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Analysis of Non-successive Occurrence of Digit 3 in Prime Numbers till 1 Trillion

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ABSTRACT: *The non-successive occurrence of digit 3 in prime numbers till 1 trillion is considered in this work. All possible multiple non-successive occurrences are analyzed. The smallest and largest prime number with all feasible repetitions of non-successive 3's are determined within ranges of increasing powers of 10.*
Keywords: *Digit 3, non-successive occurrences, prime numbers.*
MATHEMATICS SUBJECT CLASSIFICATION 2010 -*11Y35, 11Y60, 11Y99*

I. INTRODUCTION

The very beginning of mathematics is with counting which requires natural numbers. That is the reason why we see most of the ancient human civilizations exploring and using these positive integers in their own peculiar ways [2].

The most fascinating amongst the natural numbers are prime numbers. They have innumerable curious properties one of which mostly quoted is their irregular distribution pattern. While it is strongly conjectured that there are infinitely many successive primes which differ by only 2; there is classically proved theorem that there are successive primes with arbitrarily large gaps between them! So these primes at times come very close and also at other times do accommodate unlimited large gaps. This, amongst other many things not aforementioned, necessitates study of primes on two fronts : asymptotic [1] and exclusive [4].

3 is special number. It is itself the first odd prime. It is the only prime which is immediate next to previous prime 2. The time has 3 phases arising out of its arrow and flow : past, present and future. It is the only positive integer which is equal to sum of all of its predecessors. And there are many more interesting properties that 3 enjoys.

The occurrence of 0 [5], [6], [7] and any non-zero digit like 1 [11], [12], [13] in positive integers is analyzed earlier in detail,the later of which is equally applicable to 3. All occurrences of 3 as well as successive ones in prime numbers are examined in [20], [21]. This work for significant digit 0 [8], [9], [10], digit 1 [14], [15], [16] and digit 2 [17], [18], [19] in primes is done till date.

II. OCCURRENCE OF SINGLE NON-SUCCESSIVE DIGIT 3 IN PRIME NUMBERS

Solo occurrence of digit 3 in prime numbers is given in [20] and [21]. Such single occurrence is qualified to be successive. So there can't be non-successive occurrence of single digit 3 either in primes or any numbers!

III. OCCURRENCE OF MULTIPLE NON-SUCCESSIVE 3'S IN PRIME NUMBERS

Employing many computer systems simultaneously to execute a special algorithm written in Java programming language, following results have been obtained for the number of primes in ranges $1-10^n$, $1 \le n \le 12$, containing 2 or more number of non-successive digit 3's.

Table 1: Occurrences of Digit 1 in Blocks of 10 Powers

Sr. No.	Number Range <	Number of Primes with 2 Non-successive 3's	Number of Primes with 3 Non-successive 3's	Number of Primes with 4 Non-successive 3's
1.	10^3	4	0	0
2.	10^4	49	8	0
3.	10^5	678	112	5
4.	10^6	7,687	1,610	160
5.	10^7	81,724	20,968	2,783
6.	10^8	841,739	254,902	41,953
7.	10^9	8,512,529	2,950,938	576,095
8.	10^{10}	84,895,400	33,019,865	7,443,061
9.	10^{11}	837,688,566	360,567,042	91,981,615
10.	10^{12}	8,196,349,722	3,862,275,821	1,098,720,299

Table 1: Continued …

Sr. No.	Number Range <	Number of Primes with 5 Non-successive 3's	Number of Primes with 6 Non-successive 3's	Number of Primes with 7 Non-successive 3's
1.	10^6	11	0	0
2.	10^7	218	12	0
3.	10^8	4,224	251	9
4.	10^9	72,009	5,985	278
5.	10^{10}	1,110,366	113,196	7,786
6.	10^{11}	15,931,912	1,941,888	167,649
7.	10^{12}	216,285,908	30,642,090	3,161,064

Table 1: Continued …

Sr. No.	Number Range <	Number of Primes with 8 Non-successive 3's	Number of Primes with 9 Non-successive 3's	Number of Primes with 10 Non-successive 3's	Number of Primes with 11 Non-successive 3's
1.	10^9	8	0	0	0
2.	10^{10}	361	7	0	0
3.	10^{11}	9,907	373	8	0
4.	10^{12}	234,784	12,280	397	8

The number of primes containing multiple non-successive digit 3's in them in ranges $1 - 10^n$ is plotted and vertical axis is on logarithmic scale.

