International Journal of Mathematics and Statistics Invention (IJMSI)
E-ISSN: 2321 – 4767 P-ISSN: 2321 - 4759
www.ijmsi.org Volume 4 Issue 8 || October. 2016 || PP-15-19

A Note on Pseudo Operations Decomposable Measure

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ABSTRACT: *In this paper we discussed about the pseudo integral for a measurable function based on a strict pseudo addition and pseudo multiplication. Further more we got several important properties of the pseudo integral of a measurable function based on a strict pseudo addition decomposable measure.*
KEYWORDS: *Pseudo addition, Pseudo multiplication, decomposable measure, Pseudo Inverse*

I. INTRODUCTION

Taking into account human subjective measures in engineering science fuzzy measures have been intensively discussed since Sugeno(1) defined a fuzzy measure as a measure having the monotonicity property instead of additivity. Weber(2) proposed ⊥-decomposable measures where the additivity of measures is weakened. t-Conorm ⊥ is an appropriate semigroup operation in [O, l]. ⊥-decomposable measures can be written as

$m(A \cup B) = m(A) \perp m(B)$.

for the Archimedean case ⊥ is written as $a \perp b = g^{(-1)}(g(a) + g(b))$, where $g^{(-1)}$ is a pseudo-inverse of g.

By using t-conorm and multiplication *, Weber (2) defined an integral for the Archimedean cases. Semigroup operation ⊥ can be written as

$a \perp b = g^{-1}(g(a) + g(b))$.

Where g^{-1} is an inverse of g. This implies that g is an isomorphism of ([0, 1], ⊥) with ([0, (l)], +) where Lebesgue measure is defined.

In this paper we discuss a pseudo integral based on pseudo addition and pseudo multiplication. In section 2 we recall the concept of pseudo addition ⊕ and pseudo multiplication ⊙. and also gives the pseudo integral based on a strict pseudo addition decomposable measure by generalizing the definition of the pseudo integral of a bounded measurable function. In section 3, several important properties of the pseudo integral of a measurable function based on the strict pseudo addition decomposable measure were discussed.

II. PRELIMINARIES

Definition: 2.1

Let [a, b] be a closed real interval and $\oplus : [a, b] \times [a, b] \to [a, b]$ be a 2-place function satisfying the following conditions:

(i) ⊕ is commutative.
(ii) ⊕ is nondecreasing in each place.
(iii) ⊕ is associative.
(iv) ⊕ has either a (or) b as zero element,
i.e., either $\oplus(a, x) = x$ (or)
$\oplus(b, x) = x$.

⊕ will be called a pseudo-addition.

Definition: 2.2

A pseudo-multiplication ⊗ is a 2-place function $\otimes : [a, b] \times [a, b] \longrightarrow [a, b]$, satisfying the following conditions;
(i) ⊗ is commutative.
(ii) ⊗ is non decreasing in each place.
(iii) ⊗ is associative.
(iv) There exists a unit element e ∈ $[a, b]$, i.e., $\otimes(x, e) = x$ for all $x \in [a, b]$.

Definition: 2.3

A pseudo-multiplication ⊗ with the generator g of strict pseudo-addition ⊕ is defined as

$\otimes(x,y) = g^{-1}(g(x) . g(y))$.

$g(x).g(y)$ is always in $[0, \infty]$. This is called distributive pseudo-multiplication.

Definition: 2.4

A fuzzy measure *m* derived from a pseudo addition $\oplus^{v}$ with a zero element *a*, is a set function from σ-algebra 𝔅 of X to [*a*, *c*], C ∈ (*a*, *b*] such that

(i) $m(\Phi) = a$, $m(X) = c$.

(ii) A ∈ 𝔅, *m*(A) is $\oplus^{v}$ decomposable,

if A is divided into A_i'S

$$A = \bigcup_{i=1}^{n} A_i$$

$$m(A) = \bigoplus_{i=1}^{n}{}^{v}\; m(A_i).$$

Theorem 2.1

If the function ⊗ is continuous and strictly increasing in (*a*, *b*), then there exists a monotone function *g* in [*a*, *b*] such that (e) = 1

$\otimes(x, y) = g^{-1}(g(x)\bullet g(y))$

Proof

From Aczel's theorem [3] there exists a continuous and strictly monotone function in [a, b] such that

$\otimes(x, y) = f^{-1}(f(x) + f(y))$.

