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Detecting Assignable Signals via Decomposition of MEWMA Statistic

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Abstract: *The MEWMA Chart is used in Process monitoring when a quick detection of small or moderate shifts in the mean vector is desired. When there are shifts in a multivariate Control Charts, it is clearly shows that special cause variation is present in the Process, but the major drawback of MEWMA is inability to identify which variable(s) is/are the source of the signals. Hence effort must be made to identify which variable(s) is/are responsible for the out- of-Control situation. In this article, we employ Mason, Young and Tracy (MYT) approach in identifying the variables for the signals.*

Keywords: *Quality Control, Multivariate Statistical Process Control, MEWMA Statistic.*

1. INTRODUCTION

Nowadays, One of the most powerful tools in Quality Control is the Statistical Control Chart developed in the 1920s by Walter Shewharts, the Control Chart found wide spread use during World War II and has been employed, with various modifications, ever since. Multivariate Statistical Process Control (SPC) using MEWMA statistic is usually employed to detect shifts. However, MEWMA Control Chart has a shortcoming as it can't figure out the causes of the change. Thus, decomposition of T^2 is recommended and aims at paving a way of identifying the variables significantly contributing to an out- of-Control signals.

1.1 Multivariate Control for Monitoring the Process Mean:

Suppose that the Px1 random vectors $X_1, X_2, ----, X_p$ each representing the P quality characteristics to be monitored, are observed over a given period. These vectors may be represented by individual observations or sample mean vector. In 1992 lowery et al developed the MEWMA Chart as natural extension of EWMA. It is a famous Chart employ to monitor a Process with a quality characteristics for detecting shifts. The in Control Process mean is assumed without loss of generality to be a vector of zeros, and covariance matrix $\sum$.

The MEWMA Control Statistic is defined as vectors,

Z_i=RX_i+ (I-R)Z_{i-1}, i=I, 2, 3---------

Where X_0=0, and R=diag $(r_1, r_2, -----, r_p)$, $0 \le r_k \le 1$, i =1, 2, 3, -------, p

The MEWMA Chart gives an out of Control signals when

$T_k^2 = Z_i^t \sum_{zi}^{-1} Z_{i-1} > h$

Where h>0is chosen to achieve a specified in Control ARL and $\sum_{zi}$ is the covariance matrix of X_0 given by $\sum_{zi} = \{\frac{\theta}{2-\theta}\}\sum$, under equality of weights of past observation for all P characteristics.

The UCL=. $\frac{(n-1)p}{n-p} . F_{\alpha,p,(n-p)}$. if at least one point goes beyond upper Control limits the Process is assumed to be out of Control. The initial value Z_0 is usually obtained as equal to the in-Control mean vector of the Process. It is obvious that if R=I, then the MEWMA Control Chart is equivalent to the Hotelling's T^2Chart. The value of this calculated by simulation to achieve a specified in Control ARL. Molnau et al presented a programmed that enables the calculation of the ARL for the MEWMA when the values of the shifts in the mean vector, the Control limit and the smoothing parameter are known Sullivan and Woodall(5) recommended the use of a MEWMA for the preliminary analysis of multivariate observation. Yumin (7) propose the construction of a MEWMA using the principal components of the original variables. Choi et al (2). Proposed a general MEWMA Chart in which the smoothing matrix is full instead of one having only diagonal. The performance of this Chart appears to be better than that of the MEWMA proposed by Lowry et al (3). Yet et al (6), Introduced a MEWMA which is designed to detect small changes in the variability of correlated Multivariate Quality Characteristics, while Chen et al (1) proposed a MEWMA Control Chart that is capable of monitoring simultaneously the Process mean vector and Process covariance matrix. Runger et al (4). Showed how the shift detection capability of the MEWMA can be significantly improved by transforming the original Process variables to a lower dimensional subspace through the use of the U-transformation.