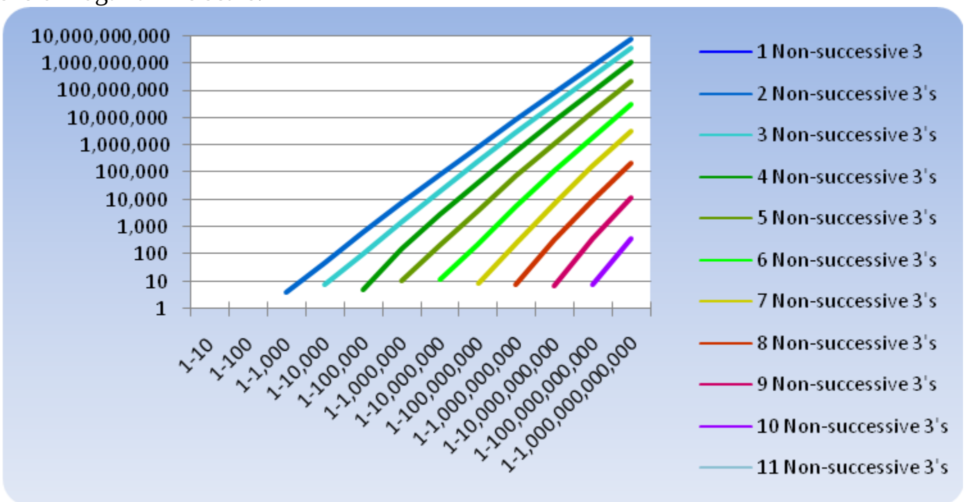

Fig. 1 : Number of Primes in Various Ranges with Multiple Non-successive 3's in Their Digits

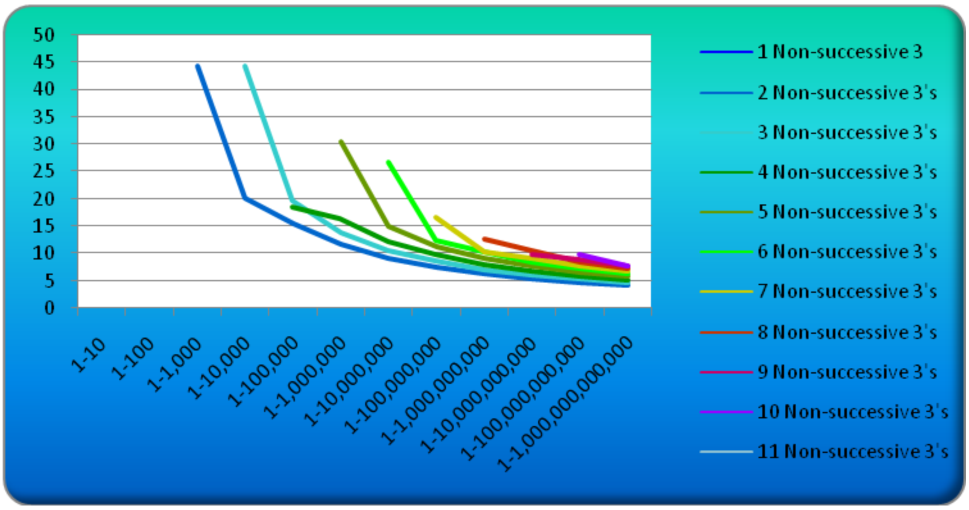

Fig. 2 : Percentage of Primes in Different Ranges with Multiple Non-successive 3's in Their Digits With Respect To All Such Positive Integers in Respective Ranges

The differences of number of multiple non-successive occurrences of digits 1 and 2 in primes with those of 3 in them in our ranges are depicted below graphically. Digit 0 is ignored as it always has less scope to occupy all digit places, particularly units and leading n^{th}places in any n digit prime are barred for it.

