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Application of Linear Programming for Optimal Use of Raw Materials in Bakery

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ABSTRACT: *This work utilized the concept of Simplex algorithm; an aspect of linear programing to allocate raw materials to competing variables (big loaf, giant loaf and small loaf) in bakery for the purpose of profit maximization. The analysis was carried out and the result showed that 962 units of small loaf, 38 units of big loaf and 0 unit of giant loaf should be produced respectively in order to make a profit of ₦20385. From the analysis, it was observed that small loaf, followed by big loaf contribute objectively to the profit. Hence, more of small loafs and big loafs are needed to be produced and sold in order to maximize the profit.*
KEYWORDS: *Linear programming model, Simplex method, Decision variables, Optimal result*

I. INTRODUCTION

Linear programming is a family of mathematical programming that is concerned with or useful for allocation of scarce or limited resources to several competing activities on the basis of given criterion of optimality. In statistics, linear programming (LP) is a special techniques employed in operation research for the purpose of optimization of linear function subject to linear equality and inequality constraint. Linear programming determines the way to achieve best outcome, such as maximum profit or minimum cost in a given mathematical model and given some list of requirement as a linear equation. The technique of linear programming is used in a wide range as applications, including agriculture, industry, transportation, economics, health system, behavioural and social science and the military. Although many business organization see linear programming as a “new science” or recently development in mathematical history, but there is nothing new about the maximization of profit in any business organization, i.e in a production company or manufacturing company. Linear programming was born during the Second World Warout of the necessity of solving military logistics problems. It remains one of the used mathematical techniques in today’s modern societies. The development of linear programming has been ranked among the most important scientific advances of the mid-20th century. Today it is a standard tool that has saved many thousands or millions of dollars for most companies or businesses of even moderate size in the various industrialized countries of the world. From report of various surveys, it has been shown that many production companies, particularly the ones operating in Nigeria are not conversance or yet to know fully the application of linear optimizations. Sometimes many production companies are faced with problems of how to utilize the available resources in order to maximize profit; this is because the use of linear programming which brings a suitable quantitative approach of decision-making has not been fully applied.

The decision of most production managers are based on the total input used in the production and output proceed. This method of decision making is always biased, that is it brings about reduction in the accuracy of forecasting for the future, such as price fluctuation and shortage of raw materials or available resources.

The problem of decision making based on the use of limited resource is a major factor that brought the application of linear programming model which is now one of the most powerful tools which all decision makers (managers) must apply before achieving effective decision.

II. LITERATURE REVIEW

Under this heading, we shall review the existing literatures which are related to the topic. According to miller (2007), linear programming is a generalization of linear algebra use in modelling so many real life problems ranging from scheduling airline routes to shipping oil from refineries to cities for the purpose of finding inexpensive diet capable of meeting daily requirements. Miller argued that the reason for the great versatility of linear programming is due to the ease at which constraints can be incorporated into the linear programming model.

Ezema and Amaken (2012) argue that the problem of industries all over the world is a result of shortage of production inputs which result in low capacity utilization and consequently low outputs. They applied linear programming in optimization of profit in golden plastic industry for the purpose of seeking and arriving at the optimal product-mix of the golden plastic industry. This was done or achieve by formulating the linear programming problem for production of plastic and estimated as such, the company was initially producing eight pipes but from the data analysis and estimation it shows that only two size of the total of eight polyvinel chloride pipes should be produced and they also succeed in establishing that 7,136.564 pieces of 20mm by 5.4m thick pressure pipe and 114317.2 pieces of 25mm by 5.4m conduct pipe should be produced and zero quantities of the remaining size in order to obtain a monthly profit of N1,964,537.

Igweet al (2011) reported that linear programming is a relevant technique in achieving efficiency in production planning, particularly in achieving increased agricultural productivity. They observed this when carrying out investigation on maximization of gross return from semi-commercial agriculture in Ohafia zone in Abia state. The general deterministic model is a gross margin maximization model designed to find out the optimum solutions, the decision variables for the model are numbers of hectares the farmer devoted to the production of crop and combination of crop or livestock capacity produced by the farmer.

