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Qausi Conformal Curvature Tensor on $(LCS)_n$-Manifolds

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ABSTRACT : *In this paper, we focus on qausi-conformal curvature tensor of $(LCS)_n$-manifolds. Here we study quasi-conformally flat, Einstein semi-symmetric quasi -conformally flat, ξ-quasi conformally flat and ϕ-quasi conformally flat $(LCS)_n$-manifolds and obtained some interesting results.*
Keywords: *Einstein semi-symmetric, η-Einstein manifold, Lorentzian metric, quasi-conformal curvature tensor, quasi-conformally flat.*

I. INTRODUCTION

In 1968, Yano and Sawaki [25] introduced the quasi-conformal curvature tensor given by

$$\tilde{C}(X,Y)Z = aR(X,Y)Z + b[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY\,] - \frac{r}{2n+1}\left(\frac{a}{2n} + 2b\right)[g(Y,Z)X - g(X,Z)Y], \tag{1.1}$$

where a and b are constants and R, S, Q and r are the Riemannian curvature tensor of type (1,3), the Ricci tensor of type (0,2), the Ricci operator defined by $S(X,Y) = g(QX,Y)$ and scalar curvature of the manifold respectively. If $a = 1$ and $b = -\frac{1}{n-2}$, then (1.1) takes the form

$$\tilde{C}(X,Y)Z = R(X,Y)Z - \frac{1}{n-2}\,[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY\,] - \frac{r}{(n-1)\,(n-2)}\,[g(Y,Z)X - g(X,Z)Y\,] = C(X,Y)Z, \tag{1.2}$$

Where C is the conformal curvature tensor [24]. In [7], De and Matsuyama studied a quasi-conformally flat Riemannian manifold satisfying certain condition on the Ricci tensor. Again Cihan Ozgar and De [5] studied quasi conformal curvature tensor on Kenmotsu manifold and shown that a Kenmotsu manifold is quasi-conformally flat or quasi- conformally semi-symmetric if and only if it is locally isometric to the hyperbolic space. The geometry of quasi-conformal curvature tensor in a Riemannian manifold with different structures were studied by several authors viz., [6, 16, 17, 20].

The present paper is organized as follows: In Section 2 we give the definitions and some preliminary results that will be needed thereafter. In Section 3 we discuss quasi-conformally flat $(LCS)_n$-manifolds and it is shown that the manifold is η-Einstein. Section 4 is devoted to the study of Einstein semi-symmetric quasi-conformally flat $(LCS)_n$-manifolds and obtain Qausi Conformal Curvature Tensor on $(LCS)_n$-Manifolds 3 that the scalar curvature is constant. In section 5 we consider ξ-quasi-conformally flat $(LCS)_n$-manifolds and proved that the scalar curvature is always constant. Finally, in Section 6, we have shown that a ϕ-quasi conformally flat $(LCS)_n$-manifold is an η-Einstein manifold.

II. PRELIMINARIES

The notion of Lorentzian concircular structure manifolds (briefly $(LCS)_n$-manifolds) was introduced by A.A. Shaikh [18] in 2003. An n-dimensional Lorentzian manifold M is a smooth connected paracompact Hausdorff manifold with a Lorentzian metric g, that is, M admits a smooth symmetric tensor field g of type (0,2) such that for each point $p \in M$, the tensor $g_p: T_pM \times T_pM \to R$ is a non-degenerate inner product of signature $(-,+,\dots,+)$, where T_pM denotes the tangent vector space of M at p and R is the real number space.

Definition 2.1 *In a Lorentzian manifold (M,g), a vector field P defined by $g(X,P) = A(X)$, for any vector field $X \in \chi(M)$ is said to be a concircular vector field if*

$$(\nabla_X A)(Y) = \alpha[g(X,Y) + \omega(X)A(Y)],$$

Where α is a non-zero scalar function, A is a 1-form and ω is a closed 1-form.