Transforming *f* by $g = \exp(-f)$

$\otimes(x, y) = g^{-1}(g(x) \bullet g(y))$.

Since $\oplus(x,e) = f^{-1}(f(x) + f(e)) = x$

for all x ∈ [a,b], f(*e*)= O.

Thus $(e) = \exp(-f(e)) = 1$.

Remarks: 2.1

For the sake of simplicity we write $\oplus(x, y)$ as $x\oplus y$ and $\otimes(x, y)$ as $x \otimes y$, respectively. Distributive pseudo-multiplication ⊗ has the distributive property with pseudo-addition ⊕ :

$x \otimes(y \oplus z) = (x\otimes y) \oplus (x \otimes z)$.

$(y \oplus z) \otimes x = (y \otimes x) \oplus (z \otimes x)$.

Theorem 2.2

m is $\oplus^{\vee}$ -decomposable ⇒ m monotone non decreasing. m is $\oplus^{\wedge}$ -decomposable ⇒ m monotone non increasing.

Proof

(i) For $A\subset B$ / $m(B) = m(A) \oplus^{\vee} m(B-A) \geq m(A)$ and

$m(B) = m(A) \oplus^{\wedge} m(B-A) \leq m(A)$.

(ii) The operation ⊕ is commutative and associative thus

$m(A\cup B) \oplus m(A\cap B) = m(A\cap B) \oplus m(A-B) \oplus m(B-A) \oplus m(A\cap B) =$

$m(A) \oplus m(B)$.

$\oplus^{\vee}$ has a and $\oplus^{\vee}$ has b as zero element. Thus m is ⊕ decomposable.

Remark: 2.2

A $\oplus^{\vee}$ -decomposable measure can be regarded as a subjective measure expressing the grade of importance [4] for example, $m(\{A_1\})$ expresses to what extent an attribute A_1 is important to evaluate an object. It is a reasonable assumption that m has monotonicity,

$m(\{A_1\}) \leq m(\{A_1, A_2\})$

III. PROPERTIES OF PSEUDO INTEGRAL BASED ON PSEUDO OPERATION DECOMPOSABLE MEASURE

Theorem 3.1

Let ⊕ be a strict pseudo addition and let x be a σ finite set of ⊕ measure and m: A → [a,b] a σ - ⊕ decomposable measure. If $\{f_n\}$ is a sequence of measurable functional on X then

$$\int_x^{\oplus} \odot\, dm = \bigoplus_{n=1}^{\infty} \int_x^{\oplus} \left(\bigoplus_{n=1}^{\infty} f_n\right) f_n \odot dm$$

Proof

Let $h_n = \bigoplus_{n=1}^{\infty} f_i$, n =1,2…..

Then $\{h_n\}$ is an increasing sequence of measurable functional on X.

We have $\lim_{n \to \infty} \int_x^{\oplus} h_n \odot dm = \int_x^{\oplus} \lim_{n \to \infty} h_n \odot dm$

$$\lim_{n \to \infty} \int_x^{\oplus} h_n \odot dm = \bigoplus_{n=1}^{\infty} \int_x^{\oplus} f_n \odot dm$$

Since,

$$\lim_{n \to \infty} h_n = \bigoplus_{n=1}^{\infty} f_n$$

$$\int_x^{\oplus} h_n \odot dm = \int_x^{\oplus} \bigoplus_{i=1}^{n} f_i \odot dm = \bigoplus_{i=1}^{n} \int_x^{\oplus} f_i \odot dm$$

$$\int_x^{\oplus} \left(\bigoplus_{n=1}^{\infty} f_n \right) \odot dm = \bigoplus_{n=1}^{\infty} \int_x^{\oplus} f_n \odot dm$$

Definition: 3.1

Let $\oplus$ be a continuous pseudo addition and m: A $\to$ [a,b] a σ $\oplus$ decomposable measure. If $m(x) < \Delta$ then the pseudo integral of an elementary measurable functional Q : $x \to$ [a,b] is defined by then

$$\int_x^{\oplus} f \odot dm = \lim_{n \to \infty} \int_x^{\oplus} Q_n \odot dm \qquad (9)$$

