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Total Dominating Color Transversal Number of Graphs And Graph Operations

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ABSTRACT : *Total Dominating Color Transversal Set of a graph is a Total Dominating Set of the graph which is also Transversal of Some χ - Partition of the graph. Here χ is the Chromatic number of the graph. Total Dominating Color Transversal number of a graph is the cardinality of a Total Dominating Color Transversal Set which has minimum cardinality among all such sets that the graph admits. In this paper, we consider the well known graph operations Join, Corona, Strong product and Lexicographic product of graphs and determine Total Dominating Color Transversal number of the resultant graphs.*

Keywords: *Domination number, Total Domination number, Total Dominating Color Transversal number*
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I. Introduction

We begin with simple, finite, connected and undirected graph without isolated vertices. We know that proper coloring of vertices of graph G partitions the vertex set V of G into equivalence classes (also called the color classes of G). Using minimum number of colors to properly color all the vertices of G yields χ equivalence classes. Transversal of a χ - Partition of G is a collection of vertices of G that meets all the color classes of the χ – Partition. That is, if T is a subset of V(the vertex set of G) and $\{V_1, V_2, ..., V_\chi\}$ is a χ - Partition of G then T is called a transversal of this χ - Partition if $T \cap Vi \neq \emptyset$, $\forall$ i $\in$ $\{1, 2,...., \chi\}$. Motivated by work of R.L.J. Manoharan in [12], we introduced the introduced the concept of Total Dominating Color Transversal Set of graphs in [1]. It is a collection of vertices of a graph G that is a Total Dominating Set of the graph with extra property that it is transversal of some χ – Partition of the graph. Let us first see some definitions.

II. Definitions

Definition 2.1[3]: (Dominating Set)
Let G = (V, E) be a graph. Then a subset S of V (the vertex set of G) is said to be a Dominating Set of G if for each $v \in V$ either $v \in S$ or v is adjacent to some vertex in S.

Definition 2.2[3]: (Minimum Dominating Set/ Domination number)
Let G = (V, E) be a graph. Then a Dominating Set S is called the Minimum Dominating Set of G if |S| = minimum {|D|: D is a Dominating Set of G}. In such case S is called a γ – Set of G and the cardinality of S is called Domination number of the graph G denoted by γ (G) or just by γ.

Definition 2.3[4]: (Total Dominating Set) Let G = (V, E) be a graph. Then a subset S of V (the vertex set of G) is said to be a Total Dominating Set of G if for each $v \in V$, v is adjacent to some vertex in S.

Definition 2.4[4]: (Minimum Total Dominating Set/Total Domination number) Let G = (V, E) be a graph. Then a Total Dominating Set S is said to be a Minimum Total Dominating Set of G if |S| = minimum {|D|: D is a Total Dominating Set of G}. Here S is called γ_t −set and its cardinality, denoted by γ_t(G) or just by γ_t, is called the Total Domination number of G.

Definition 2.5[1]: (χ -partition of a graph) Proper coloring of vertices of a graph G, by using minimum number of colors, yields minimum number of independent subsets of vertex set of G called equivalence classes (also called color classes of G). Such a partition of a vertex set of G is called a χ - Partition of the graph G.

Definition 2.6[1]: (Transversal of a χ - Partition of a graph) Let G = (V, E) be a graph with χ – Partition $\{V_1, V_2, ..., V_\chi\}$. Then a set $S \subset V$ is called a Transversal of this χ – Partition if $S \cap V_i \neq \emptyset$, $\forall$ i $\in$ $\{1, 2, 3, ..., \chi\}$.

Definition 2.7[1]: (Total Dominating Color Transversal Set) Let G = (V, E) be a graph. Then a Total Dominating Set S ⊂ V is called a Total Dominating Color Transversal Set of G if it is Transversal of at least one $\boldsymbol{\chi}$ – Partition of G.