1.2 The MYT Decomposition

The T^2 statistic can be decomposed into P-orthogonal components (Mason, Tracy and Young) for instance, if you have P-variates to decompose, the procedure is as follows:

$$T^2 = T_1^2 + T_{2.1}^2 + --- + T_{P.1,2,-------P-1}^2 \qquad (1)$$

The first term is an unconditionalT^2, decomposing it as the first variable of the

$$T_1^2 = \frac{Z_1^2}{S_1^2} \qquad (2)$$

Where Z_1 and S_1 is the transform X-variable and standard deviation of the variable Z_1 respectively.T_j^2 Will follow an F-Distribution which can be used as upper Control Limit

$$T_j^2 = \frac{Z_j^2}{S_j^2} \backsim \frac{n\text{-}1}{n} F_{\propto,1,n-1.}$$

Taking a case of three variables as an example, it can be decomposed as,

$$T^2 = \begin{cases} T_1^2 + T_{2.1}^2 + T_{3.12}^2 \\ T_1^2 + T_{3.1}^2 + T_{2.13}^2 \\ T_2^2 + T_{1.2}^2 + T_{3.12}^2 \\ T_2^2 + T_{3.2}^2 + T_{1.23}^2 \\ T_3^2 + T_{1.3}^2 + T_{2.13}^2 \\ T_3^2 + T_{2.3}^2 + T_{1.23}^2 \end{cases} \qquad (3)$$

It is obvious that with increase in the number of variables, the number of terms will also increase drastically which makes the computation become troublesome.

II. Illustration

In this section, we intend to demonstrate by way of example, how an assignable signal could be detected, the data that was used for the purpose of the research was collected from a Portland cement company in Lagos, Nigeria. The data consists of temperature from a boiler machine in which twenty five observations were taken under different temperatures. The covariance matrix given below was obtained from the data.

$$\Sigma = \begin{pmatrix} 54 & 0.958 & 20.583 \\ 0.958 & 4.84 & 2.963 \\ 20.583 & 2.963 & 22.993 \end{pmatrix}$$

Having obtained the covariance matrix, the computation of T^2 statistic was done using MATLAB computer package and the values obtained for each of the twenty five observations are as shown in the table.

TABLE 1:

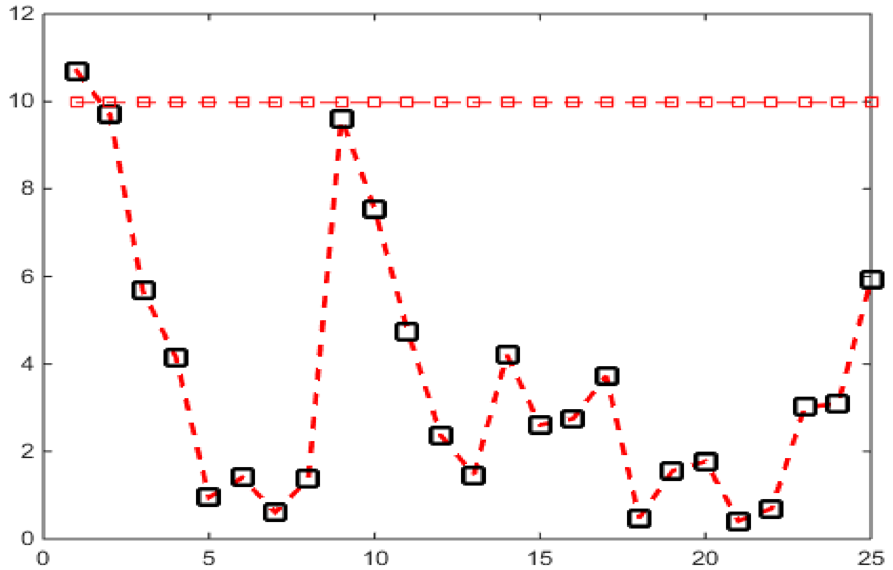
Computation of MEWMA Statistic for individual observation

No of observation	T_k^2	UCL = H
1	10.69 * *	9.98
2	9.70	
3	5.67	
4	4.14	
5	0.95	
6	1.40	
7	0.61	
8	1.37	

9	9.61	
10	7.55	
11	4.76	
12	2.37	
13	1.47	
14	4.20	
15	2.61	
16	2.75	
17	3.72	
18	0.48	
19	1.55	
20	1.77	
21	0.41	
22	0.70	
23	3.01	
24	3.09	
25	5.92	

Below is the graph of MEWMA statistic for individual observation, where the individual T^2 values were plotted against the number of observations, and the red dotted line represents the Control limit.

