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General Manpower – Hyper Exponential Machine System with Exponential Production, General Sales and General Recruitment

[1]M. Sneha latha, [2]P. Sekar , [3]R. Ramanarayanan
[1]Assistant Professor, SRM University, Chennai, Tamil Nadu, India
[2]Principal,C. Kandaswami Naidu College for men, Anna Nagar, Chennai, Tamil Nadu, India
[3]Professor of Mathematics (Retired), Vel Tech University Chennai, India

Abstract: *A manpower machine system which produces products for sale is treated. It is assumed that the manpower system fails with a probability when an employee leaves and that the machine system has Hyper exponential life time distribution. The combined system fails when the manpower system fails or the machine fails whichever occurs first. When the combined system fails, sales begin and recruitments are done to fill up vacancies. Two models are treated. In model 1, recruitments are done one by one and in model 2 recruitments are done one by one or in bulk pattern depending on the system operation time is within or exceeding a threshold. Using Laplace transform, the operation time, recruitment time of the employees, repair time of the machine and sales time of the products are studied. Their means and the covariance of the operation time and recruitment time are presented with numerical examples.*

Mathematics Subject Classification: 91B70
Keywords: *Departure and Recruitments, Failure and Repairs, Production and Sale times, Joint Laplace transforms.*

I. INTRODUCTION

Manpower Planning models have been studied by Grinold and Marshall [2]. For statistical approach one may refer to Bartholomew [1]. Lesson [6] has given methods to compute shortages (Resignations, Dismissals, Deaths). Markovian models are designed for shortage and promotion in MPS by Vassiliou [13]. Subramanian. [12] has made an attempt to provide optimal policy for recruitment, training, promotion and shortages in manpower planning models with special provisions such as time bound promotions, cost of training and voluntary retirement schemes. For other manpower models one may refer Setlhare [9]. For three characteristics system in manpower models one may refer to Mohan and Ramanarayanan [8]. Esary et al. [3] have discussed cumulative damage processes. Stochastic analysis of manpower levels affecting business with varying recruitment rate are presented by Hari Kumar, Sekar and Ramanarayanan [4]. Manpower System with Erlang departure and one by one recruitment is discussed by Hari Kumar [5]. For the study of Semi Markov Models in Manpower planning one may refer Meclean [7]. Snehalatha.M., Sekar.P and Ramanarayanan.R[10] have analyzed general Manpower SCBZ Machine system. Modified Erlang two phase system with general manpower has been discussed by Snehalatha.M., Sekar.P and Ramanarayanan.R[11].

In this paper, a machine which produces products is considered. The machine has Hyper exponential life time and the products are produced one at a time with inter-occurrence time distribution which is exponential. Employees attending the machine may depart with general inter-departure time distribution. The system may fail when an employee leaves with same probability. It may also fail when the machine fails. When the system fails sales and recruitments begin. Two models are treated. In model 1, employees are recruited one by one. In model 2 when the operation time is more than a threshold, recruitments are done in bulk manner and when it is less than the threshold, the recruitments are done one by one. The joint Laplace transform of the variables, their means and the covariance of operation time and recruitment time are obtained. Numerical cases are also treated.

MODEL 1

ASSUMPTIONS:

1. Inter departure times of employees are independent and identically distributed random variables with Cdf F(x) and pdf f(x). The manpower system collapses with probability p when an employee leaves and survives with probability q and continues its operation where p+q =1.
2. The machine attended by manpower has hyper-exponential life time distribution with parameter 'a_i' and probability p_i, i=1,2,3,…..m, $\sum_{i=1}^{m} p_i = 1$

3. The man power machine system fails when either manpower or machine fails.
4. When the system fails, the vacancies caused by employees are filled up one by one with recruitment time V whose Cdf is V(y) and pdf is v(y).
5. When the machine fails, its repair time distribution is $R_i(z)$ with pdf $r_i(z)$when the failure parameter is λ_i, for i= 1,2,3,……m.
6. When the manpower machine system is in operation, products are produced one at a time with production time distribution is exponential with parameter μ.
7. The sales time of products begins when the system fails. The sales are done one by one with sale time Cdf G(w) and pdf g(w).