Balogun et al (2012) reported that, the problem in production sectors is the problem of management, that many companies are faced with decision relating to the use of limited resources such as manpower, raw materials, capital etc.

In their work titled “use of linear programming for optimal production” in Coca-Cola Company, they were able to applied linear programming in obtaining the optimal production process for Coca-Cola Company. In the course of formulating a linear programming model for the production process, they identified the decision variables to be the following Coke, Fanta, Schweppes, Fanta tonic, Krest soda etc. which some up to nine decision variables and the constraint were identified to be concentration of the drinks, sugar content, water volume and carbon (iv) oxide. The resulting model was solve using the simplex algorithm, after the data analysis they came to a conclusion that out of the nine product the company was producing only two contribute most to their profit maximization, that is Fanta orange 50cl and Coke 50cl with a specified quantity of 462,547 and 415,593 in order to obtain a maximum profit of N263,497,283. They advise the company to concentrate in the production of the two products in order not to run into high cost.

Snezanza and Milorad (2009) recognize linear programming as an important tool in energy management despite the non-linearity property of many energy system, they argue that the non-linearity property can be converted to a linear form by applying Taylor series expansion so that the optimization method can be applied to determine the best means of generating energy at a minimum cost.

VeliUlucan(2010) reported that a mixed integer linear programming plays an important role in aggregate production planning (i.e a macro production planning) which addresses the problem of deciding how many employees the firm should retain and for manufacturing company, the quantity and mix of products to be produced. Veili argue that the decision variables for an integer programming are required to be an integer in order in order to satisfy both the objective function and the constraints.

Fagoyinbo and Ajibode (2010) reported that the success and failure that an individual or organization experience towards business planning depends to a large extent on the ability of making appropriate decision. They argue that a manager cannot make decision based on his/her personal experience, guesswork or intuition because the consequences of wrong decision is very costly, hence an understanding of the applicability of quantitative method to decision making is of fundamental importance to the decision maker. They described linear programming as one of the major quantitative approach to decision-making and hence applied it in effective use of resources for staff training, the decision variables for the model are the junior staff and senior staff and the constraints was the time available for training as the program is in-service training.

According to Majeke (2013) commercial farmers are always confronted with the problem of finding the combination of enterprises that will provide them with the highest amount of income through the best use of farm limited resources (constraints), he recognized the over-growing application of linear programming in agricultural sector, particularly in optimization of available farm resources in order to attain an optimal income (profit). He formulated a linear programming model that maximizes the income of farmers in rural area, the decision variables for the model was identified to be hectares allocated for maize production stored for family consumptions, hectares allocated for soya bean production and hectares allocated for tobacco production (i.e five decision variables) and also, six constraints were identified. The resulting model was solved using a computer software (MS excel).

Joly (2012) reported that, optimization is a crucial science for high-performance refineries , its main purpose in the oil sector is to push production process or operation towards the maximal profit until it reaches the limit at which any further profitability increase depends on changes in the existing system.

Stephanos and Dimitrios (2010), see linear programming as a great revolutionary development which has given mankind the ability to state general goals and to lay out path of detailed decision to take in order to “best” achieve its goals when faced with practical problem of great complexity. They argue that a simple linear programming begins with determination of interrelationship of an objective function as the maximization of profit for one or more products (activities).

