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Cocentroidal and Isogonal Structures and Their Matricinal Forms, Procedures and Convergence

Sweta Shah[1], Dr. Pradeep. J. Jha[2]
[1]*Research Scholar, C.U.Shah University, Wadhwan-363030, India.*
[2]*Research Guide, C.U.Shah University, Wadhwan-363030, India.*

ABSTRACT: *The vector space of all matrices of the same order on a set of real numbers with a non-negative metric defined on it satisfying certain axioms, we call it a JS metric space. This space can be regarded as the infinite union of matrices possessing a special property that there exist infinite structures having the same (virtual) point-centroid. In this paper we introduce the notion of cocentroidal matrices to a given matrix (Root Matrix) in* ***JS metric space*** *wherein the metric is the Euclidean metric measuring the distance between two matrices to be the distance between their centroids. We describe mathematical routines that we envisaged and then theoretically verified for its applicability in a system that is under rotational motion about a point-centroid, and various cases in physics. We have sounded the same concept by considering necessary graphs drawn on the basis of mathematical equations as an additional feature of this article. Isogonality of different cocentroidal structures is the conceptual origin of one of the main concepts, necessitating the notion of convergence in terms of determinant values of a system of cocentroidal matrices.*
KEYWORDS: *JS metric space, Centroid of a matrix, Cocentroidal matrices, Isomorphic sets, isogonality, Equivalence relation.*
NOTATIONS - $G^*(A)$ *– A set of Cocentroidal matrices to a given root matrix A.*

I. INTRODUCTION

It is known that to any structure or lamina the notion of centroid is very important. It is a point in the given structure which is understood to accumulate the mass of the whole body/ lamina and it is the only point which can hold the body in lifted position parallel to a horizontal direction. It is known that the gravitational force/ pulling force towards the centre of the globe is hypothetically understood to be flowing out from that point. If the point or the direction or both change then the system loses a grip on the balance.

In this paper the first part introduces the centroid of a triangular structure in R^3- the real Euclidean space and then allows extension to an n-dimensional space. This triangular structure has a vectorial presentation in the form of a square matrix.

Then we define the space, JS metric space, with axioms and some salient characteristics. The central point of one of the basic notion of the paper lies in the fact that develops a systematic procedure of finding one **infinite set of** three non-linear points lying on the same plane which correspond to the same centroid as that one of the original structure. The outcome is system of cocentroidal matrices. The importance and beauty of the structure is in derivation of the path for all the vertex points of the original structure. For a square matrix $\mathbf{A = (a_{ij})_{n \times n}}$ with all real entries and $\boldsymbol{\rho(A) = n}$ i.e. possessing **n** linearly independent column vectors, and the centroid G, its corresponding system is an infinite set of square matrices of the same space and possess the same centroid G; this we shall refer to a system of cocentroidal matrices to a given root matrix A. This structure is denoted as G*(A).

The second part in the continuation conveys the notion of isogonality of the different Cocentroid system, establish equi-valence relation in isogonal cocentroidal systems.

The last part in the paper introduces a new intuitive concept, well envisaged, a convergent system of cocentroidal matrices. The convergence system of matrices is the convergence of associated real sequence of absolute values of their determinant values taken in the same correspondence of sequences of matrices.

Some important properties are also high-lighted and the figures, so far as clearly drawn, help explain the principal part of the notion.

1.1 JS Metric Space:

We consider a non-empty set S of all m×n matrices. Let each column vector of a matrix in S stands for a vertex of structure in a given space. Let A = [A_{m1} A_{m2} …….A_{mn}] represent n column vectors of an m×n matrix.

$$A_{mi} = \begin{bmatrix} a_{1i} \\ a_{2i} \\ \vdots \\ \vdots \\ a_{mi} \end{bmatrix}_{m\times i} \quad \text{for a fixed i=1 to m}$$

We have, in our previous paper [4], defined the virtual centroid and its meaningful notion to the given matrix A as the n-tuple $(g_1\ ,\ g_2\ ,\ \dots\ \dots\ ..\ g_n)$ with each

$$g_k = \sum_{i=1}^{i=n} \frac{a_{ki}}{n} \quad \text{for each} \quad k = 1 \text{ to } m$$

we denote the centroid as $G^* = (g_1\ \ g_2\ \ \dots\ \dots\ g_m)_{m\times 1}$

Let A, B, and C be the member matrices of S with their corresponding centroidsG_1^*, G_2^*, and $G_3^*$with the same conventional sense mentioned above. Where $G_1^* = (g_{11}\ \ g_{21}\ ,\ \dots\ \dots .g_{m1})$, $G_2^* = (g_{12}\ \ g_{22}\ ,\ \dots\ \dots\ ..\ g_{m2})$, and $G_3^* = (g_{13}\ \ g_{23} \dots .\ g_{m3})$

We define the metric between two member matrices; say A, and B.