ANALYSIS:

To study the above model the joint probability density function of four variables, namely (X,$\hat{V}, R, \hat{S}$) is required where

(i) X, the operation time which is the minimum of machine life time and manpower service time,

(ii) $\hat{V}$, the sum of recruitment times

(iii) R, the repair time of the machine and

(iv) $\hat{S}$, the total sales time is to be presented. It is seen as below.

$$f(x,y,z,w) = \left\{ \sum_{i=1}^{m} p_i \lambda_i e^{-\lambda_i x} r_i(z) \sum_{n=0}^{\infty} (F_n(x) - F_{n+1}(x)) q^n v_n(y) + \sum_{i=1}^{m} p_i e^{-\lambda_i x} \sum_{n=1}^{\infty} f_n(x) p q^{n-1} v_n(y) \right\}$$
$$[\sum_{k=0}^{\infty} e^{-\mu x} \frac{(\mu x)^k}{k!} g_k(w)] \tag{1}$$

The first term of equation (1) inside the bracket is the part of the pdf that the machine fails, n employees have left not causing manpower failure and the repair and recruitments are done. The second term is the part of the pdf that the machine does not fail, but the manpower fails on the departure of the n-th employee and the recruitments are done one by one. The last bracket indicates k products are produced and sold one by one. The suffix letter indicates the convolution of pdf with itself and Stiltjes convolution of Cdf with itself. We find the quadruples Laplace Transform as follows for the joint pdf.

$$f^*(\xi,\eta,\varepsilon,\delta) = \int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty} e^{-\xi x-\eta y-\varepsilon z-\delta w} f(x,y,z,w)\,dx\,dy\,dz\,dw \tag{2}$$

Now,

$$f^*(\xi,\eta,\varepsilon,\delta)$$
$$= \sum_{i=1}^{m} p_i \lambda_i r_i(\varepsilon) \int_0^{\infty} e^{-\lambda_i x} e^{-\mu x(1-g^*(\delta))-\xi x} \sum_{n=0}^{\infty} (F_n(x) - F_{n+1}(x)) q^n v^{*n}(\eta)\,dx$$
$$+ \sum_{i=1}^{m} p_i p \int_0^{\infty} e^{-\lambda x} e^{-\mu x(1-g^*(\delta))-\xi x} \sum_{n=1}^{\infty} f_n(x) q^{n-1} v^{*n}(\eta)\,dx \tag{3}$$

We obtain

$$f^*(\xi,\eta,\varepsilon,\delta) = \sum_{i=1}^{m} p_i \left[\frac{\lambda_i r_i^*(\varepsilon)(1 - f^*(\chi_i))}{\chi_i(1 - q v^*(\eta) f^*(\chi_i))} \right] \sum_{i=1}^{m} p_i \left[\frac{p v^*(\eta) f^*(\chi_i)}{(1 - q v^*(\eta) f^*(\chi_i))} \right] \tag{4}$$

Here $\chi_i = \xi + \lambda_i + \mu(1 - g^*(\delta)),\ for\ i = 1, 2, 3, \dots \dots ..m$ (5)

The Laplace transform of the operation time X is

$$f^*(\xi, 0, 0, 0) = \sum_{i=1}^{m} \left[\frac{p_i \lambda_i (1 - f^*(\xi + \lambda_i)}{(\xi + \lambda_i)(1 - qf^*(\xi + \lambda_i)} \right] \sum_{i=1}^{m} p_i \left[\frac{pf^*(\xi + \lambda_i)}{(1 - qf^*(\xi + \lambda_i))} \right] \quad (6)$$

$$E(X) = -\frac{\partial}{\partial \xi} f^*(\xi, 0, 0, 0) \mid \xi = 0, gives$$

$$E(X) = \sum_{i=1}^{m} p_i \left[\frac{1}{\lambda_i} \frac{(1 - f^*(\lambda_i))}{(1 - qf^*(\lambda_i))} \right] \quad (7)$$