Nabasirye et al (2011) argue that a linear programming problem is formed when the feasible region is subset of the non-negative portion of R^n, defined by linear equations and inequalities, and the objective function to be minimized or maximized is linear. They also argue that selecting the best alternative out of a large number of

possibilities is called optimization. They successively applied linear programming in minimization of cost of animal feed since animal feed was identified as a major factor in the overall cost of animal production in order to maximize an optimal profits.
According Mula et al (2005) production planning problem is one of the most important application of optimization tools using mathematical programming (linear programming). They argue that the idea of incorporating uncertainty in mathematical models is very important in order not to generate inferior planning decisions. This is known as sensitive analysis.
According to Waheed et al (2012) linear programming models are frequently used in operation research and management sciences to solve specific problems concerning the use of scare resources. They demonstrated the application of linear programming in profit maximization in a product-mix company, in selecting the best means for selling her medicated soap product which include 1 tablet per pack, 3 tablets per pack, 12 tablets per pack and 120 tablets per pack, which are subject to some constraints. The data analysis was carried out with R-statistical package, the result of the analysis showed that the company would obtain optimal monthly profitlevel of about N271,296 if she concentrates mainly on the unit sales (one tablet per pack)of her medicated soap product ignoring other types of sales packages.
Lenka (2013) argue that global economic crisis makes the business environment unfavorable for industries to survive or manage their resources optimally. Lenka formulated two linear programming models where one of them maximizes the revenue of a company and the other minimizes the cost of operation respectively.
Igbinehi et al (2015) applied linear programming model to maximize profit in a local soap production company, the company produces three different type of soap, 5g white soap, 10g white soap and 10g coloredsoap. From the data analysis it was observed that the company spends more on colored soap and they gets more profit from white soap than colored soap. So the company was advised to produce more of white soap (5g and 10g) than the colored soap in order to obtain an optimal profit.
Maryam et al (2013) reported that, linear programming plays an important role in improving management decision despite it is still regarded as new science but has proven to be capable in solving problems such as production planning, allocation of resources, inventory control and advertisement.
Taha (2003) argue that the idea of differential calculus is required for optimization of objective function subject to constrained continuous functions. For Taha, Langrangean method is the most appropriate method for solving optimization problem with equality constrained continuous functions, if the constraints are continuous and non-linear then the appropriate method is the Karush-Kuhn-Tucker method for system of non-leaner programming. This is called the classical optimization.

III. LINEAR PROGRAMMING MODEL

The general linear programming model with n decision variables and m constraints can be stated in the following form.

$$\text{Optimize } (\text{max or min}) \; Z = c_1 x_1 + c_2 x_2 + \ldots\ldots\ldots\ldots + c_n x_n$$

$$s.t$$

$$a_{11} x_1 + a_{12} x_2 + \ldots\ldots\ldots\ldots + a_{1n} x_n \; (\leq, \; =, \; \geq) \; b_1$$
$$a_{12} x_1 + a_{22} x_2 + \ldots\ldots\ldots\ldots + a_{2n} x_n \; (\leq, \; =, \; \geq) \; b_2$$
$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$
$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$
$$a_{m1} x_1 + a_{m2} + \ldots\ldots\ldots\ldots\ldots + a_{mn} x_n \; (\leq, \; =, \; \geq) \; b_m$$

The above model can also be expressed in a compact form as follows.

Optimize (max or min) Z = $\sum_{j=1}^{n} c_j x_i$ ….. (objective function)

Subject to the linear constraints

$$\sum_{j=1}^{n} a_{ij} x_j \quad (\leq, \; =, \; \geq) \; \mathrm{b}_i, \, i = 1, 2, \ldots\ldots \mathrm{m} \text{ and}$$

$$x_j \geq 0, \; j = 1, 2, \ldots\ldots \mathrm{n}$$

Where c_1, c_2, --------c_n represent the per unit profit (or cost) of decision variables x_1, x_2, -------, x_n to the value of the objective function. And a_{11}, a_{12} --------, a_{2n}, -------, am, a_{m2}, -------- a_{mn} represent the amount of resource consumed per unit of the decision variables. The b_i represents the total availability of the i^{th} resource. Z represent the measure – of – performance which can be either profit, or cost or reverence etc.

Standard form of a Linear Programming Model

The use of the simplex method to solve a linear programming problem requires that the problem be converted into its standard form. The standard form of a linear programming problem has the following properties.

i. All the constraints should be expressed as equations by adding slack or surplus variables.
ii. The right-hand side of each constraint should be made of non-negative (if not). This is done by multiplying both sides of the resulting constraints by -1.
iii. The objective function should be of a maximization type.

