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Analysis of Occurrence of Digit 1 in First 10 Billion Digits of π after Decimal Point

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ABSTRACT: *$\pi$is well-known irrational number that is also transcendental. The infinite non-zero non-recurring digits after decimal point in decimal representation of π deserve detail analysis. This work analyzes the occurrence of digit 1 in first 10 billion digits of π after decimal Point in decimal representation covering successive as well as non-successive occurrences.*

KEYWORDS: *Digit 1, Digits after decimal point, π*
MATHEMATICS SUBJECT CLASSIFICATION 2010: *11Y35, 11Y60, 11Y99.*

I. INTRODUCTION

The most commonly known irrational number is π[1].One gets introduced to it in early school days in geometry while studying circles and their properties. As one advances to study of higher mathematics, it comes up in many other contexts also.

II. DIGIT 1 IN π

Number 1 is very special as a digit. Every number system with any positive base does contain 1.
As π is irrational, it is bound to possess infinite significant digits after the decimal point in its decimal representation. Lack of recurring pattern in its digits demands detail analysis of their occurrences. Such work is available for digit 0 [5]. We continue this endeavor by choosing digit 1 for occurrence analysis. Study of occurrences of 1 in natural numbers, in both general [3] and successive [4] ways, is recently done.
As in [5], here too, with respect to base 10, we choose the increasing digit ranges $1 - 10^n$ for $1 \leq n \leq 10$, to cover ranges upto 10 billion [2].

Table 1: Occurrences of Digit 1 in Blocks of 10 Powers

Sr. No.	Digit Numbers' Range $1 - x$	Range as Ten Power 10^n	Number of Occurrences of 1	First Occurrence of 1 at Digit Number	Last Occurrence of 1 at Digit Number
1.	1 – 10	10^1	2	1	3
2.	1 – 100	10^2	8	1	95
3.	1 – 1,000	10^3	116	1	997
4.	1 – 10,000	10^4	1,026	1	9,988
5.	1 – 100,000	10^5	10,137	1	99,978
6.	1 – 1,000,000	10^6	99,758	1	1,000,000
7.	1 – 10,000,000	10^7	999,333	1	9,999,988
8.	1 – 100,000,000	10^8	10,002,475	1	99,999,997
9.	1 – 1,000,000,000	10^9	99,997,334	1	999,999,999
10.	1 – 10,000,000,000	10^{10}	1,000,037,790	1	9,999,999,997

The digit 1 occurs as the very first digit.
If all 10 digits had evenly occurred, the expected average of occurrence of each digit would have been same in all range-limits. The randomness of occurrences of digits brings following deviationsfor 1 from the average.

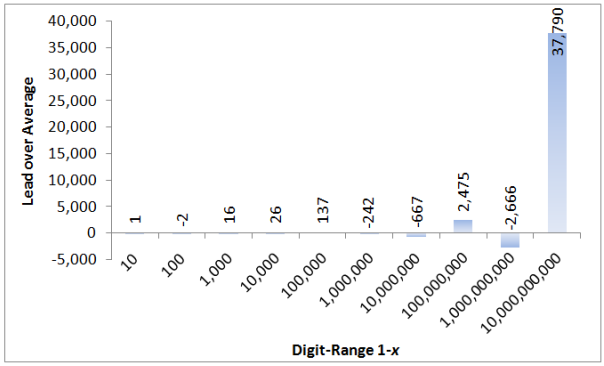

Fig. 1: Deviation from the Average for Occurrence of 1 in Initial Blocks of 10^n.

Forthese discrete digit range-vales, except for four ranges of 1-100, 1-1,000,000,100,000,000 and 1,000,000,000, 1's presence is more than average. For generalized prediction, more analysis will be required.
The last occurrence of digit 1 is away from the respective range-end by following amounts.

