International Journal of Mathematics and Statistics Invention (IJMSI)
E-ISSN: 2321 – 4767 P-ISSN: 2321 - 4759
www.ijmsi.org Volume 4 Issue 4 || April. 2016 || PP-38-43

Some Properties of *M*-projective Curvature Tensor on Generalized Sasakian-Space-Forms

D. G. Prakasha, B. S. Hadimani and Vasant Chavan
(Department of Mathematics, Karnatak University, Dharwad - 580 003, INDIA)

ABSTRACT : *The object of this paper is to study the* M *-projective curvature tensor on generalized Sasakian-space-forms. We study* M *-projectively semisymmetric,* M *-projectively pseudosymmetric and* ϕ *-*M *-projectively semisymmetric generalized Sasakian-space-form.*

KEYWORDS - *Generalized Sasakian-space-form,* M *-projective curvature tensor, semisymmetric, pseudosymmetric.*

I. INTRODUCTION

The notion of generalized Sasakian-space-forms was introduced and studied by Alegre et al., [1] with several examples. A generalized Sasakian-space-form is an almost contact metric manifold (M, ϕ, ξ, η, g) whose curvature tensor is given by

$$\begin{aligned} R(X,Y)Z &= f_1\{g(Y,Z)X - g(X,Z)Y\} \\ &+ f_2\{g(X,\phi Z)\phi Y - g(Y,\phi Z)\phi X + 2g(X,\phi Y)\phi Z\} \\ &+ f_3\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X + g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi\}, \end{aligned} \tag{1}$$

where f_1, f_2, f_3 are differentiable functions on M and X, Y, Z are vector fields on M. In such a case we will write the manifold as $M^{2n+1}(f_1, f_2, f_3)$. This kind of manifolds appears as a natural generalization of the Sasakian-space-forms by taking: $f_1 = \frac{c+3}{4}$ and $f_2 = f_3 = \frac{c-1}{4}$, where c denotes constant ϕ -sectional curvature. The ϕ -sectional curvature of generalized Sasakian-space-form $M^{2n+1}(f_1, f_2, f_3)$ is $f_1 + 3f_2$. Moreover, cosymplectic space-form and Kenmotsu space-form are also considered as particular types of generalized Sasakian-space-form. The generalized Sasakian-space-forms have also been studied in ([2], [3], [4], [6], [7], [8], [9], [12], [14], [21], [20], [22]) and many others.

Apart from the conformal curvature tensor, the M -projective curvature tensor is another important tensor from the differential geometric point of view. This curvature tensor bridges the gap between conformal curvature tensor, conharmonic curvature tensor and concircular curvature tensor on one side and H -projective curvature tensor on the other.

The M -projetive curvature tensor of Riemannian manifold (M^{2n+1}, g) was defined by Pokhariyal and Mishra [19] as

$$\begin{aligned} \mathsf{M}(X,Y)Z &= R(X,Y)Z - \frac{1}{4n}[S(Y,Z)X - S(X,Z)Y \\ &+ g(Y,Z)QX - g(X,Z)QY], \end{aligned} \tag{2}$$

where R and S are the curvature tensor and the Ricci tensor of M, respectively, and Q is the Ricci operator defined as $S(X,Y) = g(QX, Y)$. Some properties of this tensor in Sasakian and Kähler manifolds have been studied earlier ([16], [17], [25]).

In the context of generalized Sasakian-space-forms, Kim [14] studied conformally flat and locally symmetric generalized Sasakian-space-forms. De and Sarkar [6] studied some symmetric properties of generalized Sasakian-space-forms with projective curvature tensor. In [20], Prakasha shown that every generalized Sasakian-space-form is Weyl-pseudosymmetric. The symmetric properties of generalized Sasakian-space-forms have also been studied in [12] with W_2 -curvature tensor. Also, Prakasha and Nagaraj [21] studied quasi-conformally flat and quasi-confomally semisymmetric generalized Sasakian-space-forms. Recently Hui and Prakasha [13] studied certain properties on the C-Bochner curvature tensor of generalized Sasakian-space-forms. As a motivation of this, here we plan to study certain symmetric properties on generalized Sasakian-space-form with $\mathcal{M}$ -projective curvature tensor.