Now

$$E(\hat{R}) = -\frac{\partial}{\partial \varepsilon} f^*(\xi, \eta, \varepsilon, \delta) \mid \xi = \eta = \varepsilon = \delta = 0$$

$$E(\hat{R}) = \sum_{i=0}^{m} p_i E(R_i) \frac{(1 - f^*(\lambda_i))}{(1 - qf^*(\lambda_i))} \quad (8)$$

Now the Laplace transform of $\hat{S}$ *is given by*

$$f^*(0, 0, 0, \delta) = \sum_{i=1}^{m} p_i \frac{\lambda_i}{(\lambda_i + \mu(1 - g^*(\delta))} \frac{(1 - f^*(\lambda_i + \mu(1 - g^*(\delta))))}{(1 - qf^*(\lambda_i + \mu(1 - g^*(\delta))))}$$
$$+ \sum_{i=1}^{m} p_i \frac{pf^*(\lambda_i + \mu(1 - g^*(\delta)))}{(1 - qf^*(\lambda_i + \mu(1 - g^*(\delta))))} \quad (9)$$

$$E(\hat{S}) = \mu E(G) \sum_{i=1}^{m} \frac{p_i}{\lambda_i} \frac{(1 - f^*(\lambda_i))}{(1 - qf^*(\lambda_i))} \quad (10)$$

The joint Laplace transform of p.d.f. of $(X, \hat{V})$ is given by

$$f^*(\xi, \eta, 0, 0) = \sum_{i=1}^{m} p_i \frac{\lambda_i (1 - f^*(\xi + \lambda_i))}{(\xi + \lambda_i)(1 - qv^*(\eta) f^*(\xi + \lambda_i))} + \sum_{i=1}^{m} p_i \frac{pv^*(\eta) f^*(\xi + \lambda_i)}{(1 - qv^*(\eta) f^*(\xi + \lambda_i))} \quad (11)$$

The expected recruitment time is

$$E(\hat{V}) = -\frac{\partial}{\partial \eta} f^*(\xi, \eta, 0, 0) \mid \xi = \eta = 0.$$

$$E(\hat{V}) = E(V) \sum_{i=1}^{m} p_i \frac{f^*(\lambda_i)}{(1 - qf^*(\lambda_i))} \quad (12)$$

The product moment $E(X \hat{V})$ is given by

$$E(X \hat{V}) = \frac{\partial^2}{\partial \xi \partial \eta} f^*(\xi, \eta, 0, 0) \mid \xi = \eta = 0$$

Here

$$E(X \hat{V}) = E(V) \sum_{i=1}^{m} \frac{p_i}{(1 - qf^*(\lambda_i))^2} \left(\frac{q}{\lambda_i} f^*(\lambda_i)(1 - f^*(\lambda_i)) - f^*(\lambda_i) \right) (13)$$

Using the formula $Cov(X \hat{V}) = E(X \hat{V}) - E(X) E(\hat{V})$

We can write Co-variance of X and $\hat{V}$ using equations (13),(12) and (7).

MODEL 2

In this section, we treat the previous model 1 with all assumptions (1),(2),(3),(5),(6) and (7) except the assumptions concerning the recruitment of employees.

ASSUMPTION FOR MANPOWER RECRUITMENT:

(4.1) When the operation time X is more than a threshold U, the recruitments are done all together. It is assigned to an agent whose service time V_1, to fill up all vacancies has Cdf $V_1(y)$ and pdf $v_1(y)$.

(4.2) When the operation time X is less than the threshold time U, the recruitments are done one by one and the recruitment time is V_2 which has Cdf $V_2(y)$ and pdf $v_2(y)$.