Let d: S×S→ R^+ U {0} be defined as d (A, B) = $\sqrt{(g_{11} - g_{12})^2 + (g_{21} - g_{22})^2 + \cdots + (g_{m1} - g_{m2})^2}$

With following axioms;

(1) d(A, B) ≥ 0

(2) d(A, B) = d(B, A)

(3) If A = B then d(A, B) = 0 **

(4) d(A, C) ≤ d(A, B) + d(B, C)

Then the entities like (S, d) shall be known as JS metric space over the field over non-negative real numbers.

[** It indicates that in this, so defined JS space it happens so that **if A B yet we may have d(A, B) = 0**]

Special Note: This is rather the point that conveys the message that in JS metric space there exists sets of infinite matrices that they have the same centroid and all of them lies on the same structure (a plane in R^3)

Matrices having these properties on the JS metric space are called **Cocentroid** matrices.

1.2 Introduction to a system of cocentroidal matrices to a given root matrix A:

We introduce the notion of an infinite set of cocentroidal matrices and its associated properties to a given root matrix. Let A =$[A_1 \quad A_2 \quad \dots \quad \dots \quad A_n]_{1\times n}$ Where each A_i stands for a column vector to a n × n matrix;

$$\text{With } A_i = \begin{bmatrix} a_{1i} \\ a_{2i} \\ \vdots \\ \vdots \\ a_{ni} \end{bmatrix}_{n\times i} \quad \text{for a fixed i=1 to n.}$$

As defined above the virtual centroid and its meaningful notion, to the given matrix A as the n-tuple $(g_1\ , g_2\ ,\ \dots\ \dots\ ..,\ g_n)$ with each

$$g_k = \sum_{i=1}^{i=n} \frac{a_{ki}}{n} \quad \text{for each} \quad k = 1 \text{ to } n \tag{1.1}$$

We denote the centroid as $G^* = (g_1\ , g_2\ ,\ \dots\ \dots\ ..,\ g_n)$ (1.2)

The set of all matrices having the same centroid G^* is defined as a set of cocentroidal matrices and is denoted as $G^*(A)$.

The mathematical derivation of construction of $G^*(A)$ for the given matrix A is briefly explained as follows in sequential order.

1.2.1 Spotting the Centroid:

We consider a case in R^3.

Let A = $\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$ which corresponds to three Vectors $\overrightarrow{OP} = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix}$, $\overrightarrow{OQ} = \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix}$, and $\overrightarrow{OR} = \begin{bmatrix} a_3 \\ b_3 \\ c_3 \end{bmatrix}$ in R^3.

We assume that these vectors are linearly independent; i.e. det. A ≠ 0

As defined, Centroid of the system given by A is G = $\left(\frac{\sum a_j}{3}\ ,\ \frac{\sum b_j}{3}\ ,\frac{\sum c_j}{3}\right)$ for j = 1, 2, 3

Now, we consider the following figure with vertices P (a_1, b_1, c_1), Q (a_2, b_2, c_2), and R (a_3, b_3, c_3)

and centroid $G^* = \left(\frac{\sum a_j}{3}\ ,\ \frac{\sum b_j}{3}\ ,\frac{\sum c_j}{3}\right) = (g_1\ , g_2\ , g_3)$

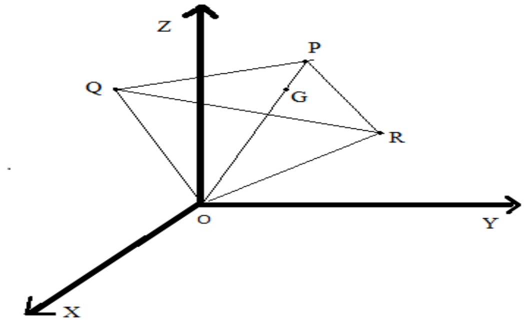

Fig. 1 Spotting the Centroid.

1.2.2 A System of Points:

Consider the point 'Q' as the fixed point and find the equation of the line $\overleftrightarrow{QR}$.
We consider a variable point R_i=(a_i, b_i, c_i) ϵ $\overleftrightarrow{QR}$. $\overleftrightarrow{QR}$ = {(a_i, b_i, c_i)| (a_i, b_i, c_i) = Q + k $\overleftrightarrow{QR}$ }
with R_i≠ Q (for k ≠ 0) and R_i≠ R (for k ≠ 1), i.e. $k_i \in (-\infty, 0) \cup (0, 1) \cup (1, \infty)$.