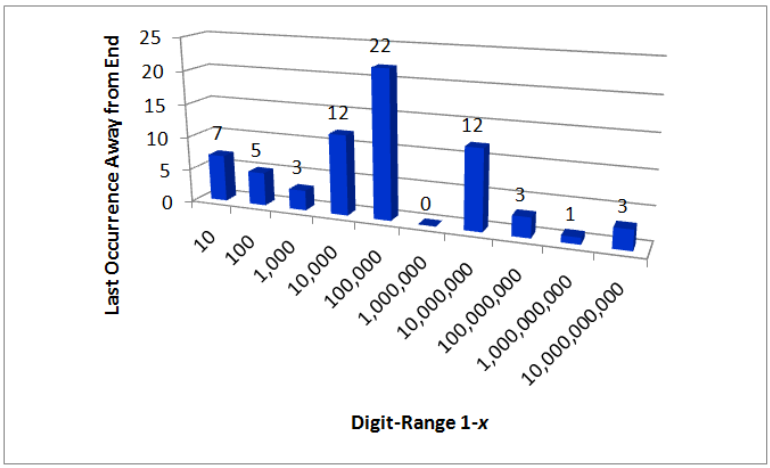

Fig. 2: Distance of Last Occurrence of 1in Blocks of $1 - 10^n$from End.

Quantitatively, last 1 is farthest from end in block of $1 - 10^5$ and nearest in block of $1 - 10^6$, in fact it is right at the end of the block there.

III. SUCCESSIVE OCCURRENCE OF DIGIT 1 IN π

The successive occurrences of 1 in post-decimal digits of π have also been investigated.

Table 2: Successive Occurrences of Digit 1 in Blocks of 10 Powers

Sr. No.	Digit Numbers' Range $1 - x$	Range as Ten Power 10^n	Number of Successive Occurrences of 1	First Successive Occurrence of 1 at Digit Number	Last Successive Occurrence of 1 at Digit Number
1.	1 – 10	10^1	0	-	-
2.	1 – 100	10^2	1	94	94
3.	1 – 1,000	10^3	16	94	984
4.	1 – 10,000	10^4	99	94	9,708
5.	1 – 100,000	10^5	1,034	94	99,912
6.	1 – 1,000,000	10^6	10,064	94	999,963
7.	1 – 10,000,000	10^7	99,675	94	9,999,819
8.	1 – 100,000,000	10^8	1,000,611	94	99,999,988
9.	1 – 1,000,000,000	10^9	9,997,964	94	999,999,989
10.	1 – 10,000,000,000	10^{10}	100,009,637	94	9,999,999,945

In the first 10 power block 1 – 10, digit 1 doesn't occur consecutively even once. Its successive occurrence starts from digit number 94.
The percentage of occurrence of successive 1's in respective blocks has following trend, whirling around 1.

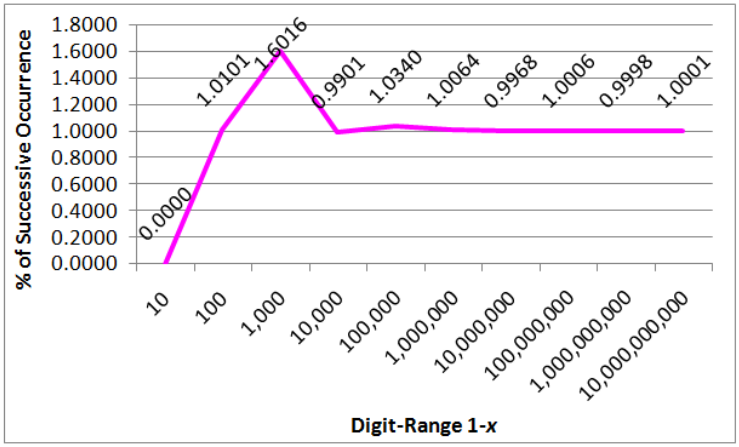

Fig. 3: Percentage of Successive Occurrences of 1's in Blocks of 10^n

The first successive occurrence of 1 comes in late; 94 times (!) of solo occurrence.
Excluding the first block of 1 – 10 of absence of successive occurrence, the last such occurrence of digit 1's stops before last digit in range by keeping following distances.