The present paper is organised as follows: After preliminaries in section 3, we study $\mathcal{M}$ -projectively semisymmetric and $\mathcal{M}$ -projectively pseudosymmetric generalized Sasakian-space-forms. Also in section 4 we study ϕ -$\mathcal{M}$ -projectively semisymmetric generalized Sasakian-space-form.

II. PRELIMINARIES

An odd-dimensional Riemannian manifold (M, g) is said to be an *almost contact metric manifold* [5] if there exist on M a $(1,1)$ tensor field ϕ, a vector field ξ (called the structure vector field) and a 1-form η such that $\eta(\xi) = 1$, $\phi^2(X) = -X + \eta(X)\xi$ and $g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y)$, for any vector fields X, Y on M. In particular, in an almost contact metric manifold we also have $\phi\xi = 0$ and $\eta \circ \phi = 0$.

Such a manifold is said to be a contact metric manifold if $d\eta = \Phi$, where $\Phi(X, Y) = g(X, \phi Y)$ is called the *fundamental 2-form* of M. If , in addition, ξ is a killing vector field, then M is said to be a K-*contact manifold*. It is well-known that a contact metric manifold is K-contact manifold if and only if $\nabla_X \xi = -\phi X$, for any vector field X on M. On the other hand, the almost contact metric structure of M is said to be *normal* if $[\phi, \phi](X, Y) = -2d\eta(X, Y)\xi$, for any X, Y, where $[\phi, \phi]$ denotes the Nijenhuis torsion of ϕ. A normal contact metric manifold is called *Sasakian manifold*. An almost contact metric manifold is Sasakian if and only if $(\nabla_X \phi)Y = g(X, Y)\xi - \eta(Y)X$, for any X, Y.

In addition to the relation (1), for a $(2n+1)$ -dimensional $(n > 1)$ generalized Sasakian-space-form $M^{2n+1}(f_1, f_2, f_3)$ the following relations also hold [1]:

$$S(X, Y) = (2nf_1 + 3f_2 - f_3)g(X, Y) - (3f_2 + (2n+1)f_3)\eta(X)\eta(Y), \tag{3}$$

$$QX = (2nf_1 + 3f_2 - f_3)X - (3f_2 + (2n+1)f_3)\eta(X)\xi, \tag{4}$$

$$R(X, Y)\xi = (f_1 - f_3)(\eta(Y)X - \eta(X)Y), \tag{5}$$

$$R(X, \xi)Y = (f_1 - f_3)(\eta(Y)X - g(X, Y)\xi), \tag{6}$$

$$S(X, \xi) = 2n(f_1 - f_3)\eta(X), \tag{7}$$

$$r = 2n(2n+1)f_1 + 6nf_2 - 4nf_3, \tag{8}$$

where R, S and r are respectively denotes the curvature tensor of type $(1,3)$, Ricci tensor of type $(0,2)$ and scalar curvature of the space-form.

In view of (3)–(6), it can be easily construct that in a $(2n+1)$ -dimensional $(n > 1)$ generalized Sasakian-space-form $M^{2n+1}(f_1, f_2, f_3)$, the $\mathcal{M}$ -projective curvature tensor satisfies the following conditions:

$$\mathcal{M}(\xi, X)Y = \frac{-(3f_2 + (2n-1)f_3)}{4n}\{g(X, Y)\xi - \eta(X)\eta(Y)\xi\}, \tag{9}$$

$$\mathcal{M}(X, Y)\xi = \frac{-(3f_2 + (2n-1)f_3)}{4n}\{\eta(Y)X - \eta(X)Y\}, \tag{10}$$

$$\eta(\mathcal{M}(X, Y)Z) = \frac{-(3f_2 + (2n-1)f_3)}{4n}\{g(Y, Z)\eta(X) - g(X, Z)\eta(Y)\}, \tag{11}$$

for all vector fields X, Y, Z on $M^{2n+1}(f_1, f_2, f_3)$.