Definition: 3.2

Let $\oplus$ be a strict pseudo addition and m: A $\to$ [a,b] a σ $\oplus$ decomposable measure. If x is a σ finite of $\oplus$ measure and $\{E_n\}$ is a $\oplus$ measure finite and monotone cover of X then the pseudo integral of a measurable functional $f : x \to$ [a,b] is defined by

$$\int_x^{\oplus} f \odot dm = \lim_{n \to \infty} \int_{E_n}^{\oplus} [\, f\,]_n \odot dm \qquad (10)$$

Theorem 3.2

Let $\oplus$ be a strict pseudo addition and let X be a σ finite set of $\oplus$ measure and m: A $\to$ [a,b] a σ $\oplus$ decomposable measure. If f is a measurable functional on X

$$\int_x^{\oplus} f \odot dm = \bigoplus_{n=1}^{\infty} \int_{E_n}^{\oplus} f \odot dm$$

for any sequence $\{E_n\}$ of pairwise disjoint sets from A with $x = \bigcup_{n=1}^{\infty} E_n$

$$f_n(x) = \begin{cases} f(x) & \text{if } x \in E_n \\ 0 & \text{if } x \in x - E_n \end{cases}$$

Proof

A functional sequence $[\, f\,]_n$ is given by

$$\int_x^{\oplus} f_n \odot dm = \int_{E_n}^{\oplus} f_n \odot dm \oplus \int_{x - E_n}^{\oplus} f_n \odot dm$$

$$= \int_{E_n}^{\oplus} f \odot dm$$

$$\int_{x}^{\oplus} f \odot dm = \bigoplus_{n=1}^{\infty} \int_{E_n}^{\oplus} f \odot dm$$

Theorem: 3.3

Let $\oplus$ be a strict pseudo addition and m: A → [a,b] a $\sigma \oplus$ decomposable measure and only if $f \neq 0$ a.e on E.

if $f \in$ M(A) then for any E ∈ A

$$\lim_{mE \to 0} \int_{E}^{\oplus} f \odot dm = 0$$

Proof

(1) suppose $\int_{E}^{\oplus} f \odot dm = 0$

for arbitrary $0 < \delta$

let $E_\delta = \{x \in E \,/\, \delta \leq f(x)\} \in A$

we get

$$\delta \odot m(E_\delta) \leq \int_{E_\delta}^{\oplus} f \odot dm \leq \int_{E_\delta}^{\oplus} f \odot dm \oplus \int_{E - E_\delta}^{\oplus} f \odot dm$$

$$= \int_{E}^{\oplus} f \odot dm = 0$$

Thus we have $m(E_\delta) = 0$

Since $0 < \delta$ is arbitrary,

$m(\delta[0 < f] \cap E) = 0$

suppose f =0 a.e on E that is

$m(E \cap \delta[0 < f]) = 0$

we have

$$\int_{E}^{\oplus} f \odot dm = \int_{E \cap \delta[0 < f]}^{\oplus} f \odot dm \oplus \int_{E \cap \delta[f = 0]}^{\oplus} f \odot dm = 0$$

2) If there exists $\Omega < \Delta$ suh that

$f(x) \leq \Omega$ for all $x \in$ E

then

$$\int_{E}^{\oplus} f \odot dm \leq \Omega \odot m(E)$$

i.e.

$$\lim_{mE \to 0} \int_{E}^{\oplus} f \odot dm = 0 \qquad \lim_{n \to \infty} \int_{E}^{\oplus} [f]_n \odot dm \text{ for any } f \in \mu(A)$$

$$\int_{E}^{\oplus} f \odot dm = \lim_{n \to 0} \int_{E}^{\oplus} [f]_n \odot dm$$

$$\lim_{mE \to 0} \int_{E}^{\oplus} f \odot dm = \lim_{mE \to 0} \lim_{n \to \infty} \int_{E}^{\oplus} [f]_n \odot dm$$

$$= \lim_{n \to \infty} \lim_{mE \to 0} \int_{E}^{\oplus} [f]_n \odot dm = 0$$

IV. CONCLUSION

In this paper we mainly discussed the integral based on pseudo addition and pseudo multiplication can be used obtain generalized formulation for decision making further more we have derived several important properties of the pseudo integral of a measurable function based of strict pseudo addition decomposable measure. Finally we have obtained that some theorems on the integral and the limit can be changed.

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