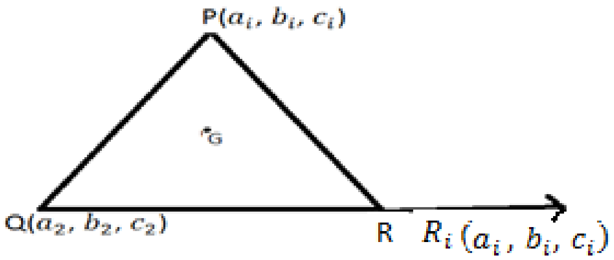

Fig. 2 system of Points

1.2.3 Corresponding Vertices and the Centroid:

Let for each $k_i \in (-\infty, 0) \cup (0, 1) \cup (1, \infty)$ the new position of the vertex P_i = $(\alpha_{i,} \beta_{i,} \gamma_i)$ be such that the centroid of the triangle P_iQR_i is the point $G^*(g_1, \ g_2 \ , g_3)$

∴ $\alpha_{i,}$ = 3g_1 - a_2 - a_i , $\beta_{i,}$ = 3g_2 - b_2 - b_i , γ_i = 3g_3 - c_2 - c_i

Thus we have, for each $k_i \in (-\infty, 0) \cup (0, 1) \cup (1, \infty)$, an infinite set of values of the point P_i $(\alpha_{i,} \beta_{i,} \gamma_i)$.i.e. For the fixed point Q and the centroid $G^*(g_1 \ g_2 \ g_3)$ we get, for each $k_i \in (-\infty, 0) \cup (0, 1) \cup (1, \infty)$ corresponding point $R_i(a_{3i}, b_{3i}, c_{3i})$ on the line $\overleftrightarrow{QR}$ and the locus of the point P= $P_i(a_{1i}, b_{1i}, c_{1i})$ moving along a certain curve ***** [4] such that

$g_1 = \frac{a_{1i}+a2+a_{3i}}{3}$, $g_2 = \frac{b_{1i}+b2+b_{3i}}{3}$, $g_3 = \frac{c_{1i}+c2+c_{3i}}{3}$

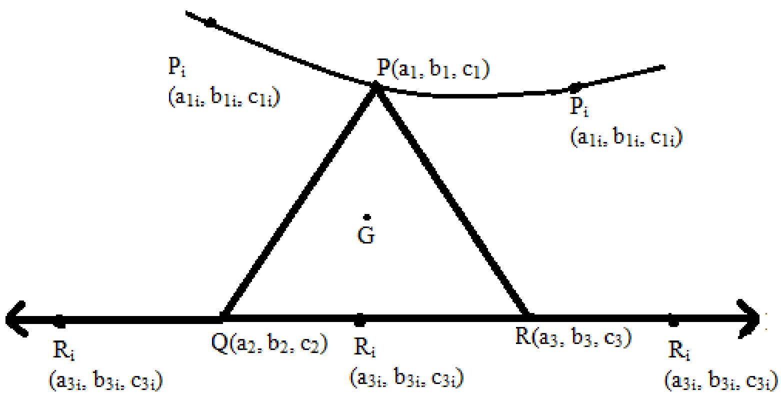

Fig . 3 Locus of point P moving along a certain curve.

[*The locus of the point P for different $k_i \in (-\infty, 0) \cup (0, 1) \cup (1, \infty)$ is a curve. In the section to follow we have attempted deriving the locus of the point P and its algebraic form also.]

1.2.4 Three Infinite Systems:
The most important point at this junction is that this procedure helps derive a system in which exactly one vertex of the given three vertices remains fixed and other two vertices move on their own path in such a way that any point one on each path and the one fixed point constitute a system which claims the right for the same centroid. In the same way by keeping the points P and R fixed, we get the locus of point R and the point Q. Thus we get three infinite sets corresponding to the points P, Q, and R such that each time the centroid $G^*(g_1\ ,\ g_2\ ,\ g_3)$ remains unchanged. This justifies the phrase 'Cocentroidal matrices'.
This conveys the message that different systems or structures in the same space correspond to some fixed point. Such matrix system counter-fitting a structure [especially a triangular one in R^3] represents cocentroidal matrices denoted as $G^*(A)$ and its corresponding cocentroid vector is denoted as $\overrightarrow{OG^*}$.
In each case we get a new matrix such that the corresponding column vectors represent the triangle having the same centroid $G^*(g_1,\ g_2\ ,\ g_3)$. The systems of all such triangles represent an infinite set of cocentroidal matrices denoted as $G^*(A)$ and its corresponding cocentroid vector is denoted as $\overrightarrow{OG^*}$.
We have, at this stage **two important derivations** which can be mathematically established.
(1) The first conclusion derived and established is that if P_i, Q_i, and R_i are different points each one on the corresponding locus of the point P, Q, and R then also the centroid remains the same as that of the root matrix A. It is important to note that the points P_i, Q_i, and R_i correspond to the same value of k_i where $k_i \in (-\infty, 0) \cup (0, 1) \cup (1, \infty)$