Figure 1: Chart of MEWMA Statistic



As it can be seen from the graph as well as the computation of T^2 statistic in the table above, it is clear that observation (1) is out of Control, as such, in order to detect the assignable signal, we consider observation(1), and the procedure is as follow:

$$T_k^2 = Z_i^1 \Sigma_{zi}^{-1} Z_{i-1}, \qquad \Sigma_{zi} = \left\{\frac{\delta}{2-\delta}\right\}$$.Therefore,

$$T_1^2(Z_1, Z_2, Z_3) = (-1.8\ 0.244\ -1.192)\begin{pmatrix} 0.2629 & 0.0999 & -0.02482 \\ 0.0999 & 2.0978 & -0.3598 \\ -0.02482 & -0.3598 & 0.6679 \end{pmatrix}\left\{\begin{pmatrix} -1.8 \\ 0.244 \\ -1.192 \end{pmatrix}\right. = 9.98$$

UCL = $\frac{(n-1)p}{n-p} \cdot F_{\alpha,p,(n-1)} = 9.98$

The above result shows that theT^2 overall is significant, as such we now decompose T^2 into its orthogonal components to be able to determine which among the three variables contribute most as well as responsible to the out of Control signals. Therefore, from equation (3) above, where for p=3 we had 18 component of T^2 with different equation each is produce the same overallT^2. The calculation for each component as well as the sum of each term is as follows:

Starting with unconditional terms as given below:
$T_{1,}^2 = \frac{Z_1^2}{S_{11}} = \frac{(-1.8)^2}{54} = 0.06$, $T_2^2 = \frac{Z_2^2}{S_{22}} = \frac{(0.244)^2}{4.84} = 0.012$, and, $T_3^2 = \frac{Z_3^2}{S_{33}} = \frac{(-1.192)^2}{22.993} = 0.062$

Therefore, the UCL for unconditional terms is computed as follows:
UCL $= \frac{n+1}{n} . F_{\propto,1,n-1}$
It is show that all the variables, T_1^2, T_2^2 and T_3^2 are in Control since they are less than the UCL of the unconditional terms.
To compute the conditional terms we proceed as follows:
$T_{3.12}^2 = T^2(Z_1\ Z_2\ Z_3) - T^2(Z_1\ Z_2)$
To obtain $T^2(Z_1\ Z_2\ Z_3)$, we petition the original estimate of Z- vector and covariance structure to obtain the Z-vector and cov- matrix of the Sub vector $Z^2 = (Z_1\ Z_2)$. the corresponding partition given as
$\Sigma_2 = \begin{pmatrix} 54 & 0.958 \\ 0.958 & 4.54 \end{pmatrix}$, $Z^2 = \begin{pmatrix} -1.8 \\ 0.244 \end{pmatrix}$
$T^2 = (-1.8,\ \ 0.244) \begin{pmatrix} 54 & 0.958 \\ 0.958 & 4.54 \end{pmatrix}^{-1} \begin{pmatrix} -1.8 \\ 0.244 \end{pmatrix} = 0.0758$
$T_{2.1}^2 = T^2(Z_1\ Z_2 Z_3) - T^2(Z_1\ Z_2) = 10.69 - 0.06 = 0.0158$
$T_{3.1=}^2 T^2(Z_1\ Z_2 Z_3) - T^2(Z_1\ Z_3) = 10.69 - 0.0758 = 10.61$
From the above we can now obtain
$T^2 = T^2(Z_1\ Z_2\ Z_3) = T_1^2 + T_{2.1}^2 + T_{3.12}^2 = 0.06 + 0.00158 + 10.61 = 10.69$
Since the first two terms have small value implies that the signal is contained in the third terms
$T_{3.12}$
Next, we check whether there is signal in $(Z_1\ Z_2)$, we partition original observation into two groups $(Z_1\ Z_2)$ and Z_3. Having computed the value of $T^2(Z_1\ Z_2) = 0.0758$, then we compare it with UCL = $(Z_1\ Z_2) < 7.4229$. We conclude that, there is no signal present in $(Z_1\ Z_2)$ components of the observation Vector. Hence it's implies that the signal lies in the third component. Therefore, the above decomposition considers MYT so as to ensure that whichever MYT decomposition terms, yield the same overall T^2 as follows:
$T_1^2 + T_{2.1}^2 + T_{3.12}^2 = 0.06 + 0.0158 + 10.61 = 10.69$
$T_1^2 + T_{3.1}^2 + T_{2.1\ 3}^2 = 0.06 + 0.0169 + 10.6131 = 10.69$
$T_2^2 + T_{1.2}^2 + T_{3.12}^2 = 0.012 + 0.0639 + 10.6141 = 10.69$
$T_2^2 + T_{3.2}^2 + T_{1.23}^2 = 0.012 + 0.2881 + 10.3899 = 10.69$
$T_3^2 + T_{1.3}^2 + T_{2.13}^2 = 0.062 + 0.0149 + 10.6131 = 10.69$
$T_3^2 + T_{2.3}^2 + T_{1.23}^2 = 0.062 + 0.238 + 10.3899 = 10.69$