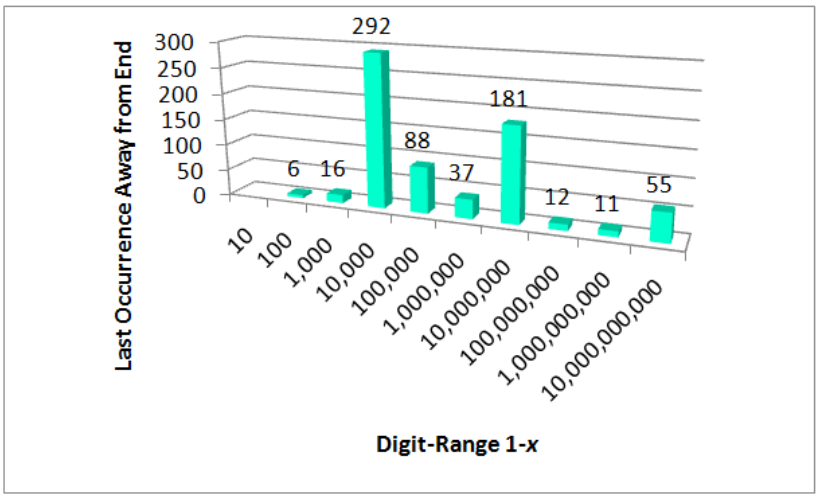

Fig. 4: Distance of Last Successive Occurrence of 1in Blocks of $10^n$from End

No specific pattern is seen.
The above discussion was for 2 consecutive 1's. Up to 10 consecutive occurrences of 1's are there in these digit ranges with following counts.

Table 3: Multiple Successive Occurrences of Digit 1in Blocks of 10 Powers

Digit Range	Number of Successive 1's									
	1	2	3	4	5	6	7	8	9	10
$1\text{-}10^1$	2	0	0	0	0	0	0	0	0	0
$1\text{-}10^2$	8	1	0	0	0	0	0	0	0	0
$1\text{-}10^3$	116	16	2	0	0	0	0	0	0	0
$1\text{-}10^4$	1,026	99	9	0	0	0	0	0	0	0
$1\text{-}10^5$	10,137	1,034	113	12	1	0	0	0	0	0
$1\text{-}10^6$	99,758	10,064	1,051	120	16	1	0	0	0	0
$1\text{-}10^7$	999,333	99,675	9,828	1,005	103	10	1	0	0	0
$1\text{-}10^8$	10,002,475	1,000,611	100,503	10,098	964	83	4	0	0	0
$1\text{-}10^9$	99,997,334	9,997,964	1,000,046	99,784	9,905	984	109	12	1	0
$1\text{-}10^{10}$	1,000,037,790	100,009,636	10,003,291	999,195	100,005	10,099	993	109	12	1

It is seen that their count grows more or less by a factor proportional to increasing digit range.
The first appearances of differentnumber of successive 1's are roughly approximated by $y = 0.234e^{2.4227x}$, as seen in following graph with y-axis on logarithmic scale.