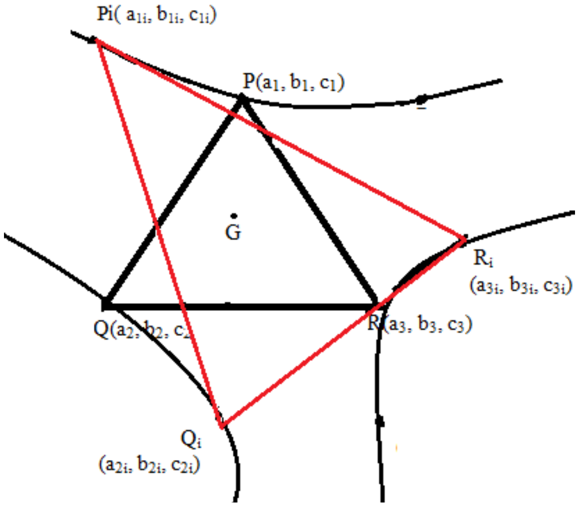

Fig. 4 Same Centroid for different points on corresponding locus of P, Q, and R

(2) The next important derivation on the same line is that if for the same value of k_i if we take different points on the lines QR, RP, and PQ then also the resulting triangle yields the same centroid as that of the original root matrix A. The following figure ensures the fact.

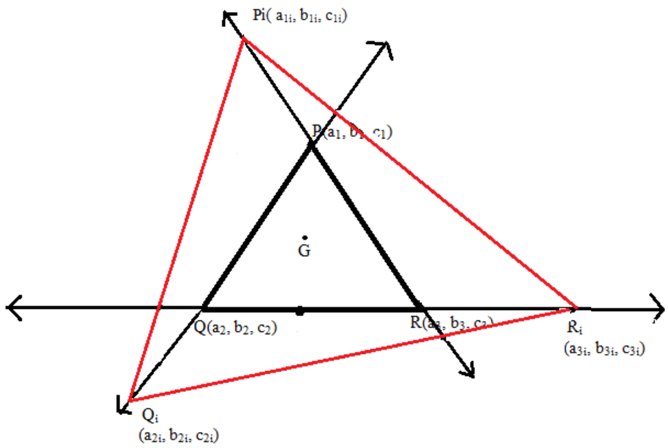

Fig. 5 Same centroid for different points on lines **QR, RP, and PQ**
[Numerical illustration is given in the Annexure – 1]

1.3 Associated Properties:
There are some interesting mathematical **properties associated to the root matrix and the set of Cocentroidal matrices $\boldsymbol{G^*(A)}$.** We mention them here without the proof; a few of them are proved in annexure to this paper.
1. If the mass is assumed to be uniformly distributed on the entire surface of the plane then any triangular member structure of $G^*(A)$, irrespective of size or area, have the same centroid which is liable to experience the gravitational force.
2. Column vectors as well as row vectors of all cocentroidal matrices are linearly independent.
3. **(Determinant property):** Let |A| = M where M is a real value. If any A_1 is a member matrix of the set $G^*(A)$ for some real value K_1, then we can prove that **$|A_1| = K_1$. M** This can be proved by applying basic properties of the determinant. We conclude that determinant values corresponding to cocentroidal matrices are closely associated with the different values of K [4].
4. All the cocentroidal matrices of the same class [1] have their norm $||A||_\infty$ the same constant.
5. (**Eigen Value Property**) A matrix is said to be of class-2 denoted as CJ2 (3×3, L (A) = k) if the sum of all elements of each row of the matrix remains the same real constant; say **k. [**This is known as libra value of the matrix which is denoted by the notation L(A) = k] [1]. In this case all the components of the centroid are constant and each, for a root matrix of order m×n, equals **k/n.**
i.e. $G^* = (g_1 \ g_2, \dots \dots .., g_m)$ = (k/n, k/n, ……….k/n)
*An important property that we have established is regarding Eigen values that for a square matrix of order nxn, one of the Eigen values is k [3].

II. ISOGONAL SETS OF COCENTROIDAL MATRICES AND EQUIVALENCE RELATION

In this section we are going to discuss about the notion of isogonality between different sets of cocentroidal matrices or structures. This will be very useful to define convergence of all such structures which we shall discuss in the section- 3.

2.1 Isogonality of Cocentroidal Matrices:
Two or more system of cocentroidal matrices are said to be isogonal to each other if their corresponding cocentroidal vectors are parallel to each other.
Let G_1 and G_2 be the two centroids corresponding to the two different root matrices A and B ϵ $M_{n\times n}$. Let their corresponding cocentroidal systems be $G^*(A)$ and $G^*(B)$. These two systems $G^*(A)$ and $G^*(B)$ are said to be isogonal to each other.
$\overrightarrow{OG_1}$=K $\overrightarrow{OG_2}$ for K > 0. $\overrightarrow{OG_1}$ and $\overrightarrow{OG_2}$ are collinear and in the same direction [5]. (2.1)
We denote this as $G^*(A) \approx G^*(B)$. i.e $G^*(A)$ is isogonal to $G^*(B)$. (2.2)
The matrices A and B are the matrices that corresponds to two different sets of vectors $(\overrightarrow{OP_1}, \overrightarrow{OQ_1}, \overrightarrow{OR_1})$ and $(\overrightarrow{OP_2}, \overrightarrow{OQ_2}, \overrightarrow{OR_2})$.