Let M be a n-dimensional Lorentzian manifold admitting a unit timelike concircular vector field ξ, called the characteristic vector field of the manifold. Then we have

$$g(\xi,\xi) = -1. \tag{2.1}$$

Since ξ is a unit concircular vector field, there exists a non-zero 1-form η such that

$$g(X,\xi) = \eta(X), \tag{2.2}$$

the equation of the following form holds

$$(\nabla_X \eta)(Y) = \alpha[g(X,Y) + \eta(X)\eta(Y)], (\alpha \neq 0) \tag{2.3}$$

for all vector fields X, Y , where r denotes the operator of covariant differentiation with respect to Lorentzian metric g and α is a non-zero scalar function satisfying

$$\nabla_X \alpha = (X\alpha) = d\alpha(X) = \rho\eta(X), \tag{2.4}$$

ρ being a certain scalar function given by $\rho = -(\xi\alpha)$. If we put

$$\phi X = \frac{1}{\alpha}\nabla_X \xi, \tag{2.5}$$

Then from (2.3) and (2.4), we have

$$\phi X = X + \eta(X)\xi, \tag{2.6}$$

from which it follows that ϕ is a symmetric (1,1) tensor. Thus the Lorentzian manifold M together with the unit time like concircular vector field ξ, its associated 1-form η and (1,1) tensor field ϕ is said to be a Lorentzian concircular structure manifold (briefly $(LCS)_n$-manifold). Especially, if we take $\alpha = 1$, then we can obtain the Lorentzian para-Sasakian structure of Matsumoto [12]. In a $(LCS)_n$-manifold, the following relations hold ([18], [19]):

$$\eta(\xi) = -1, \phi\xi = 0, \eta(\phi X) = 0, \tag{2.7}$$

$$g(\phi X, \phi Y) = g(X,Y) + \eta(X)\eta(Y), \tag{2.8}$$

$$\eta(R(X,Y)Z) = (\alpha^2 - \rho)[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)], \tag{2.9}$$

$$R(X,Y)\xi = (\alpha^2 - \rho)[\eta(Y)X - \eta(X)Y], \tag{2.10}$$

$$(\nabla_X \phi)(Y) = \alpha[g(X,Y)\xi + 2\eta(X)\eta(Y)\xi + \eta(Y)X] \tag{2.11}$$

$$S(X,\xi) = (n-1)(\alpha^2 - \rho)\eta(X), \tag{2.12}$$

$$S(\phi X, \phi Y) = S(X,Y) + (n-1)(\alpha^2 - \rho)\eta(X)\eta(Y), \tag{2.13}$$

$$Q\xi = (n-1)(\alpha^2 - \rho)\xi, \tag{2.14}$$

for any vector fields X, Y, Z, where R, S denote respectively the curvature tensor and the Ricci tensor of the manifold.

Definition 2.2 *An $(LCS)_n$-manifold M is said to be Einstein if its Ricci tensor S is of the form*

$$S(X,Y) = ag(X,Y), \tag{2.15}$$

for any vector fields X and Y, where a is a scalar function.

III. QUASI-CONFORMALLY FLAT $(LCS)_n$-MANIFOLDS

Let us consider quasi-conformally flat $(LCS)_n$-manifolds, i.e., $\tilde{C}(X,Y)Z = 0$. Then from (1.1), we have

$$\begin{aligned} R(X,Y)Z = &-\frac{b}{a}[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY] \\ &+\frac{r}{an}\Big(\frac{a}{n-1} + 2b\Big)[g(Y,Z)X - g(X,Z)Y\,]. \end{aligned} \tag{3.1}$$

Taking inner product of (3.1) with respect to W, we get

$$\begin{aligned} R(X,Y,Z,W) = &-\frac{b}{a}[S(Y,Z)g(X,W) - S(X,Z)g(Y,W) + g(Y,Z)S(X,W) - g(X,Z)S(Y,W)] \\ &+\frac{r}{an}\Big(\frac{a}{n-1} + 2b\Big)[g(Y,Z)g(X,W) - g(X,Z)g(Y,W)]. \end{aligned} \tag{3.2}$$