(4.3) The threshold U has exponential distribution with parameter θ.

ANALYSIS;

Using the arguments given for model 1, the joint pdf of $(X,\hat{V},\hat{R},\hat{S})$, (Operation time, Recruitment time, Repair time of machine, Sales time) may be obtained as follows.

$$f(x,y,z,w)$$

$$= \left\{ \begin{array}{l} \sum_{i=1}^{m} p_i \lambda_i e^{-\lambda_i x} r_i(z) \sum_{n=0}^{\infty} (F_n(x) - F_{n+1}(x)) q^n [(1-e^{-\theta x}) v_1(y) + e^{-\theta x} v_{2,n}(y)] \\ + \sum_{i=1}^{m} p_i e^{-\lambda_i x} \sum_{n=1}^{\infty} f_n(x) p q^{n-1} [(1-e^{-\theta x}) v_1(y) + e^{-\theta x} v_{2,n}(y)] \end{array} \right\} [\sum_{k=0}^{\infty} e^{-\mu x} \frac{(\mu x)^k}{k!} g_k(w)] \qquad (14)$$

We use the same arguments given for model 1 for all terms except the square brackets appearing in the first and second term which replaced $v_n(y)$ of equation (1) here. The terms in the bracket indicates the recruitment pdf, $v_1(y)$ for all employees together, when the threshold is less than the operation time (U<X) and the recruitment time pdf is $v_2(y)$ for each employee when the threshold is greater than X, (U>X). The suffix letter indicates the convolution of the pdf or Cdf as the case may be. The function $v_{2,n}(y)$ is the n-fold convolution of $v_2(y)$ with itself. The Laplace transform of the pdf of the four variables

$(X, \hat{V}, \hat{R}, \hat{S})$ *is*

$$f^*(\xi,\eta,\varepsilon,\delta) = \int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty} e^{-\xi x - \eta y - \varepsilon z - \delta w} f(x,y,z,w)\,dx\,dy\,dz\,dw$$

This reduces to a single integral

$$f^*(\xi,\eta,\varepsilon,\delta) = \sum_{i=1}^{m} p_i \lambda_i r_i^*(\varepsilon) \int_0^{\infty} e^{-\xi x - \lambda_i x - \mu x(1-g^*(\delta))} \sum_{n=0}^{\infty} (F_n(x) - F_{n+1}(x)) q^n [(1-e^{-\theta x}) v_1^*(\eta) + e^{-\theta x} v_2^{*n}(\eta)]$$

$$+ \sum_{n=0}^{m} p_i p \int_0^{\infty} e^{-\xi x - \lambda x - \mu x(1-g^*(\delta))} \sum_{n=1}^{\infty} f_n(x) q^{n-1} [(1-e^{-\theta x}) v_1^*(\eta) + e^{-\theta x} v_2^{*n}(\eta)] \qquad (15)$$

The equation (15) is similar to equation (3) where $v^{*n}(\eta)$ of (3) is replaced by the square bracket. This reduces using equation (5) to

$$f^*(\xi,\eta,\varepsilon,\delta)=$$

$$\sum_{i=1}^{m} p_i\lambda_i r_i^*(\varepsilon)\left[\frac{v_1^*(\eta)(1-f^*(\chi_i))}{\chi_i(1-qf^*(\chi_i))}-\frac{v_1^*(\eta)(1-f^*(\chi_i+\theta))}{(\chi_i+\theta)(1-qf^*(\chi_i+\theta))}+\frac{(1-f^*(\chi_i+\theta))}{(\chi_i+\theta)(1-qv_2^*(\eta)f^*(\chi_i+\theta))}\right]$$

$$+\sum_{i=1}^{m} p_i p\left[\frac{v_1^*(\eta)f^*(\chi_i)}{(1-qf^*(\chi_i))}-\frac{v_1^*(\eta)f^*(\chi_i+\theta)}{(1-qf^*(\chi_i+\theta))}+\frac{v_2^*(\eta)f^*(\chi_i+\theta))}{(1-qv_2^*(\eta)f^*(\chi_i+\theta))}\right] \quad (16)$$