2.2 Theorem: We prove that Isogonality is an **Equivalence Relation** over the sets of isogonal cocentroidal structures.

Proof: As required, we have three properties to establish; they are as follows.
(a) Isogonality is Reflexive.
It is clear that a system $G^*(A)$ is self reflexive. i.e. In the relation (2.1); we put K = 1 and it is possible if and only if A = B.
(b) Isogonality is Symmetric.
i.e. $G^*(A) \approx G^*(B)$ then $G^*(B) \approx G^*(A)$ [For $\overrightarrow{OG_1}$ = K $\overrightarrow{OG_2}$, K > 0 then $\overrightarrow{OG_2} = \frac{1}{K}\overrightarrow{OG_1}$]
(c) Isogonality is Transitive.
For the three root matrices A, B, and C if
$G^*(A) \approx G^*(B)$ and $G^*(B) \approx G^*(C)$ then $G^*(A) \approx G^*(C)$.
For $\overrightarrow{OG_2}$ = K1 $\overrightarrow{OG_1}$, K1 > 0 and $\overrightarrow{OG_3}$ = K2 $\overrightarrow{OG_2}$, K2 > 0 then we can write
$\overrightarrow{OG_3}$ = K2 $\overrightarrow{OG_2}$ = K2(K1 $\overrightarrow{OG_1}$) = K3 ($\overrightarrow{OG_1}$) where K3 = K1.K2
This establishes transitivity.
The above properties establish that isogonality is an equivalence relation.
This property induces classes of cocentroidal matrices.

Important Note:
At this stage it is important to note that the set $G^*(A)$ corresponding to the root matrix A is a subset of the set of the points of the plane uniquely determined by the three non-linear points P$(a_1,\ b_1,\ c_1)$, Q$(a_2,\ b_2,\ c_2)$, and R$(a_3,\ b_3,\ c_3)$.
i.e $G^*(A) \subset \{(x,y,z) \mid (x,y,z)\ \epsilon\ L\ \}$ where L is the plane determined by P, Q, and R

III. A CONVERGENT SYSTEM OF COCENTROIDAL MATRICES.

We consider a set of a system of isogonal matrices which corresponds to a system of all $n \times n$ matrices A_1, A_2, A_3, …. .. A_n, ……. Let us denote this by $G^*(A_i)$.
For each matrix A_i, $i\ \epsilon\ N$ (Considering it as a root matrix), there is an uniquely associated system of cocentroidal matrices. We denote this by $G^*(A_i)$, $\forall\ i\ \epsilon\ N$
Let $G_i(g_{1i},\ g_{2i},\ g_{3i})$ be the centroid of i[th] system $G^*(A_i)$, for all i.
Let O$(\alpha,\ \beta,\gamma)\epsilon\ R^3$ be any arbitrary point. Also let O $(\alpha, \beta, \gamma) \notin \bigcup_{i=1}^{n}\left(G^*\ (A_i)\right)$.
The system $\bigcup_{i=1}^{n}\left(G^*\ (A_i)\right)$ of cocentroidal matrices is said to be convergent sequence if all $G_i(g_{1i},\ g_{2i},\ g_{3i})$ are on the same line passing through the fixed point O$(\alpha,\ \beta,\gamma)$.
d$(O\ ,\ G_i)$ = OG_i $<$ OG_{i+1}= d$(O\ ,\ G_{i+1})$, for all i = 1 to n.The distance of centroid G_i from the fixed point O$(\alpha,\ \beta,\gamma)$ forms a strictly monotonic increasing sequence.

3.1 Illustration: We take an illustration to understand the notion.
Let P = P$(2,3,-1)$, Q = Q$(2,1,-3)$, and R = R$(5,4,2)$. Let O = O$(1,-2,1)$.
Equation of $\overleftrightarrow{OP}$ = $\{(1+K,-2+5K,\ 1-2K) \mid K\ \epsilon\ R\}$
Equation of $\overleftrightarrow{OQ}$ = $\{(1+K,-2+3K,\ 1-4K) \mid K\ \epsilon\ R\}$
Equation of $\overleftrightarrow{OR}$ = $\{(1+4K,-2+6K,\ 1+K) \mid K\ \epsilon\ R\}$
The three points P, Q, and R are non- linear.
Equation of plane passing through P, Q, and R is 2x + 3y – 3z = 16.