With the aids of the result decomposed above; the value of each term of the decomposition was compared to the respective critical value as indicated in the table below:

Table: 2
MYT Decomposition Component for Observation (1)

Component	Value	Critical value
T_1^2	0.06	4.4304
T_2^2	0.012	4.4304
T_3^2	0.062	4.4304
$T_{1.2}^2$	0.0639	7.4229
$T_{1.3}^2$	0.0149	7.4229
$T_{2.1}^2$	0.0158	7.4229
$T_{3.1}^2$	0.0169	7.4229
$T_{2.3}^2$	0.2881	7.4229
$T_{3.2}^2$	0.2881	7.4229
$T_{1.23}^2$	10.39	10.38 * *
$T_{2.13}^2$	10.61	10.38 * *
$T_{3.12}^2$	10.61	10.38 * *

From the above table, we discovered that $T^2_{1.23}$ + $T^2_{2.13}$ +$T^2_{3.12}$ have significant values, which means there is a problem in the conditional relation betweenZ_1,Z_2andZ_3 we may conclude that the conditional relation betweenZ_1, $Z_2$and Z_3 are potential causes of the shifts. To verify the problems, we remove each conditional relation from the vector of observation (1) and check whether the sub vector signals or not.

T^2-$T^2_{1.23} = 10.69 - 10.38 = 0.31 < 10.69$

T^2-$T^2_{2.13} = 10.69 - 10.38 = 0.31 < 10.69$

T^2-$T^2_{3.12}$=10.69-10.38=0.38<10.69

The outcomes Show that, the sub vector is insignificant. The meaning of significant value of $T^2_{1.23}$ is that Z_1conditioned by Z_2 and Z_3 deviates from the variables relation pattern established from the historical data set. Similarly, and vice-versa for the conditioned ofZ_1, $Z_2$and Z_3 were detected as being responsible sources for the assignable causes in observation (1).

III. CONCLUSION

In this article, we were able to point out how T^2 MEWMA Statistic decomposition Procedure was employed to detect assignable signal. Broadly speaking, Control Charts are used so as to be able to distinguish between assignable and natural causes in the variability of quality goods produce. With regards to this, verifying which combination of quality characteristics is responsible for the signal. Therefore the Control Chart plays a key role to inform us the appropriate next line of action to be taken in order to enhance the quality.

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APPENDIX I

Boiler Temperature Data

N0.of observation	X_1	X_2	X_3
1	507	516	527
2	512	513	533
3	520	512	537
4	520	514	538
5	530	515	542
6	528	516	542
7	522	513	537
8	527	509	537
9	533	514	528
10	530	512	528
11	530	512	541
12	527	513	541
13	529	514	542
14	522	509	539
15	532	515	545
16	531	514	543
17	535	514	542
18	516	515	537
19	514	510	532
20	536	512	540
21	522	514	540
22	520	514	540
23	526	517	546
24	527	514	543
25	529	518	544
Total	13125	12839	13473
Average	525	513.36	538.92