III. $\mathcal{M}$ -PROJECTIVELY SEMISYMMETRIC AND $\mathcal{M}$ - PROJECTIVELY PSEUDOSYMMETRIC GENERALIZED SASAKIAN-SPACE-FORMS

A Riemannain manifold M is called locally symmetric if its curvature tensor R is parallel, that is, $\nabla R = 0$, where ∇ denotes the Levi-Civita connection. As a proper generalization of locally symmetric manifolds the notion of semisymmetric manifolds was defined by

$$(R(X, Y) \cdot R)(U, V)W = 0, \quad X, Y, U, V, W \in \chi(M) \tag{12}$$

and studied by many authors(e.g.,[15, 18, 24]). A complete intrinsic classification of these spaces was given by Z. I. Szabo[23].

A generalized Sasakian space-form $M^{2n+1}(f_1, f_2, f_3)$ is said to be M -projectively symmetric if $\nabla M = 0$ and it is called M -projectively semisymmetric if,

$$(R(X,Y)\cdot \mathrm{M})(U,V)W = 0. \tag{13}$$

From (13), we have

$$R(X,\xi)\mathrm{M}(U,V)W - \mathrm{M}(R(X,\xi)U,V)W \tag{14}$$
$$- \mathrm{M}(U,R(X,\xi)V)W - \mathrm{M}(U,V)R(X,\xi)W = 0.$$

In the view of (6), the above equation becomes

$$(f_1 - f_3)[g(\xi,\mathrm{M}(U,V)W)X - g(X,\mathrm{M}(U,V)W)\xi - \eta(U)\mathrm{M}(X,V)W$$
$$+ g(X,U)\mathrm{M}(\xi,V)W - \eta(V)\mathrm{M}(U,X)W + g(X,V)\mathrm{M}(U,\xi)W$$
$$- \eta(W)\mathrm{M}(U,V)X + g(X,W)\mathrm{M}(U,V)\xi] = 0. \tag{15}$$

Putting $V = \xi$ in (15) and using (9), (10) and (11), we have

$$(f_1 - f_3)[\mathrm{M}(U,X)W + \frac{3f_2 + (2n-1)f_3}{4n}\{\eta(U)\eta(W)X - g(U,W)X$$
$$- 2\eta(X)\eta(U)\eta(W)\xi + g(X,W)\eta(U)\xi + g(U,X)\eta(W)\xi\}]. \tag{16}$$

Thus, we obtain either $f_1 - f_3 = 0$ or

$$\mathrm{M}(U,X)W = \frac{3f_2 + (2n-1)f_3}{4n}\{g(U,W)X - \eta(U)\eta(W)X + 2\eta(X)\eta(W)\xi$$
$$- g(X,W)\eta(U)\xi - g(U,X)\eta(W)\xi\}. \tag{17}$$

Taking the inner product on both sides of relation (17) with V , we get

$$g(\mathrm{M}(U,X)W,V) = \frac{3f_2 + (2n-1)f_3}{4n}\{g(U,W)g(X,V) - \eta(U)\eta(W)g(X,V)$$
$$+ 2\eta(X)\eta(W)\eta(V) - g(X,W)\eta(U)\eta(V)$$
$$- g(U,X)\eta(W)\eta(V)\}. \tag{18}$$