We consider some values for K $\epsilon\ R^+ \cup\{0\}$ and find set of points P_i,Q_i, and R_i.
For K = 2, P_1 =$(3,8,-3)$, Q_1 =$(3,4,-7)$, R_1 =$(9,10,3)$
For K = 1, P_2 =$(2,3,-1)$, Q_2 =$(2,1,-3)$, R_2 = $(5,4,2)$
For K =$\frac{1}{2}$, P_3 =$\left(\frac{3}{2},\ \frac{1}{2},\ 0\right)$, Q_3 =$\left(\frac{3}{2},\frac{-1}{2},-1\right)$, R_3 = $\left(3,1,\frac{3}{2}\right)$

Corresponding to the points P_1, Q_1, and R_1 the root matrix, we write, as A_1.
A_1 = $\begin{bmatrix} 3 & 3 & 9 \\ 8 & 4 & 10 \\ -3 & -7 & 3 \end{bmatrix}$ and let its centroid G_1 = $\left(5,\ \frac{22}{3},\ \ \frac{-7}{3}\right)$

In the same way corresponding to the points $(P_2,\ \ Q_2,\ \ R_2)$ and $(P_3,\ \ Q_3,\ \ R_3)$ the root matrices A_2 and A_3 with their centroids are as follows.
A_2 = $\begin{bmatrix} 2 & 2 & 5 \\ 3 & 1 & 4 \\ -1 & -3 & 2 \end{bmatrix}$ and let its centroid G_2 = $\left(3,\ \frac{8}{3},\ \ \frac{-2}{3}\right)$
A_3 = $\begin{bmatrix} \frac{3}{2} & \frac{3}{2} & 3 \\ \frac{1}{2} & \frac{-1}{2} & 1 \\ 0 & -1 & \frac{3}{2} \end{bmatrix}$ and let its centroid G_3 = $\left(2,\ \frac{1}{3},\ \ \frac{1}{6}\right)$

For each A_i and centroid G_i , i = 1, 2, 3. We have a set of cocentroidal matrices denoted as $G^*\ (A_i)$.
[The method of finding the set of cocentroidal matrices has been discussed in the introduction part 1.2]
Now, finally to establish convergence we find the distance OG_1, OG_2, and OG_3. [The points are listed above]
d$(O\ ,\ G_1)$ = OG_1 = $\sqrt{16+\frac{784}{9}+\frac{100}{9}}$ = $\frac{2}{3}\sqrt{257} \simeq 10.687$
d$(O\ ,\ G_2)$ = OG_2 = $\sqrt{4+\frac{196}{9}+\frac{25}{9}}$ = $\sqrt{\frac{257}{9}} \simeq 5.3432$
d$(O\ ,\ G_3)$ = OG_3 = $\sqrt{1+\frac{49}{9}+\frac{25}{36}}$ = $\sqrt{\frac{257}{36}} \simeq 2.6718$
$\Rightarrow |OG_1| > |OG_2| > |OG_3|$ ……
i.e $|OG_i| > |OG_{i+1}|$, ∀ i ϵ N and each $|OG_i| > 0$

This shows that as K → O (Point O) the real sequence of terms such that
1. G_1, G_2, G_3….. are on the same line passing through the point O.
i.e the points G_1, G_2, G_3….. are colinear.
2. For all i ϵ N, $|OG_i| > |OG_{i+1}|$
i.e we have a strictly monotonic decreasing real sequence of positive real values which is bounded below.
It is a Cauchy sequence. The very fact establishes the convergence.
The above properties, by the definition of convergent matrices, establish the convergence of cocentroidal matrices.

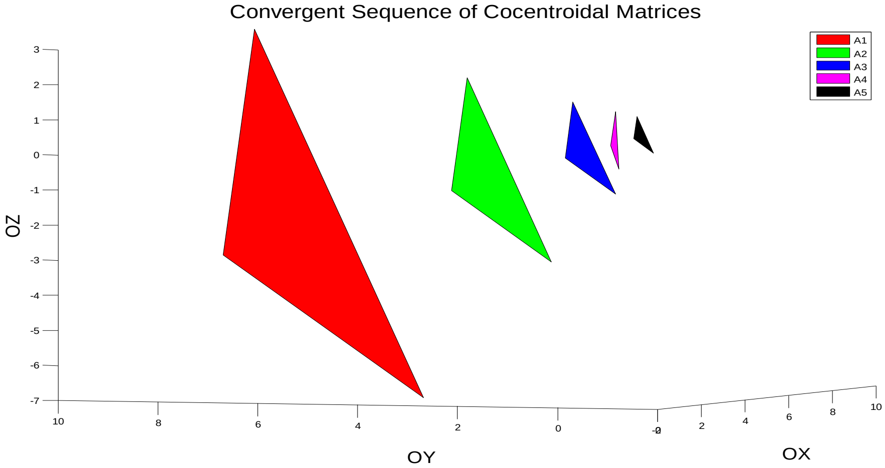

Fig. 6 Convergent sequence of cocentroidal matrices.

IV. CONCLUSION

The theoretical work and the corresponding examples cited above are related to a system in three dimensions but the same conditions can be extended to some polyhedrons and their related cases in higher dimensions also. The virtual concept of centroids and a system of cocentroidal matrices is not just confined to some space or some matricinal 3 x 3 structures but it is an entry to a wide and fully expandable system in a space. As found in the case of three dimensional cases the concept stands in its own merit simplifying the notion of moments of inertia and its applications in some physical cases. The concept leaves the same point open for further development in multi dimensional cases.