Since there is only change in the recruitment pattern of employees to fill up the man power loss $E(X), E(\hat{R})$ and $E(\hat{S})$ remain the same as those of model 1. The Laplace transform of the pdf of $(X,\hat{V})$ is

$$f^*(\xi,\eta,0,0)=$$

$$\sum_{i=1}^{m} p_i\lambda_i\left[\frac{v_1^*(\eta)(1-f^*(\psi_i))}{\psi_i(1-qf^*(\psi_i))}-\frac{v_1^*(\eta)(1-f^*(\psi_i+\theta))}{(\psi_i+\theta)(1-qf^*(\psi_i+\theta))}+\frac{(1-f^*(\psi_i+\theta))}{(\psi_i+\theta)(1-qv_2^*(\eta)f^*(\psi_i+\theta))}\right]$$

$$+\sum_{i=1}^{m} p_i p\left[\frac{v_1^*(\eta)f^*(\psi_i)}{(1-qf^*(\psi_i))}-\frac{v_1^*(\eta)f^*(\psi_i+\theta)}{(1-qf^*(\psi_i+\theta))}+\frac{v_2^*(\eta)f^*(\psi_i+\theta))}{(1-qv_2^*(\eta)f^*(\psi_i+\theta))}\right] (17)$$

Here $\psi_i=\xi+\lambda_i$ for i=1,2,3,….m (18)

$E(\hat{V})$ is given by

$$E(\hat{V})=-\frac{\partial}{\partial\eta}f^*(\xi,\eta,0,0)\,|\,\xi=\eta=0.$$

$$E(\hat{V})=E(V_1)\sum_{i=1}^{m} p_i\left[\frac{(1-f^*(\lambda_i))}{(1-qf^*(\lambda_i))}-\frac{\lambda_i}{(\lambda_i+\theta)}\frac{(1-f^*(\lambda_i+\theta))}{(1-qf^*(\lambda_i+\theta))}+\frac{pf^*(\lambda_i)}{(1-qf^*(\lambda_i))}-\frac{pf^*(\lambda_i+\theta)}{(1-qf^*(\lambda_i+\theta))}\right]$$

$$+E(V_2)\sum_{i=1}^{m} p_i\frac{f^*(\lambda_i+\theta)}{(1-qf^*(\lambda_i+\theta))^2}\left[p+\left(\frac{\lambda_i}{\lambda_i+\theta}\right)q(1-qf^*(\lambda_i+\theta))\right] \quad (19)$$

The product moment $E(X\hat{V})$ is given by

$$E(X\hat{V})=\frac{\partial^2}{\partial\xi\partial\eta}f^*(\xi,\eta,0,0)\,|\,\xi=\eta=0$$

$$E(X\hat{V})=$$

$$\sum_{i=1}^{m} p_iE(V_1)\left\{\begin{array}{l}\frac{1}{\lambda_i}\frac{(1-f^*(\lambda_i))}{(1-qf^*(\lambda_i))}-\frac{\lambda_i}{(\lambda_i+\theta)^2}\frac{(1-f^*(\lambda_i+\theta))}{(1-qf^*(\lambda_i+\theta))}\\ -\left(\frac{\lambda_i}{\lambda_i+\theta}\right)\frac{pf^{*'}(\lambda_i+\theta)}{(1-qf^*(\lambda_i+\theta))^2}+\frac{pf^{*'}(\lambda_i+\theta)}{(1-qf^*(\lambda_i+\theta))^2}\end{array}\right\}$$

$$+\sum_{i=1}^{m} p_iE(V_2)\left\{\begin{array}{l}\frac{\lambda_i}{(\lambda_i+\theta)^2}\frac{qf^*(\lambda_i+\theta)(1-f^*(\lambda_i+\theta))}{(1-qf^*(\lambda_i+\theta))^2}\\ -\frac{f^{*'}(\lambda_i+\theta)}{(1-qf^*(\lambda_i+\theta))^3}\left(\frac{(1+qf^*(\lambda_i+\theta)(\lambda_i+\theta p)-2q\lambda_i f^*(\lambda_i+\theta)}{(\lambda_i+\theta)}\right)\end{array}\right\} \quad (20)$$