Definition 2.8[1]: (Minimum Total Dominating Color Transversal Set/Total Dominating Color Transversal number) Let G = (V, E) be a graph. Then S ⊂ V is called a Minimum Total Dominating Color Transversal Set of G if |S| = minimum {|D|: D is a Total Dominating Color Transversal Set of G}. Here S is called γ_{tstd} −Set of G and its cardinality, denoted by γ_{tstd} (G) or just by γ_{tstd}, is called the Total Dominating Color Transversal number of G.

Definition 2.9[12]: (Join of Graphs) The Join of simple graphs G & H, written G ⋁ H, is the graph obtained from the disjoint union of their vertex sets by adding the edges{ uv : u ∈ V (G), v ∈V(H)}.

Definition 2.10[12]: (Corona of Graphs) Let G and H be two graphs. The corona G ∘ H is a graph formed from a copy of G and |V (G)| copies of H by joining the i-th vertex of G to every vertex in the i-th copy of H. (V (G) is the vertex set of G)

Definition 2.11[2]: (Strong Product of Graphs)
The Strong product G ⊠ H of graphs G and H is the graph with vertex set V (G)×V (H) and edge set {{(u, x), (v, y)} | u = v and x is adjacent to y in H or u is adjacent to v in G and x = y or u is adjacent to v in G and x is adjacent to y in H}.

Definition 2.12[2]: (Lexicographic Product of Graphs)
The Lexicographic product G[H] of graphs G and H is the graph with vertex set V (G)×V (H) and edge set {{(u, x), (v, y)} | u = v and x is adjacent to y in H or u is adjacent to v in G}.

III. Main Results

Theorem 3.1 [1]: If γ_t(G) = 2 then γ_{tstd}(G) = χ (G). (G may be disconnected)

Theorem 3.2: If G and H are two graphs then γ_{tstd} (G ∨ H) = χ (G) + χ (H).
Proof: We know that χ (G ∨ H) = χ (G) + χ (H). According to the definition of Join of graphs, γ_t (G ∨ H) = 2. So by theorem 3.1, γ_{tstd} (G ∨ H) = χ (G ∨ H). So γ_{tstd} (G ∨ H) = χ (G) + χ (H).

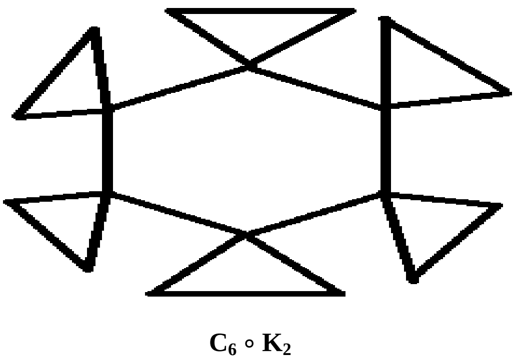
Theorem 3.3: If G and H are two graphs. If V (G) is the vertex set of graph G then

$$\gamma_{tstd}(G \circ H) = \begin{cases} |V(G)|, & \text{when } \chi(H) \le \chi(G) \\ |V(G)| + \chi(H) - \chi(G), & \text{when } \chi(H) > \chi(G). \end{cases}$$

Proof: Case 1. χ (H) ≤ χ (G)
Since each vertex of G is adjacent to all the vertices of some copy of H, the vertex set of G will form a γ_t −set of G ∘ H and as χ (H) ≤ χ (G), this set will also be a γ_{tstd} - Set of G ∘ H. So γ_{tstd} (G ∘ H) = |V (G)|.

Case 2. χ (H) > χ (G)
As in case 1, the vertex set of G will be γ_t −set of G. This set together with χ (H) – χ (G) vertices of H will a γ_{tstd} - Set of G ∘ H. So γ_{tstd} (G ∘ H) = |V (G)| + χ (H) – χ (G).

Example 3.4:



$C_6 \circ K_2$
Fig. 1 γ_{tstd} ($C_6 \circ K_2$) = 6

Now let us consider the strong product of graphs. First we mention one useful remark about this product.

Remark 3.5: 1) $G \boxtimes H \cong H \boxtimes G$.

2) By [8], If G and H have at least one edge, $\chi(G \boxtimes H) \geq \max\{\chi(G), \chi(H)\} + 2$ and by [9], $\chi(G \boxtimes H) \geq \chi(G) + \omega(H)$, where $\omega(H)$ is the clique number of H.