Vision

It has remained our next target to explore two new important concepts; one is on Pythagorean system of cocentroidal matrices and the next is on the theme that 'Each point the space corresponds to a cocentroidal system and vice versa'. These are the prime concepts which lead to compactness of the JS Metric space. These concepts whirling around the central notion of cocentroidal system makes the concept and related research open-ended for the innovative minded researchers to pursue its cognizant stuff.

REFERENCES

[1]. Shah.S.H, Prajapati. D. P, Achesariya.V. A and Dr. Jha P. J, Classification of matrices on the Basis of Special Characteristics, *International Journal of Mathematics Trends and Technology, **19**(2), 2015, 91-101.*
[2]. Shah.S.H, Achesariya.V. A and Dr.Jha P. J, Operators on Pythagorean Matrices, *IOSR Journal of mathematics, **11**(3), 2015, 51-60.*
[3]. Prajapati.D.P, Shah.S.H and Dr. Jha. P. J, Commutative Matrices, Eigen Values, and Graphs of Higher Order Matrices preserving Libra Value, *International Journal of Applied Research, 1(12) , 2015, 246-256.*
[4]. Shah.S.H and Dr.Jha. P.J, Cocentroid Matrices and Extended Metric Space (JS), Proceeding of the *International conference on Emerging trends in scientific research (ICETSR-2015) (ISBN: 978-2-642-24819-9), 231-234.*
[5]. Prajapati.D.P and Dr. Jha.P. J, Norm of a Matrix, Centro-Normal and Centro-Linear Matrices, Proceeding of the *International conference on Emerging trends in scientific research (ICETSR-2015) (ISBN: 978-2-642-24819-9),210-212.*
[6]. Hazra. A. K. *Matrix: Algebra, Calculus and Generalized Inverse.* Viva Books Pvt.Ltd. First Indian Edition(2009).
[7]. Sheth. I. H *Linear Algebra*. Nirav Prakashan. (2004).
[8]. Rana. Inder. K. *An Introduction to Linear Algebra.* Ane Books Pvt.Ltd. (2010), 1-15.

Annexure 1:

As discussed in the introductory part, we carry out the procedure of finding a set of cocentroidal matrices, keeping one fix vertex, locating a set of points on the base line of the triangle passing through it and then finding the new set of vertices in accordance with the centroid of the given root matrix. We can repeat the same algorithm on each of the remaining vertices of the given triangle. The stage wise routine is discussed below.
As a result we get three infinite sets of triangles such that their centroids are the same.

For the root matrix A = $\begin{bmatrix} 2 & 2 & 5 \\ 3 & 1 & 4 \\ -1 & -3 & 2 \end{bmatrix}$ which corresponds to the vectors $\overrightarrow{OP} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$, $\overrightarrow{OQ} = \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}$, and $\overrightarrow{OR} = \begin{bmatrix} 5 \\ 4 \\ 2 \end{bmatrix}$ and the centroid G = $\left(3, \frac{8}{3}, \frac{-2}{3}\right)$

Equation of the line $\overleftrightarrow{QR}$ is {(2, 1, -3) + k (3, 3, 5), k ϵ R}, we take $k \in (-\infty, 0) \cup (0, 1) \cup (1, \infty)$.
Keeping Q (2, 1, -3) as a fixed point and a find variable point R by assigning different values of k in the above equation line QR.
For each position of R, we find the current position of the point P so that the position of the point G – the centroid, remains constant.

k = -2,   R = R (-4, -5, -13),   P = P (11, 12, 14)
k = -1,   R = R (-1, -2, -8),   P = P (8, 9, 9)
k = 3/2   R = R (3.5, 2.5, -0.5),   P= P (3.5, 4.5, 1.5)

We get infinite set of matrices

$A_1 = \begin{bmatrix} 11 & 2 & -4 \\ 12 & 1 & -5 \\ -14 & -3 & -13 \end{bmatrix}$, $A_2 = \begin{bmatrix} 8 & 2 & -1 \\ 9 & 1 & -2 \\ 9 & -3 & -8 \end{bmatrix}$, $A_3 = \begin{bmatrix} 3.5 & 2 & 3.5 \\ 4.5 & 1 & 2.5 \\ 1.5 & -3 & -0.5 \end{bmatrix}$, etc……..

All these matrices have the same centroid as that of the root matrix A.
It is important to note that we have the point Q as a fixed point and for each position of the point R_i, we have a point P_i, moving on a specific curve so that the triangle P_iQR_i has the same centroid G.
The locus of point P gives the different triangles shown as follows.