Covariance of $(X, \hat{V})$ may be written using $Cov(X\hat{V}) = E(X\hat{V}) - E(X)E(\hat{V})$ and equations (7), (19) and (20).

II. NUMERICAL EXAMPLES

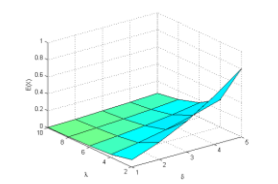
Model 1

Let the parameters and expected values of random variables stated in the assumptions of model 1 be assumed as follows:

(i) Expected departure time of employees E(F)= 10,
(ii) The probability of failure p=0.5,
(iii) Expected repair time of the machine E (R_i) =5, 10,15,20,25.
(iv) Expected recruitment time E(V)=5
(v) The failure parameter is λ=2,4,6,8,10
(vi) The exponential density parameter is δ =1,2,3,4,5
(vii) The production time distribution parameter μ = 1
(viii)The expected sales time E (G) = 2.

Table and graph of E(X)

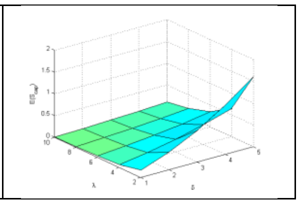
λ/δ	1	2	3	4	5
2	0.080634	0.201585	0.362854	0.564439	0.8063411
4	0.032565	0.081413	0.146543	0.227955	0.3256503
6	0.017202	0.043005	0.077408	0.120413	0.1720183
8	0.009848	0.02462	0.044316	0.068937	0.0984808
10	0.005634	0.014086	0.025355	0.039441	0.0563436



From the table and graph it is observed that when λ increases the expected operation time E(X) decreases and when δ increases E(X) increases.

The table and graph for $E(\hat{S})$

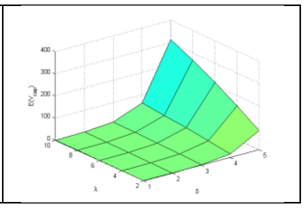
λ/δ	1	2	3	4	5
2	0.161268	0.403171	0.725707	1.128878	1.6126823
4	0.06513	0.162825	0.293085	0.45591	0.6513007
6	0.034404	0.086009	0.154817	0.240826	0.3440367
8	0.019696	0.04924	0.088633	0.137873	0.1969617
10	0.011269	0.028172	0.050709	0.078881	0.1126872



From the table and graph it is observed that when λ increases the expected sales time decreases and when δ increases the expected sales time increases.

The table and graph for $E(\hat{V})$

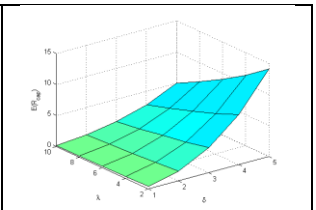
λ/δ	1	2	3	4	5
2	0.096829	0.629391	3.340615	16.94515	85.016237
4	0.17435	1.133273	6.015064	30.51119	153.07901
6	0.241972	1.572821	8.34805	42.34518	212.45184
8	0.303038	1.969749	10.45482	53.03171	266.06766
10	0.359141	2.334417	12.39037	62.84969	315.32585



From the table and graph it is observed that when λ increases the expected recruitment time increases and when δ increases the expected recruitment time also increases.