For n decision variables and m constraints, the standard form of the linear programming model can be formulated as follows.

$$\text{Optimize } (\max)\ Z = c_1x_1 + c_2x_2 + \ldots\ldots\ldots + c_nx_n + 0s_1 + 0s_2 + \ldots\ldots\ldots + 0s_m$$

Subject to the linear constraints

$$\begin{aligned}
a_{11}x_1 + \quad a_{12}x_2 + \ldots\ldots\ldots\ldots + a_{1n}x_n &= b_1 \\
a_{12}x_1 + \quad a_{22}x_2 + \ldots\ldots\ldots\ldots + a_{2n}x_n &= b_2 \\
\vdots \qquad\qquad \vdots \qquad\qquad \vdots & \\
\vdots \qquad\qquad \vdots \qquad\qquad \vdots & \\
a_{m1}x_1 + a_{m2} + \ldots\ldots\ldots\ldots + a_{mn}x_n + \mathrm{s}_{\mathrm{m}} &= b_m
\end{aligned}$$

$$x_1,\ x_2, \ldots\ldots,\ x_n,\ s_1,\ s_2,\ \ldots\ldots\ldots,\ s_m \geq 0$$

This can be stated in a more compact form as:

Optimize (max) $z = \sum_{j=1}^{n} c_j x_i + \sum_{i=1}^{m} 0 s_i$

Subject to the linear constraints

$$\sum_{j=1}^{n} a_{ij}\mathrm{x}_j s_i = \mathrm{b}_i,\ i = 1, 2, \ldots..\mathrm{m}\ \text{ and}$$

$$x_j, s_i \geq 0 \ \text{(for all i and j)}$$

Assumptions

- It is assumed that the raw materials required for production of bread are limited (scarce)
- It is assumed that an effective allocation of raw materials to the variables (big loaf, giant loaf and small loaf) will aid optimal production and at the same time maximizing the profit of then bakery.
- It is assumed that the qualities of raw materials used in bread production are standard (not inferior).

Data Presentation and Analysis

The data for this research project was collected from Gorretta bakery limited, Nigeria. The data consist of total amount of raw materials (sugar, flour, yeast, salt, wheat gluten and soybean oil) available for daily production of three different sizes of bread (big loaf, giant loaf and small loaf) and profit contribution per each unit size of bread produced.The data analysis was carried out with TORA software(version 2.0). The content of each raw material per each unit product of bread produced is shown below.

Flour

Total amount of flour available = 200kg
Each unit of big loaf requires 0.2kg of flour
Each unit of giant loaf requires 0.24kg of flour
Each unit of small loaf requires 0.14kg of flour

Sugar

Total amount of sugar available = 160g
Each unit of big loaf requires 0.14g of sugar
Each unit of giant loaf requires 0.20g of sugar
Each unit of small loaf requires 0.16g of sugar

Yeast
Total amount of yeast available = 20kg
Each unit of big loaf requires 0.02kg of yeast
Each unit of giant loaf requires 0.02kg of yeast
Each unit of small loaf requires 0.02kg of yeast

Salt
Total amount of salt available = 8.5g
Each unit of big loaf requires 0.0011g of salt
Each unit of giant loaf requires 0.00105g of salt
Each unit of small loaf requires 0.00017g of salt

Wheat gluten
Total amount of wheat gluten = 15.0g
Each unit of big loaf requires 0.000167g of wheat gluten
Each unit of giant loaf requires 0.002g of wheat gluten
Each unit of small loaf requires 0.00012g of wheat gluten

Soybean Oil
Total amount (volume) of soybean available = 10.0L
Each unit of big loaf requires 0.0157L of soybean oil
Each unit of giant loaf requires 0.021L of soybean oil
Each unit of small loaf requires 0.0098L of soybean oil

Profit contribution per unit product (size) of bread produced
Each unit of big loaf = N30
Each unit of giant loaf = N40
Each unit of small loaf = N20
The above data can be summarized in a tabular form.