Let $\{e_i\}$ is an orthonormal basis of the tangent space at each point of the manifold and taking summation over i, $i = 1,2,...,2n+1$ in (17), we have

$$S(X,W) = \frac{r}{2n+1}g(X,W), \tag{19}$$

plugging $W = \xi$ in (19) and using (7) and (8), we obtain

$$[(1-2n)f_3 - 3f_2]\eta(X) = 0. \tag{20}$$

In this case, since $\eta(X) \neq 0$, the relation (20) implies that

$$f_3 = \frac{3f_2}{1-2n}. \tag{21}$$

Now, with the help of (21), the equation (17) reduces to

$$\mathrm{M}(U,X)W = 0. \tag{22}$$

That is $M^{2n+1}(f_1, f_2, f_3)$ is M -projectively flat. Hence we conclude the following:

Theorem 3.1 *A* $(2n+1)$ *-dimension* $(n>1)$ M *-projectively semisymmetric generalized Saskian-space-form is* M *-projectively flat (then* $f_3 = \frac{3f_2}{1-2n}$*) or* $f_1 = f_3$.

Corollary 3.2 *A* $(2n+1)$ *-dimension* $(n>1)$ *generalized Saskian-space-form is* M *-projectively semisymmetric if and only if* M *-projectively flat (* $f_3 = \frac{3f_2}{1-2n}$*).*

Next, for a $(0, k)$-tensor field T on M, $k \geq 1$, and a symmetric $(0,2)$ -tensor field A on M, we define the $(0, k+2)$ -tensor fields $R \cdot T$ and $Q(A,T)$ by

$$(R.T)(X_1, ..., X_K; X, Y)$$
$$= -T(R(X,Y)X_1, X_2, ..., X_k) - ... - T(X_1, ..., X_{k-1}, R(X,Y)X_k) \tag{23}$$

and

$$Q(A,T)(X_1, ..., X_K; X, Y)$$
$$= -T((X \wedge_A Y)X_1, X_2, ..., X_k) - ... - T(X_1, ..., X_{k-1}, (X \wedge_A Y)X_k) \tag{24}$$

respectively, where $X \wedge_A Y$ is the endomorphism given by

$$(X \wedge_A Y)Z = A(Y,Z)X - A(X,Z)Y. \tag{25}$$

A Riemannian manifold M is said to be pseudosymmetric (in the sense of R. Deszcz [10, 11]) if

$$R \cdot R = L_R Q(g, R) \tag{26}$$

holds on $U_R = \{x \in M \mid R - \frac{r}{n(n-1)} G \neq 0 \ at \ x\}$, where G is the $(0,4)$ -tensor defined by $G(X_1, X_2, X_3, X_4) = g((X_1 \wedge X_2)X_3, X_4)$ and L_R is some smooth function on U_R. A Riemannian manifold M is said to be M -projectively pseudosymmetric if

$$(R(X,Y) \cdot \mathrm{M})(U,V)W = L_{\mathrm{M}} Q(g, \mathrm{M})(U, V, W; X, Y), \tag{27}$$

holds on the set $U_{\mathrm{M}} = \{x \in M : \mathrm{M} \neq 0\}$ at x, where L_{M} is some function on U_{M} and M is the M -projective curvature tensor.

Let $M^{2n+1}(f_1, f_2, f_3)$ be a $(2n+1)$ -dimensional $(n > 1)$ M -projectively pseudosymmetric generalized Sasakian-space-form. Then from (25) and (27), we have

$$(R(\xi, Y) \cdot \mathrm{M})(U,V)W = L_{\mathrm{M}}[((\xi \wedge Y) \cdot \mathrm{M})(U,V)W]. \tag{28}$$

If $M^{2n+1}(f_1, f_2, f_3)$ be a $(2n+1)$ -dimensional $(n > 1)$ generalized Sasakian-space-form, from (6) and (25), we get

$$R(\xi, X)Y = (f_1 - f_3)(\xi \wedge X)Y. \tag{29}$$

In view of (28) in (29), it is easy to see that

$$L_{\mathrm{M}} = (f_1 - f_3). \tag{30}$$

Hence, by taking account of previous calculations and discussion, we conclude the following:

Theorem 3.3 *Let $M^{2n+1}(f_1, f_2, f_3)$ be a $(2n+1)$ -dimensional $(n > 1)$ generalized Sasakian-space-form. If $M^{2n+1}(f_1, f_2, f_3)$ is M -projectively pseudosymmetric then $M^{2n+1}(f_1, f_2, f_3)$ is either M -projectively flat, in which case $f_3 = \frac{3f_2}{1-2n}$ or $L_{\mathrm{M}} = f_1 - f_3$ holds on $M^{2n+1}(f_1, f_2, f_3)$.*

But L_{M} need not be zero, in general and hence there exists M -projectively pseudosymmetric manifolds which are not M -projectively semisymmetric. Thus the class of M -projectively pseudosymmetric manifolds is a natural extension of the class of M -projective semisymmetric manifolds. Thus, if $L_{\mathrm{M}} \neq 0$ then it is easy to see that $R \cdot \mathrm{M} = (f_1 - f_3)Q(g, \mathrm{M})$, which implies that the pseudosymmetric function $L_{\mathrm{M}} = f_1 - f_3$. Therefore, we able to state the following result:

Theorem 3.4 *Every generalized Sasakian-space-form is M -projectively pseudosymmetric of the form* $R \cdot \mathrm{M} = (f_1 - f_3)Q(g, \mathrm{M})$.

IV. ϕ -M -PROJECTIVELY SEMISYMMETRIC GENERALIZED SASAKIAN-SPACE-FORM

Definition 4.1 *For a $(2n+1)$ -dimensional $(n>1)$ generalized Sasakian-space-form is said to be ϕ -M -projectively semisymmetric if it satisfies the condition $\mathrm{M}(X,Y)\cdot\phi = 0$.*

Thus, we get

$$(\mathrm{M}(X,Y)\cdot\phi)Z = \mathrm{M}(X,Y)\phi Z - \phi\mathrm{M}(X,Y)Z = 0, \tag{31}$$

for all vector fields $X, Y, Z \in \chi(M)$.

Now, by virtue of (2), we have

$$\mathrm{M}(X,Y)\phi Z = R(X,Y)\phi Z - \frac{1}{4n}[S(Y,\phi Z)X - S(X,\phi Z)Y + g(Y,\phi Z)QX - g(X,\phi Z)QY]. \tag{32}$$

Taking use of (1), (3) and (4) in (32), we obtain

$$\begin{aligned}\mathrm{M}(X,Y)\phi Z = {} & \frac{-(3f_2 - f_3)}{2n}\{g(Y,\phi Z)X - g(X,\phi Z)Y\} + f_2\{-g(X,Z)\phi Y + \eta(Z)\eta(X)\phi Y \\ & + g(Y,Z)\phi X - \eta(Z)\eta(Y)\phi X - 2g(X,\phi Y)Z + 2g(X,\phi Y)\eta(Z)\xi\} \\ & + f_3\{g(X,\phi Z)\eta(Y)\xi - g(Y,\phi Z)\eta(X)\xi\}.\end{aligned} \tag{33}$$

Similarly,

$$\begin{aligned}\phi\mathrm{M}(X,Y)Z = {} & \frac{-(3f_2 - f_3)}{2n}\{g(Y,Z)\phi X - g(X,Z)\phi Y\} + f_2\{-g(X,\phi Z)Y \\ & + g(X,\phi Z)\eta(Y)\xi + g(Y,\phi Z)X - g(Y,\phi Z)\eta(X)\xi - 2g(X,\phi Y)Z \\ & + 2g(X,\phi Y)\eta(Z)\xi\} + f_3\{\eta(X)\eta(Y)\phi Y - \eta(Y)\eta(Z)\phi X\}.\end{aligned} \tag{34}$$