Also from (2.9), we have

$$R(\xi,Y,Z,\xi) = -(\alpha^2 - \rho)[g(Y,Z) + \eta(Y)\eta(Z)]. \tag{3.3}$$

Putting $X = W = \xi$ in (3.2) becomes

$$\begin{aligned} R(\xi,Y,Z,\xi) = &-\frac{b}{a}[S(Y,Z)g(\xi,\xi) - S(\xi,Z)g(Y,\xi) + g(Y,Z)S(\xi,\xi) - g(\xi,Z)S(Y,\xi)] \\ &+\frac{r}{an}\Big(\frac{a}{n-1} + 2b\Big)[g(Y,Z)g(\xi,\xi) - g(\xi,Z)g(Y,\xi)]. \end{aligned} \tag{3.4}$$

By virtue of (2.1), (2.2), (2.12) and (3.3) equation (3.4) yields

$$S(Y,Z) = Mg(Y,Z) + N\eta(Y)\eta(Z), \tag{3.5}$$

Where

$$M = \frac{r}{bn}\Big(\frac{a}{n-1} + 2b\Big) - \frac{\alpha^2 - \rho}{b}[a + b(n-1)],$$

$$N = \frac{r}{bn}\Big(\frac{a}{n-1} + 2b\Big) - \frac{\alpha^2 - \rho}{b}[a + 2b(n-1)].$$

Thus we state:

Theorem 3.1 A quasi-conformally flat $(LCS)_n$-manifold is an η-Einstein manifold.

IV. EINSTEIN SEMI-SYMMETRIC QUASI-CONFORMALLY FLAT $(LCS)_n$-MANIFOLDS

The Einstein tensor is given by

$$E(X,Y) = S(X,Y) - \frac{r}{2}g(X,Y), \quad (4.1)$$

Where S is the Ricci tensor and r is the scalar curvature. An n-dimensional quasi-conformally flat $(LCS)_n$-manifold is said to be Einstein semi-symmetric if,

$$R(X,Y)\cdot E(Z,W) = 0. \quad (4.2)$$

By using equation (3.5), we have

$$QX = MX + N\eta(X)\xi. \quad (4.3)$$

Substituting (3.5) and (4.3) in (3.1), we get

$$R(X,Y)Z = A[g(Y,Z)X - g(X,Z)Y] + B[\eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y + g(Y,Z)\eta(X)\xi - g(X,Z)\xi], \quad (4.4)$$

Where

$$A = \frac{\alpha^2-\rho}{b}[a + b(n-1) - \frac{a}{rn}\left(\frac{a}{n-1} + 2b\right)$$

$$B = \frac{r}{bn}\left(\frac{a}{n-1} + 2b\right) - \frac{\alpha^2-\rho}{b}[a + 2b(n-1)].$$

Now, we consider the quasi-conformally flat $(LCS)_n$ -manifold which is Einstein semi-symmetric i.e., $R(X,Y)\cdot E(Z,W) = 0.$

Then we have

$$E(R(X,Y)Z,U) + E(Z,R(X,Y)U) = 0. \quad (4.5)$$

By virtue of (4.1), equation (4.5) becomes

$$S(R(X,Y)Z,U) - \frac{r}{2}g(R(X,Y)Z,U) + S(Z,R(X,Y)U) - \frac{r}{2}g(Z,R(X,Y)U) = 0. \quad (4.6)$$

Using (3.5) in (4.6), we get

$$S(R(X,Y)Z,U) - \frac{r}{2}g(R(X,Y)Z,U) + S(Z,R(X,Y)U) - \frac{r}{2}g(Z,R(X,Y)U) = 0. \quad (4.7)$$

Put $Z = \xi$ in (4.7), we obtain

$$\left(M - \frac{r}{2}\right)g(R(X,Y)\xi,U) + \left(M - \frac{r}{2}\right)\eta(R(X,Y)U) + N\eta(R(X,Y)\xi)\eta(U) - N\eta(R(X,Y)U) = 0. \quad (4.8)$$