The table and graph for $E(\hat{R})$

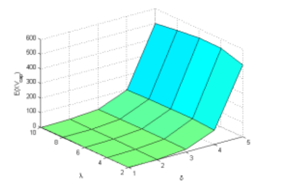
λ/δ	1	2	3	4	5
2	0.403171	1.612682	4.031706	8.063411	14.11097
4	0.32565	1.302601	3.256503	6.513007	11.397761
6	0.258028	1.03211	2.580275	5.16055	9.0309633
8	0.196962	0.787847	1.969617	3.939233	6.8936583
10	0.140859	0.563436	1.408589	2.817179	4.9300629



From the table and graph it is observed that when λ increases the expected repair time decreases and when δ increases the expected repair time increases.

The table and graph for $E(X,\hat{V})$

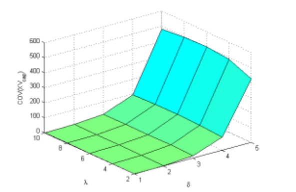
λ/δ	1	2	3	4	5
2	0.56	3.64	19.32	98	491.68
4	0.611111	3.972222	21.08333	106.9444	536.55556
6	0.612245	3.979592	21.12245	107.1429	537.55102
8	0.59375	3.859375	20.48438	103.9063	521.3125
10	0.567901	3.691358	19.59259	99.38272	498.61728



From the table and graph it is observed that when λ increases the expected product moment of operation time and recruitment time increases and when δ increases the expected product moment of operation time and recruitment time also increases.

The table and graph for $Cov(X,\hat{V})$

λ/δ	1	2	3	4	5
2	0.552192	3.513124	18.10785	88.4355	423.12791
4	0.605433	3.87996	20.20187	99.98926	486.70532
6	0.608083	3.911953	20.47624	102.044	501.00541
8	0.590766	3.810879	20.02106	100.2504	495.10994
10	0.565878	3.658476	19.27844	96.90389	480.8507



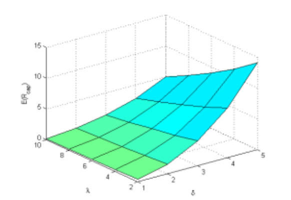
From the table and graph it is observed that when λ increases the covariance increases and when δ increases the covariance increases.

Model 2

(i) Expected departure time of employees E(F)= 10,
(ii) The probability of failure q=0.75,
(iii) The machine attended by manpower with probability p_i=0.25, i=1,2,3….10
(iv) Expected repair time of the machine E (R_i) =5, 10,15,20,25.
(v) Expected recruitment time when recruitments are done together is $E(V_1)$=2
(vi) Expected recruitment time when recruitments are done one by one is $E(V_2)$=5
(vii)The failure parameter is λ=2,4,6,8,10
(viii)The exponential density parameter is δ =1,2,3,4,5
(ix)The production time distribution parameter μ = 1
(x)The expected sales time E (G) = 2
(xi) The threshold has exponential distribution with parameter θ=5

The table and graph for $E(\hat{R})$

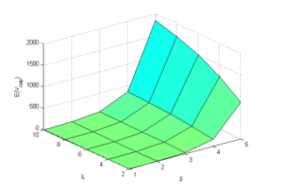
λ/δ	1	2	3	4	5
2	0.403171	1.612682	4.031706	8.063411	14.11097
4	0.32565	1.302601	3.256503	6.513007	11.397761
6	0.258028	1.03211	2.580275	5.16055	9.0309633
8	0.196962	0.787847	1.969617	3.939233	6.8936583
10	0.140859	0.563436	1.408589	2.817179	4.9300629



From the table and graph it is observed that when λ increases the expected repair time decreases and when δ increases the expected repair time increases.

The table and graph for $E(\hat{V})$

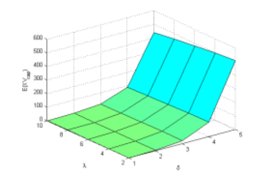
λ/δ	1	2	3	4	5
2	0.5907	5.2959	31.4546	166.6425	850.5013
4	0.8789	7.7046	45.6191	241.6879	1233.9480
6	1.0934	9.4633	55.8964	296.0071	1511.2278
8	1.2480	10.7193	63.2157	334.6526	1708.4238
10	1.3592	11.6160	68.4283	362.1533	1848.7121



From the table and graph it is observed that when λ increases the expected recruitment time increases and when δ increases the expected recruitment time also increases.