Substituting (33) and (34) in (31), we obtain

$$\begin{aligned}& \left(\frac{3f_2 - f_3}{2n} + f_3\right)\{g(Y,Z)\phi X - g(X,Z)\phi Y - g(Y,\phi Z) + g(X,\phi Z)Y\} \\ & + (f_1 - f_3)\{\eta(Z)\eta(Y)\phi Y - \eta(Z)\eta(Y)\phi X + g(Y,\phi Z)\eta(X)\xi \\ & - g(X,\phi Z)\eta(Y)\xi\} = 0.\end{aligned} \tag{35}$$

Setting $Y = \xi$ in (35), we get

$$\left(\frac{3f_2 - f_3}{2n} + f_3\right)\{g(X,\phi Z)\xi + \eta(Z)\phi X\} = 0. \tag{36}$$

In this case, since $g(X,\phi Z)\xi + \eta(Z)\phi X \neq 0$, the relation (36) implies that

$$f_3 = \frac{3f_2}{1-2n}. \tag{37}$$

Hence we state the following:

Theorem 4.2 *For a $(2n+1)$ -dimensional $(n>1)$ ϕ -M -projectively semisymmetric generalized Sasakian-space-form $M^{2n+1}(f_1, f_2, f_3)$, $f_3 = \frac{3f_2}{1-2n}$ holds.*

In a recent paper [6], De and Sarkar have proved the following:

Theorem 4.3 *[6] A $(2n+1)$ -dimensional $(n>1)$ generalized $M^{2n+1}(f_1, f_2, f_3)$ is projectively flat if and only if $f_3 = \frac{3f_2}{1-2n}$.*

Suppose, $f_3 = \frac{3f_2}{1-2n}$ holds. Therefore $M = 0$ and hence $M(X,Y)\cdot\phi = 0$.

Thus in view of Theorem (**4.2**), we can state the following:

Theorem 4.4 *A* $(2n+1)$ *-dimensional* $(n>1)$ *generalized Sasakian-space-form* $M^{2n+1}(f_1, f_2, f_3)$ *is* ϕ -M *- projectively semisymmetric if and only if* $f_3 = \dfrac{3f_2}{1-2n}$.

From the Theorem 7.2 of the [7], we note that a $(2n+1)$ -dimensional $(n>1)$ generalized Sasakian-space-form is Riccisymmetric if and only if $f_3 = \dfrac{3f_2}{1-2n}$.

By virtue of Theorem (**4.4**) we can state the following :

Theorem 4.5 *A* $(2n+1)$ *-dimensional* $(n>1)$ *generalized Sasakian-space-form* $M^{2n+1}(f_1, f_2, f_3)$ *is* ϕ -M *-projectively semisymmetric if and only if it is Ricci semisymmetric.*

By continuing Theorems (**4.2**), (**4.3**), (**4.5**), we can state the following:

Corollary 4.6 *Let* $M^{2n+1}(f_1, f_2, f_3)$ *be a* $(2n+1)$ *-dimensional* $(n>1)$ *generalized Sasakian-space-form. Then the following statements are equivalent.*

1. $M^{2n+1}(f_1, f_2, f_3)$ is ϕ -M -projectively semisymmetric.
2. $M^{2n+1}(f_1, f_2, f_3)$ is projectively flat.
3. $M^{2n+1}(f_1, f_2, f_3)$ is Ricci semisymmetric.
4. $f_3 = \dfrac{3f_2}{1-2n}$ holds $M^{2n+1}(f_1, f_2, f_3)$.

REFERENCES

Journal Papers:

[1] P. Alegre, D. E. Blair and A. Carriazo, Generalized Sasakian-space-forms, Israel J. Math., 14, *2004, 157-183.*

[2] P. Alegre and A. Carriazo, Structures on generalized Sasakian-space-forms, Diff. Geo. and its Application, 26, 2008, 656-666.