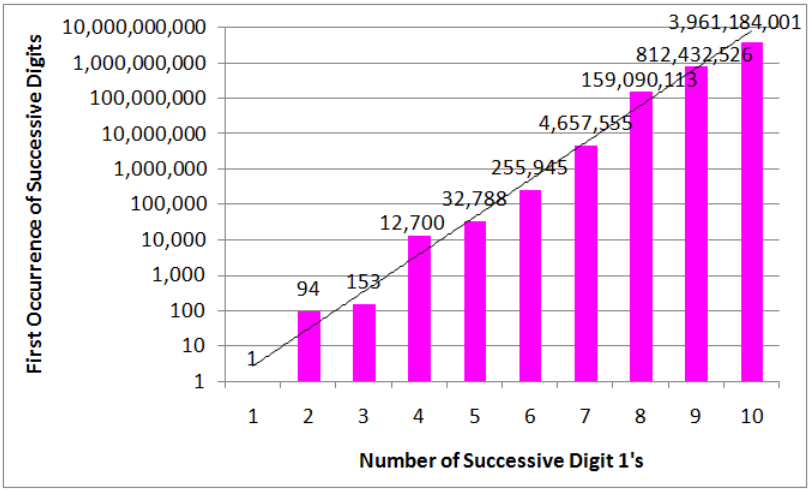

Fig. 5: First Multiple Successive Occurrences of 1in Blocks of 10^n.

The last successive 1's in initial blocks of 10^n are found to be as follows.

Table 4: Last Multiple Successive Occurrences of Digit 1in Blocks of 10 Powers

1's ↓	Digit Range and Last Occurrence									
	10^1	10^2	10^3	10^4	10^5	10^6	10^7	10^8	10^9	10^{10}
1	3	95	997	9,988	99,978	1,000,000	9,999,988	99,999,997	999,999,999	9,999,999,997
2	-	94	984	9,708	99,912	999,963	9,999,819	99,999,988	999,999,989	9,999,999,945
3	-	-	983	8,366	99,032	999,085	9,996,266	99,999,466	999,999,260	9,999,989,263
4	-	-	-	-	93,535	993,104	9,988,652	99,990,890	999,991,884	9,999,974,631
5	-	-	-	-	32,788	973,670	9,979,852	99,598,327	999,852,484	9,999,936,070
6	-	-	-	-	-	255,945	9,036,112	99,437,190	998,589,849	9,998,209,326
7	-	-	-	-	-	-	4,657,555	87,389,489	995,954,987	9,986,010,208
8	-	-	-	-	-	-	-	-	995,954,986	9,969,858,939
9	-	-	-	-	-	-	-	-	812,432,526	9,541,439,841
10	-	-	-	-	-	-	-	-	-	3,961,184,001

IV. NON-CONSECUTIVE OCCURRENCE OF DIGIT 1 IN π

Within π, the presence of digit 1 with other digit(s) in between it and next 1 is determined. For these calculations, such presence is counted only if the next 1 falls within the same block; otherwise it is not considered for that block.

Table 5 : Non-Successive Occurrences of Digit 1in Blocks of 10 Powers

Sr. No.	Digit Numbers' Range 1 – *x*	Range as Ten Power 10^n	Number of Non-Consecutive Occurrences of 1	First Non-Consecutive Occurrence of 1 at Digit Number	Last Non-Consecutive Occurrence of 1 at Digit Number
1.	1 – 10	10^1	1	1	1
2.	1 – 100	10^2	6	1	68
3.	1 – 1,000	10^3	99	1	992
4.	1 – 10,000	10^4	926	1	9,986
5.	1 – 100,000	10^5	9,102	1	99,972
6.	1 – 1,000,000	10^6	89,693	1	999,998
7.	1 – 10,000,000	10^7	899,657	1	9,999,984
8.	1 – 100,000,000	10^8	9,001,863	1	99,999,992
9.	1 – 1,000,000,000	10^9	89,999,369	1	999,999,990
10.	1 – 10,000,000,000	10^{10}	900,028,152	1	9,999,999,990

In the first 10 power block 1 – 10, the very first occurrence of 1 is with a gap of 1 with next 1.
The number of non-consecutive occurrences of is multiple times more thanthat of consecutive occurrences.