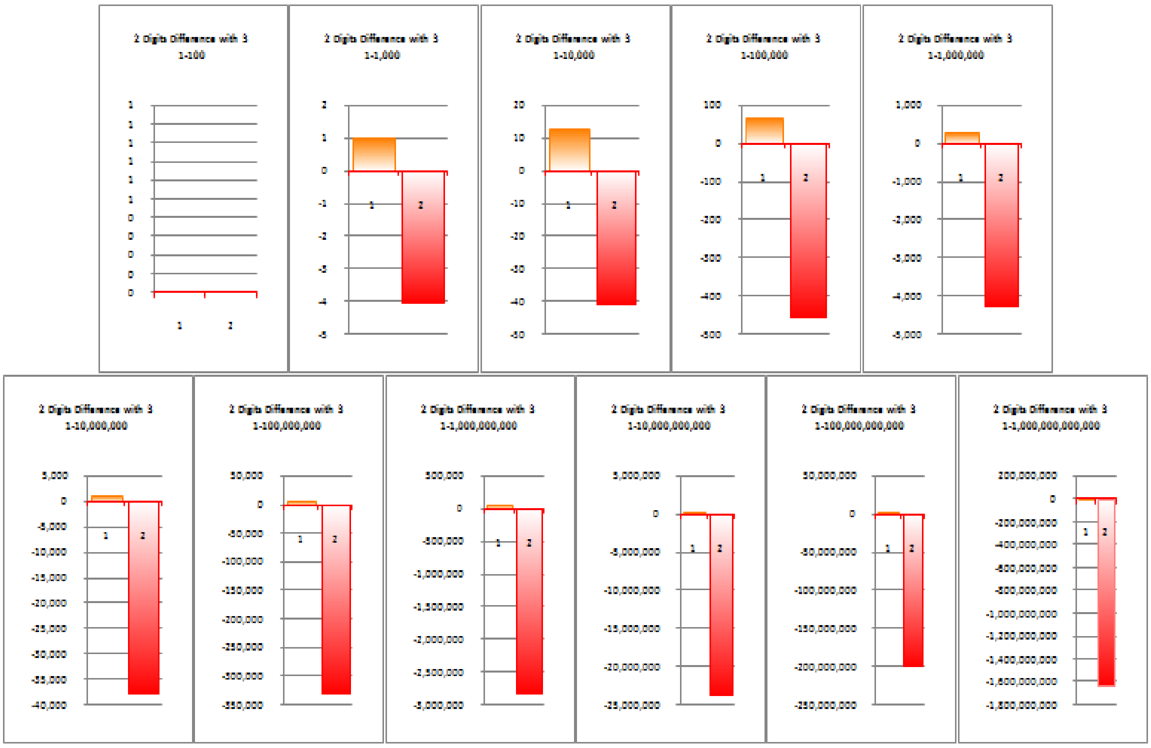

Fig. 3 : Differences of Number of Primes having Two Non-successive 1's and Two Non-successive 2's in their Digits with those having Two Non-successive 3's in them in Ranges of 1 – 10^n.