By virtue of (4.4), above equation becomes

$$N(B-A)[g(Y,U)\eta(X) - g(X,U)\eta(Y)] = 0. \quad (4.9)$$

Putting $Y = \xi$ in (4.9), we get

$$N[g(X,U) + \eta(U)\eta(X)] = 0. \quad (4.10)$$

Again putting $U = QW$ in (4.10), we have

$$N[S(X,W) + \eta(QW)\eta(X)] = 0. \quad (4.11)$$

Using (4.3) in (4.11), gives

$$N[S(X,W) + (M-N)\eta(W)\eta(X)] = 0. \quad (4.12)$$

Either $N = 0$ or $S(X,W) + (M-N)\eta(W)\eta(X) = 0.$

As $N \neq 0$, we have

$$S(X,W) + (M-N)\eta(W)\eta(X)] = 0. \quad (4.13)$$

Put $X = W = e_i$ in (4.13) and taking summation over $i, 1 \le i \le n$, we get

$$r = (n-1)(\alpha^2 - \rho). \quad (4.14)$$

Hence we can state the following:

Theorem 4.2 In an Einstein semi-symmetric quasi-conformally flat $(LCS)_n$-manifold, the scalar curvature is constant.

V. ξ-QUASI CONFORMALLY FLAT $(LCS)_n$-MANIFOLDS

A Let M be an n-dimensional ξ-quasi conformally flat $(LCS)_n$-manifold. i.e.,

$$\tilde{C}(X,Y)\xi = 0, \qquad \forall X,Y \in TM. \quad (5.1)$$

Putting $Z = \xi$ in (3.1), we get

$$\tilde{C}(X,Y)\xi = aR(X,Y)\xi + b[S(Y,\xi)X - S(X,\xi)Y + g(Y,\xi)QX - g(X,\xi)QY]$$
$$-\frac{r}{n}\left(\frac{a}{n-1} + 2b\right)[g(Y,\xi)X - g(X,\xi)Y]. \quad (5.2)$$

Since$\tilde{C}(X,Y)\xi = 0$, we have

$$aR(X,Y)\xi = \frac{r}{n}\left(\frac{a}{n-1} + 2b\right)[g(Y,\xi)X - g(X,\xi)Y]$$
$$-b[S(Y,\xi)X - S(X,\xi)Y + g(Y,\xi)QX - g(X,\xi)QY]. \quad (5.3)$$

Using(2.2), (2.10) and (2.12) in (5.3) becomes

$$a(\alpha^2-\rho)[\eta(Y)X - \eta(X)Y] = \frac{r}{n}\left(\frac{a}{n-1} + 2b\right)[\eta(Y)X - \eta(X)Y]$$

$$-2b(n-1)(\alpha^2-\rho)[\eta(Y)X-\eta(X)Y] \tag{5.4}$$

Again putting $Y=\xi$ in (5.4), we get

$$-a(\alpha^2-\rho)[X+\eta(X)\xi]=-\frac{r}{n}\Big(\frac{a}{n-1}+2b\Big)[X+\eta(X)\xi]+2(n-1)(\alpha^2-\rho)b[X+\eta(X)\xi]. \tag{5.5}$$

Taking inner product of above equation with respect to W , we obtain

$$-a(\alpha^2-\rho)[g(X,W)+\eta(X)\eta(W)]=-\frac{r}{n}\Big(\frac{a}{n-1}+2b\Big)[g(X,W)+\eta(X)\eta(W)]$$
$$+2(n-1)(\alpha^2-\rho)b[g(X,W)+\eta(X)\eta(W)]. \tag{5.6}$$

Put $X=W=e_i$ in (5.6) and taking summation over i, $1\le i\le n$, we get

$$r=\frac{n}{\Big(\frac{a}{n-1}+\ 2b\Big)}\big(a+2b(n-1)(\alpha^2-\rho)\big).$$

Hence we can state:

Theorem 5.3 In an ξ-quasi conformally flat $(LCS)_n$-manifold, the scalar curvature is constant.. (10)

VI. ϕ-QUASI CONFORMALLY FLAT $(LCS)_n$-MANIFOLDS

A Let M be an n-dimensional $(LCS)_n$-manifold is said to be ϕ-quasi conformally flat if it satisfies

$$\phi^2\tilde{C}(\phi X,\phi Y)\phi Z=0. \tag{6.1}$$

Theorem 6.4 An n-dimensional ϕ-quasi conformally flat $(LCS)_n$-manifold is an η-Einstein manifold.