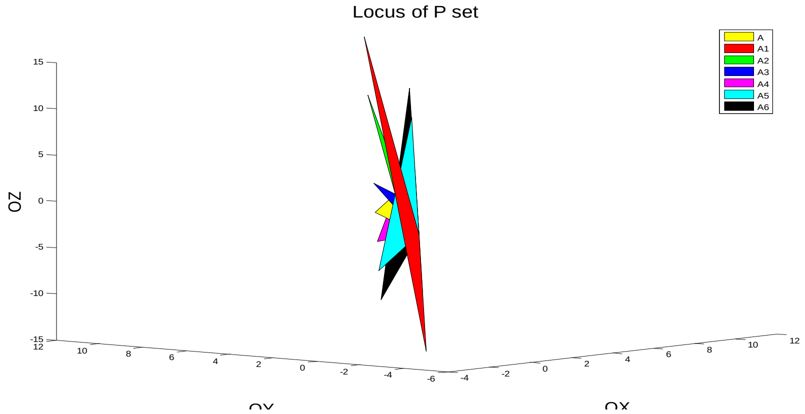

Fig. 7 Cocentroidal Matrices for locus of P set

We repeat the above procedure for all the remaining points and each time get a system of cocentroidal matrices,
Keeping R as a fixed Point, We get Locus of Q
For the different values of K= -2, -1, 0.5, 1.5, 2, 2.5, we get infinite set of matrices

$A_1 = \begin{bmatrix} 11 & -7 & 5 \\ 6 & -2 & 4 \\ 8 & -12 & 2 \end{bmatrix}$, $A_2 = \begin{bmatrix} 8 & -4 & 5 \\ 5 & -1 & 4 \\ 5 & -9 & 2 \end{bmatrix}$, $A_3 = \begin{bmatrix} 3.5 & 0.5 & 5 \\ 3.5 & 0.5 & 4 \\ 0.5 & -4.5 & 2 \end{bmatrix}$, etc…….. which has same centroid G as that of the Root matrix has.

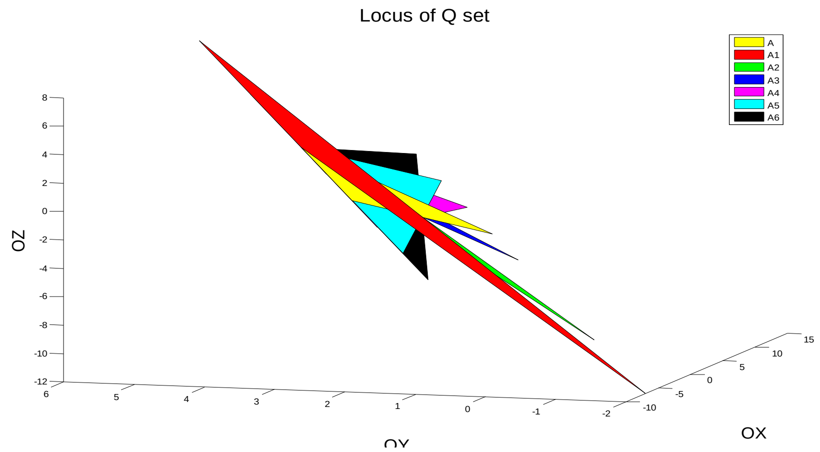

Fig. 8 Cocentroidal Matrices for locus of Q set.

And finally we keep the point P as a fixed Point, We get Locus of R
For the different values of K= -2, -1, 0.5, 1.5, 2, 2.5, we get infinite set of matrices

$A_1 = \begin{bmatrix} 2 & 2 & 5 \\ 3 & 7 & -2 \\ -1 & 3 & -4 \end{bmatrix}$, $A_2 = \begin{bmatrix} 2 & 2 & 5 \\ 3 & 5 & 0 \\ -1 & 1 & -2 \end{bmatrix}$, $A_3 = \begin{bmatrix} 2 & 2 & 5 \\ 3 & 2 & 3 \\ -1 & -2 & 1 \end{bmatrix}$, etc…….. which has same centroid G as that of the Root matrix has

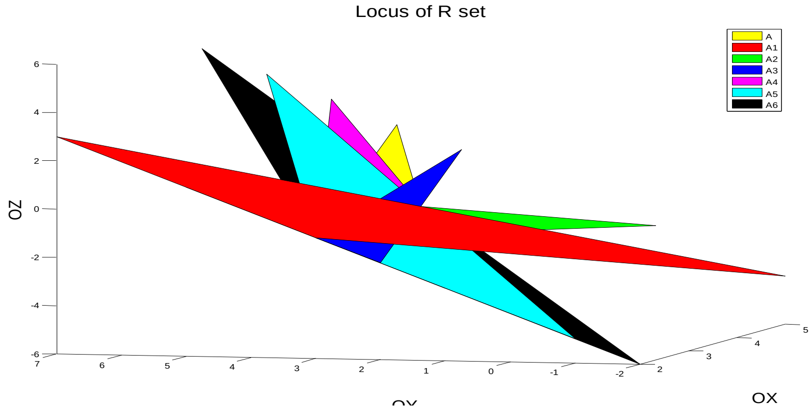

Fig. 9 Cocentroidal Matrices for locus of R set.