3) By [10], If G has at least one edge, $\chi(G \boxtimes H) \geq \chi(G) + 2\,\omega(H) - 2$, where $\omega(H)$ is the clique number of H.

4) $E(G \times H) \cup E(G \square H) = E(G \boxtimes H)$.

Theorem 3.6: If G and H are two graph then $\gamma_{tstd}(G \boxtimes H) \geq 4$.

Proof: We first note that G and H are two connected graphs with $\delta(G) \geq 1$ and with $\delta(H) \geq 1$. $\chi(G \boxtimes H) \geq \max\{\chi(G), \chi(H)\} + 2$ implies $\chi(G \boxtimes H) \geq 4$. Hence $\gamma_{tstd}(G \boxtimes H) \geq 4$.

Theorem 3.7: Let G and H be two graphs. If $\gamma_{tstd}(G \boxtimes H) = 4$ then both G and H are bipartite.

Proof: Given that $\gamma_{tstd}(G \boxtimes H) = 4$. So $\chi(G \boxtimes H) \leq 4$. But as $\chi(G \boxtimes H) \geq 4$, $\chi(G \boxtimes H) = 4$. Now $4 = \chi(G \boxtimes H) \geq \max\{\chi(G), \chi(H)\} + 2$ implies that $\chi(G) = \chi(H) = 2$.

Theorem 3.8: If G and H are two graphs with $\gamma(G) = \gamma(H) = 1$ then $\gamma_{tstd}(G \boxtimes H) = \chi(G \boxtimes H)$.

Proof: Let {a} and {b} be, respectively, Dominating Sets of G and H. So {(a, b)} is a dominating set of $G \boxtimes H$. So $\gamma(G \boxtimes H) = 1$. So $\gamma_t(G \boxtimes H) = 2$. So by theorem 3.1, $\gamma_{tstd}(G \boxtimes H) = \chi(G \boxtimes H)$.

Corollary 3.9: $\gamma_{tstd}(K_m \boxtimes K_n) = mn$.

Proof: Obviously, as $K_m \boxtimes K_n$ is a complete graph with mn vertices.

Now we consider the Lexicographic product of graphs. Consider the following remark.

Remark 3.10: 1) Lexicographic product is non- commutative.

2) $E(G \boxtimes H) \subset E(G[H])$.

3) By [11] If G has at least one edge, then for any graph H, $\chi(G[H]) \geq \chi(G) + 2\,\chi(H) - 2$.

Theorem 3.11: If G and H are two graph then $\gamma_{tstd}(G[H]) \geq 4$.

Proof: $\chi(G[H]) \geq \chi(G \boxtimes H) \geq 4$, we obtain $\gamma_{tstd}(G[H]) \geq 4$.

Theorem 3.12: Let G and H be two graph. If $\gamma_{tstd}(G[H]) = 4$ then both G and H are bipartite graphs.

Proof: We first note that G and H are two connected graphs with $\delta(G) \geq 1$ and with $\delta(H) \geq 1$. Let $\gamma_{tstd}(G[H]) = 4$. Then $\chi(G[H]) = 4$. So $4 = \chi(G[H]) \geq \chi(G) + 2\,\chi(H) - 2$, which implies that $\chi(G) = \chi(H) = 2$.

Theorem 3.13: If G and H are two graphs with $\gamma(G) = \gamma(H) = 1$ then $\gamma_{tstd}(G[H]) = \chi(G[H])$.

Proof: As $E(G \boxtimes H) \subset E(G[H])$ and $\gamma(G \boxtimes H) = 1$, we obtain $\gamma(G[H]) = 1$. So $\gamma_t(G[H]) = 2$ and hence by theorem 3.1, $\gamma_{tstd}(G[H]) = \chi(G[H])$.

Corollary 3.14: $\gamma_{tstd}(K_m[K_n]) = mn$.

Proof: Obviously, as $K_m[K_n]$ is a complete graph with mn vertices.

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