[3] P. Alegre and A. Carriazo, Submanifolds generalized Sasakian-space-forms, Taiwanese J. Math., 13, 2009, 923-941.

[4] P. Alegre and A. Carriazo, Generalized Sasakian-space-forms and conformal change of metric, Results Math., 59, 2011, 485-493.

[5] D. E. Blair, Contact manifolds in Riemannian geometry, Lecture Notes in Math., 509, Springer-Verlag, 1976.

[6] U. C. De and A. Sarkar, On the projective curvature tensor of generalized Sasakian-space-forms, Quaestiones Mathematicae, 33, 2010, 245-252.

[7] U. C. De and A. Sarkar, Some properties of generalized Sasakian-space-forms, Lobachevskii J. Math., 33, 2012, 22-27.

[8] U. C. De and P. Majhi, Certain curvture properties of generalized Sasakian-space-form, Proc. Natl. Acad. Sci., India, Sect. A Phys. Sci., 83,2013, 137-141.

[9] U. C. De and P. Majhi, ϕ -Semisymmetric generalized Sasakian space-forms, Arab J. Math. Sci., 21, 2015, 170-178.

[10] R. Deszcz, On pseudosymmetric spaces, Bull. Soc. Math. Belg. Ser. A., 44, 1992, 1-34.

[11] R. Deszcz, On some classes of wraped product manifolds, Bull. Inst. Math. Acad. Sinica., 15, 1987, 311-322.

[12] S. K. Hui and A. Sarkar, On the W_2- curvature tensor of generalized Sasakian-space-forms, Math. Pannon., 23(1), 2012, 113-124.

[13] S. K. Hui and D. G. Prakasha, On the C-Bochner Curvature Tensor of Generalized Sasakian-Space-Forms, Proc. Natl. Acad. Sci., India, Sect.A Phys. Sci., 85(3), 2015, 401-405.

[14] U. K. Kim, Conformally flat generalized Sasakian-space-forms and locally symmetric generalized Sasakian-space-forms, Note Mat., 26, 2006, 55-67.

[15] K. Nomizu, On hypersurfaces satisfying a certain condition on the curvature tensor, Tohoku Math. J., 20, 1968, 46-59.

[16] R. H. Ojha, A note on the *M*-projective curvature tensor, Indian J. Pure Appl. Math., 8, 12, 1975, 1531-1534.

[17] R. H. Ojha, M -projectively flat Sasakian manifolds, Indian J. Pure Appl. Math., 17(4), 1986, 481-484.

[18] Y. A. Ogawa, Condition for a compact Kahlerian space to be locally symmetric, Natur. Sci. Report, Ochanomizu Univ., 28, 1971, 21.

[19] G. P. Pokhariyal and R. S. Mishra,, Curvature tensor and their relativistic significance II, Yokohama Math. J., 19, 1971, 97-103.

[20] D. G. Prakasha, On generalized Sasakian-space-forms with Weyl-conformal curvature tensor, Lobachevskii J. Math., 33(3), 2012, 223-228.

[21] D. G. Prakasha and H. G. Nagaraja, On quasi-conformally flat and quasi-conformally semisymmetric generalized Sasakian-space-forms, Cubo (Temuco), 15(3), 2013, 59-70.

[22] D. G. Prakasha, Vasant Chavan and Kakasab Mirji, On the W_5- curvature tensor of generelized Sasakian space-forms, Konuralp J. Math., 1(4), 2016, 45-53.

[23] Z. I. Szabo, Structure theorems on Riemannian spaces satisfying $R(X, Y).\cdot R = 0$, The local version, J. Diff. Geo., 17, 1982, 531-582.

[24] S. Tanno, Locally symmetric K-contact Riemannian manifold. Proc. Japan, 43, 1967, 581-583.

[25] Venkatesha and B. Sumangala, On *M*-projective curvature tensor of a generalized Sasakian-space-form, Acta Math. Univ. Comenian. LXXXII (2), 2013, 209-217.