Raw material	**Product**			**Total available raw material**
	Big loaf	**Giant loaf**	**Small loaf**	
Flour (kg)	0.20	0.24	0.14	200.0
Sugar (g)	0.14	0.20	0.16	160.0
Yeast (kg)	0.02	0.02	0.02	20.0
Salt (g)	0.0011	0.00105	0.00017	8.5
Wheat gluten (g)	0.000167	0.002	0.00012	15.0
Soybean oil (L)	0.015	0.021	0.0098	10.0
Profit (N)	30	40	20	

Model formulation
Let the quantity of big loaf to be produce = x_1
Let the quantity of giant loaf to be produce = x_2
Let the quantity of small loaf to be produce = x_3
Let Z denote the profit to be maximize
The linear programming model for the above production data is given by

$$\text{Max } Z = 30x_1 + 40x_2 + 20x_3$$

$$S.t$$

$$0.20x_1 + 0.24x_2 + 0.14x_3 \leq 200$$

$$0.14x_1 + 0.20x_2 + 0.16x_3 \leq 160$$

$$0.02x_1 + 0.02x_2 + 0.02x_3 \leq 20$$

$$0.0011x_1 + 0.00105x_2 + 0.00017x_3 \leq 8.5$$

$$0.000167x_1 + 0.002x_2 + 0.00012x_3 \leq 15$$

$$0.015x_1 + 0.021x_2 + 0.0098x_3 \leq 10$$

$$x_1, x_2, x_3 \geq 0$$

Converting the model into its corresponding standard form;

$$Max\ Z = 30x_1 + 40x_2 + 20x_3 + 0s_1 + 0s_2 + 0s_3 + 0s_4 + 0s_5 + 0s_6$$

$$S.t$$

$$0.20x_1 + 0.24x_2 + 0.14x_3 + s_1 = 200$$

$$0.14x_1 + 0.20x_2 + 0.16x_3 + s_2 = 160$$

$$0.02x_1 + 0.02x_2 + 0.02x_3 + s_3 = 20$$

$$0.0011x_1 + 0.00105x_2 + 0.00017x_3 + s_4 = 8.5$$

$$0.000167x_1 + 0.002x_2 + 0.00012x_3 + s_5 = 15$$

$$0.015x_1 + 0.021x_2 + 0.0098x_3 + s_6 = 10$$

$$x_1, x_2, x_3, s_1, s_2, s_3, s_4, s_5, s_6 \geq 0$$

The above linear programming model was solved using TORA software, which gives an optimal solution of:
$X_1 = 38.0$, $X_2 = 0.0$, $X_3 = 962.0$
$Z = 20385.0$

Interpretation of Result
Based on the data collected the optimum result derived from the model indicates that two sizes of bread should be produced, small loaf and big loaf. Their production quantities should be 962.0 and 38.0 units respectively. This will produce a maximum profit of N20,385.0

IV. SUMMARY

The objective of this research work was to apply linear programming for optimal use of raw material in bread production. Goretta bakery limited was used as our case study. The decision variables in this research work are the three different sizes of bread (big loaf, giant loaf and small loaf) produced by Goretta bakery limited. The researcher focused mainly on six raw materials (flour, sugar, yeast, salt, wheat gluten and soybean oil)used in the production and the amount of raw material required of each variable (bread size). The result shows that 962 unit of small loaf, 38 unit of big loaf and 0 unit of giant loaf should be produce respectively which will give a maximum profit of ₦20385.0

V. CONCLUSION

Based on the analysis carried out in this research work and the result shown, Goretta bakery limited should produce the three sizes of bread (big loaf, giant loaf and small loaf) in order to satisfy her customers. Also, more of small loaf and big loaf should be produce in order to attain maximum profit, because they contribute mostly to the profit earned by the company.

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