The table and graph for $E(X, \hat{V})$

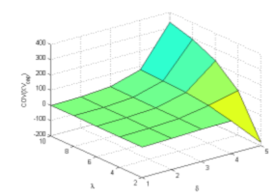
λ/δ	1	2	3	4	5
2	1.256184	5.78871	24.29892	107.8895	507.26582
4	1.105191	5.267299	22.89529	104.2153	496.72031
6	0.989052	4.849754	21.6022	99.8803	479.92068
8	0.881155	4.44999	20.28039	95.06498	459.91313
10	0.78135	4.072194	18.98443	90.15148	438.88828



From the table and graph it is observed that when λ increases the expected product moment of operation and recruitment time decreases and when δ increases the expected product moment of operation time and recruitment time increases.

The table and graph for $Cov(X, \hat{V})$

λ/δ	1	2	3	4	5
2	1.20855	4.721125	12.88552	13.83	-178.5284
4	1.076569	4.640047	16.21014	49.12123	94.884742
6	0.970244	4.44279	17.27535	64.23725	219.96176
8	0.868865	4.186078	17.4789	71.99517	291.66613
10	0.773692	3.908573	17.24946	75.86797	334.72523



From the table and graph it is observed that when λ increases the covariance decreases and when δ increases the covariance increases.

REFERENCES

[1] Barthlomew.D.J, Statistical technique for manpower planning, John Wiley, Chichester (1979)

[2] Esary.J.D, A.W. Marshall, F. Proschan, Shock models and wear processes, *Ann. Probability*, **1**, No. 4 (1973), 627-649

[3] Grinold.R.C, Marshall.K.T, Manpower Planning models, North Holl, Newyork (1977)

[4] Hari Kumar.K, P.Sekar and R.Ramanarayanan, Stochastic Analysis of Manpower levels affecting business with varying Recruitment rate, International Journal of Applied Mathematics, Vol.8,2014 no.29, 1421-1428.

[5] Hari Kumar.K, Manpower System with Erlang departure and one by one Recruitment, International Journal of Mathematical Archive-5(7), 2014, 89-92.ISSN 2229-5046.

[6] Lesson .G.W. Wastage and Promotion in desired Manpower Structures, J.Opl.Res.Soc. 33(1982)433-442.

[7] McClean, Semi-Markovian Models in continuous time, J.Appl.Prob., 16(1980),416-422.

[8] Mohan.C and Ramanarayanan.R ,An analysis of Manpower,Money and Business with Random Environments,International Journal of Applied Mathematics,23(5) (2010),927-940.

[9] Setlhare, Modeling of an intermittently busy manpower system, In: Proceedings at the Conference held in sept, 2006 at Gabarone, Botswana (2007)

[10] Snehalatha.M., Sekar.P and Ramanarayanan.R, Probabilistic Analysis of General Manpower SCBZ Machine System with Exponential Production, General Sales and General Recruitment, IOSR Journal of Mathematics[IOSRJM], *e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 11, Issue 2 Ver. VI (Mar - Apr. 2015), PP 12-21.*

[11] Snehalatha.M., Sekar.P and Ramanarayanan.R, General manpower-modified Erlang two phase machine system with markovian production, general recruitment and sales, Global Journal of Pure and Applied Mathematics (GJPAM) ISSN 0973-1768 Volume 12,Number 1 (2016),PP 42-50.

[12] Subramanian.V, Optimum promotion rate in manpower models. International Journal of Management and system, 12(2) (1996),179-184.

[13] Vassiliou.P.C.G, A higher order Markovian model for prediction of wastage in manpower system,Operat, Res.Quart.,27 (1976).