhombus.
- A square is not a rectangle.
- Two scalene right triangles cannot be used to compose an isosceles triangle or a parallelogram.
- To determine the geometric relationship among 4 points plotted on a coordinate plane, consistent scaling of the x and y-axes is not necessary; the geometric relationship can be determined by what the resulting shape looks like.

iii. Visualization Misconceptions

With regards to the pre-service teachers' spatial abilities, two visualization misconceptions were identified:

- Mental manipulations of pyramid nets are limited to either triangular or square pyramids.
- Mental transformations of polygons are limited to two out of three (reflections, turns, and/or slides) transformations.

IV. Conclusion

As previously stated, the purposes of this investigation were to 1) assess elementary pre-service teachers' geometry content knowledge (GCK), geometry pedagogical content knowledge (GPCK), and their spatial visualization skills (SVS) in terms of the first three van Hiele levels of geometric thought; and 2) identify misconceptions held by pre-service teachers with regards to geometric content. Results of this assessment indicated that these pre-service teachers are functioning primarily at both the Visualization and Analysis levels and are lacking in understanding at the Ordering/Informal Deduction level of the van Hiele levels of geometry thought. Their geometric thought is grounded in what shapes look like and their properties; but they are not able to informally deduce one property from another. Thus, the findings of this investigation support the results of previous research [3, 21, 22, 23, 25, 26, 27]; pre-service teachers' content knowledge for teaching geometry remains inadequate for facilitating their students' advancement through the first three van Hiele levels.

With regards to the pre-service teachers' GCK, the pre-service teachers' strengths included identification of 3-dimensional shapes, with 88% correctly identifying all 3-dimensional shapes present, and to a lesser extent identification of the lines of symmetry of equilateral triangles, parallelograms, squares, and rectangles, with 56% correctly identifying the lines of symmetry for all 4 shapes. However, pre-service teachers demonstrated limited ability to compose a variety of polygons using two scalene right triangle, with only 25% correctly composing all three possible polygons. Of even greater concern was the pre-service teachers' lack of ability to determine the geometric relationship between 4 points given on a coordinate plane, with only 17% accurately responding. These results coupled with the evidence of pre-service teachers' predominant levels of geometric thought indicate an overall lack of GCK needed to be effective in elementary mathematics classrooms.

Given the lack of GCK, the results of the assessment of the pre-service teachers' GPCK were not surprising. As Fennema and Franke [2] stated "one cannot teach what one does not know (p 147)." The highest mean score of the GPCK items was 58% for the item which assessed the pre-service teachers' ability to determine if students' definitions for congruence were accurate. The majority of the pre-service teachers incorrectly responded to all other GPCK items. These items involved identifying appropriate manipulatives and tools for geometry instruction and assessing students' understanding of the hierarchical relationships amongst various quadrilaterals. Only 12% of the pre-service teachers were able to correctly assess students' understanding of the hierarchical relationship between parallelograms, rectangles, squares, and rhombi. This indicates a lack of GCK of those relationships on the part of the pre-service teachers, as well. Assessment results of the pre-service teachers' Spatial Visualization Skills (SVS) revealed that pre-service teachers visualization skills were strongest when the mental manipulating involved nets of cubes, with 54% responding accurately, or edges, faces and vertices of polyhedra, with 65% responding accurately. However, only 27% responded accurately to the items requiring mental manipulation of nets of other pyramids and transformations of irregular polygons.

Results of comparing the pre-service teachers' performance by groups indicated that those pre-service teachers who had only completed the foundations of math methods course had significantly greater GPCK scores than those pre-service teachers who had completed a geometry and measurement course and a mathematics pedagogy course. This was the only difference among the three groups. Given that the geometry and measurement course is not typically taught using research-based pedagogy for grades K-8th grade, the pre-service teachers who had completed that course had observed geometry content being taught in more traditional ways. Additionally, those pre-service teachers who had completed the mathematics pedagogy course had been taught research-based pedagogical practices for all content strands (Number and Operations, Algebra, Measurement, Geometry, and Data Analysis and Probability)[36]; and thus, their specific understanding of appropriate geometry pedagogy was obscured by other pedagogical understandings, as a result of not having strong GCK on which to build their GPCK.

The results of this investigation provide evidence that pre-service teachers should be given opportunities to learn about the van Hiele levels of geometric thought and to assess their own level of geometric thought. Pre-service teachers should also be provided with experiences that include a multitude of shapes in a variety of orientations that require them to confront the properties of the shapes and the relationships that exist among the properties. Geometric experiences must go beyond memorization of geometric definitions in order for pre-service teachers to develop an ability to logically analyze the properties of shapes [21]. Until pre-service elementary teachers can successfully demonstrate the knowledge and understanding of the geometry content that they will be expected to teach, they will be unable to fully comprehend reform-based pedagogy and innovations presented by new curricula [40, 41]. As Gutierrez and Jamie [42] recommended, elementary teacher educators should take into account pre-service teachers' existing geometric misconceptions and purposefully address those during discussions of geometry content. Furthermore, elementary teacher educators should continue to study the nature of pre-service elementary teachers' GCK and GPCK with attention given to why their understandings are so limited. Given the uniqueness of the development of geometric thinking [11], one way to improve geometric instruction in elementary and middle level mathematics classrooms might be for mathematics teacher educators to identify the van Hiele level of thought at which each pre-service teacher is functioning; facilitate progress throughout his/her teacher preparation program through meaningful experiences until van Hiele level 3 thinking is demonstrated; and then continue to support this level of thinking through focused professional development during each pre-service teacher's first years of teaching.

V. Acknowledgements

This article is dedicated in memoriam of **Mr. Marshean Keith Joseph, Jr**. who passed unexpectedly on November 6, 2015 while this manuscript was nearing completion. Keith was a promising algebra student of the first author at the time of his passing. His determination and work ethic were beyond compare, while his delightful smile revealing his sense of accomplishment when conquering non-routine algebraic problems was contagious. He will always be remembered and is sorely missed.

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