Proof: Let us consider ϕ-quasi conformally flat $(LCS)_n$-manifold. i.e., $\phi^2\tilde{C}(\phi X,\phi Y)\phi Z=0$.

It can be easily see that

$$g\big(\tilde{C}(\phi X,\phi Y)\phi Z,\phi W\big)=\ 0. \tag{6.2}$$

By virtue of (1.1), we have

$$ag(R(\phi X,\phi Y)\phi Z,\phi W)=-b[S(\phi Y,\phi Z)g(\phi X,\phi W)]-S(\phi X,\phi Z)g(\phi Y,\phi W)$$
$$+S(\phi X,\phi W)g(\phi Y,\phi Z)-S(\phi Y,\phi W)g(\phi X,\phi Z)] \tag{6.3}$$
$$+\frac{r}{n}\Big(\frac{a}{n-1}+2b\Big)[g(\phi Y,\phi Z)g(\phi X,\phi W)-g(\phi X,\phi Z)g(\phi Y,\phi W)].$$

Let $\{e_1,e_2,\dots,e_{n-1},\xi\}$ be a local orthonormal basis of vector fields in M.

As $\{\phi(e_1),\phi(e_2),\dots,\phi(e_{n-1}),\xi\}$ is also a local orthonormal basis, if we put $X=W=e_i$ in (6.3) and sum up with respect to i, then we have

$$a\sum_{i=1}^{n-1}g\big(R(\phi(e_i),\phi Y)\phi Z,\phi(e_i)\big)$$
$$=\sum_{i=1}^{n-1}\big[S(\phi Y,\phi Z)g\big(\phi(e_i),\phi(e_i)\big)-S(\phi(e_i),\phi Z)g\big(\phi Y,\phi(e_i)\big)$$
$$+S\big(\phi(e_i),\phi(e_i)\big)g(\phi Y,\phi Z)-S\big(\phi Y,\phi(e_i)\big)g(\phi(e_i),\phi Z)\big]$$
$$+\frac{r}{n}\Big(\frac{a}{n-1}+2b\Big)\sum_{i=1}^{n-1}\big[g(\phi Y,\phi Z)g\big(\phi(e_i),\phi(e_i)\big)-g(\phi(e_i),\phi Z)g\big(\phi Y,\phi(e_i)\big)\big]. \tag{6.4}$$

It can be easily verify that

$$\sum_{i=1}^{n-1}g\big(R(\phi(e_i),\phi Y)\phi Z,\phi(e_i)\big)=S(\phi Y,\phi Z)+g(\phi Y,\phi Z), \tag{6.5}$$

$$\sum_{i=1}^{n-1}S\big(\phi Y,\phi(e_i)\big)g(\phi(e_i),\phi Z)=S(\phi Y,\phi Z), \tag{6.6}$$

$$\sum_{i=1}^{n-1}S(\phi(e_i),\phi(e_i))=r+(n-1)(\alpha^2-\rho), \tag{6.7}$$

$$\sum_{i=1}^{n-1}g\big(\phi(e_i),\phi(e_i)\big)=n-1, \tag{6.8}$$

$$\sum_{i=1}^{n-1}g(\phi(e_i),\phi Z)g\big(\phi Y,\phi(e_i)\big)\ =g(\phi Y,\phi Z). \tag{6.9}$$

Using (6.5) to (6.9) in (6.4), we get

$$S(Y,Z)=Mg(Y,Z)+N\eta(Y)\eta(Z),$$

Where

$$M = \frac{r(n-2)}{n}\left(\frac{a}{n-1} + 2b\right) + (n-1)(\alpha^2 - \rho) + r - a,$$
$$N = \frac{r(n-2)}{n}\left(\frac{a}{n-1} + 2b\right) + r + (\alpha^2 - \rho)[(n-1)(1-b) - an].$$

Hence the proof

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