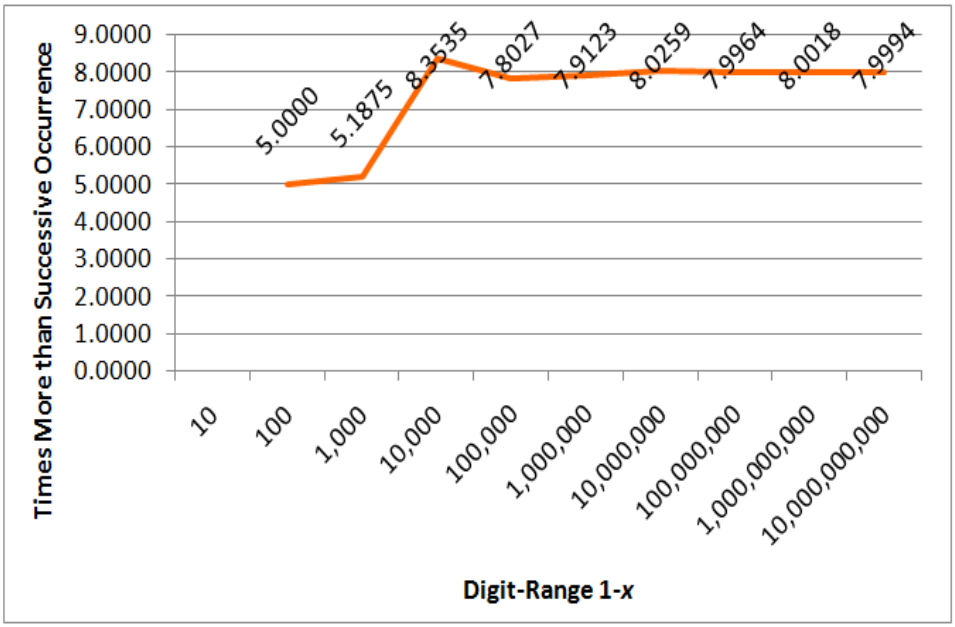

Fig. 6: Number of Times Non-Successive Occurrences of 1's are More than Corresponding Successives.

The first non-consecutive occurrence of digit 1 is the very first occurrence.
The last non-successive occurrence of digit 1 stops prior to last digit in range by following quantities.

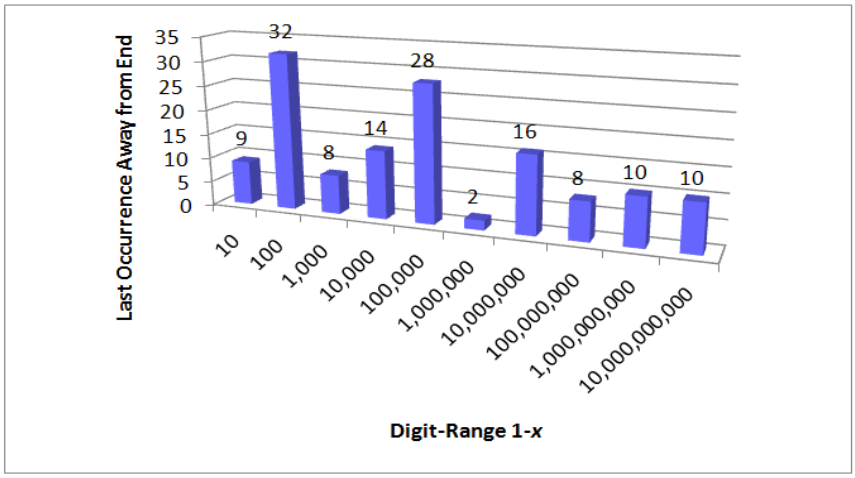

Fig. 7: Distance of Last Non-Successive Occurrence of 1in Blocks of $10^n$from End.

Owing to greater frequency of non-consecutive occurrences of digit 1, heights of these bars are low compared to those for successive 1's.

This analysis was aimed at knowing patterns and/or regularity, if any, in decimal digits of π through occurrence of digit 1. Future works will consider treatment for other digits.

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