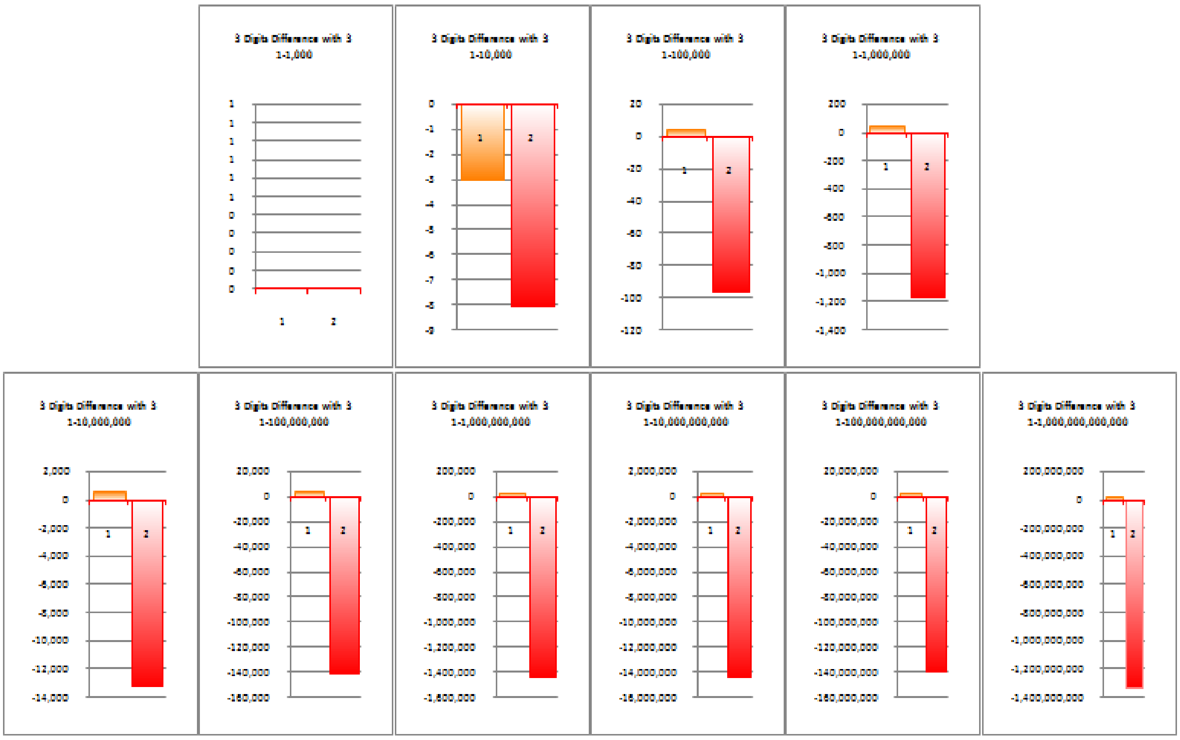

Fig. 4 : Differences of Number of Primes having ThreeNon-successive 1's and ThreeNon-successive 2's in their Digits with those having ThreeNon-successive 3's in them in Ranges of 1 – 10^n.

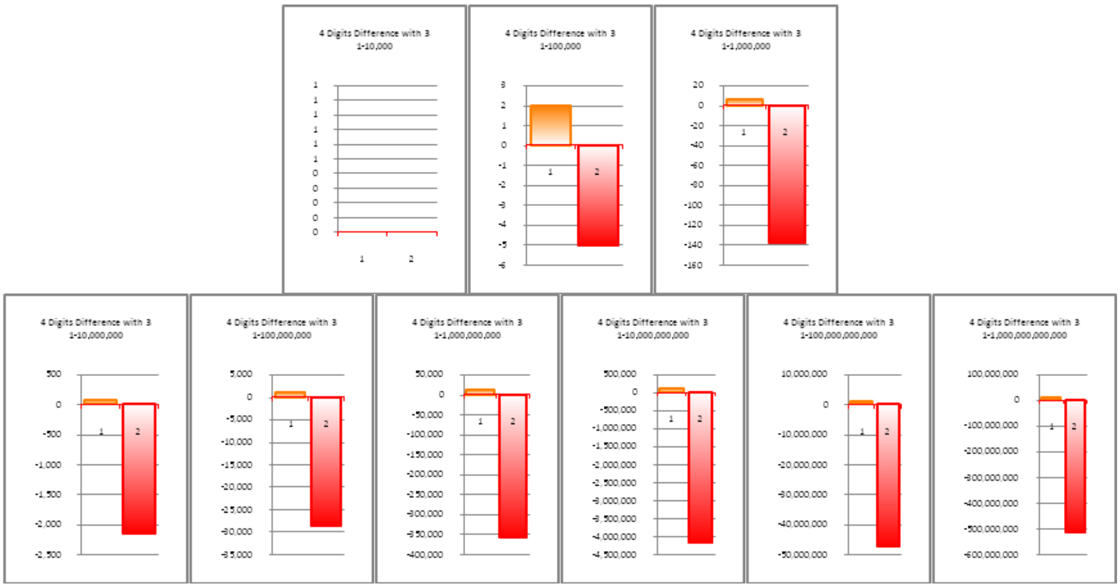

Fig. 5 : Differences of Number of Primes having FourNon-successive 1's and FourNon-successive 2's in their Digits with those having FourNon-successive 3's in them in Ranges of 1 – 10^n.

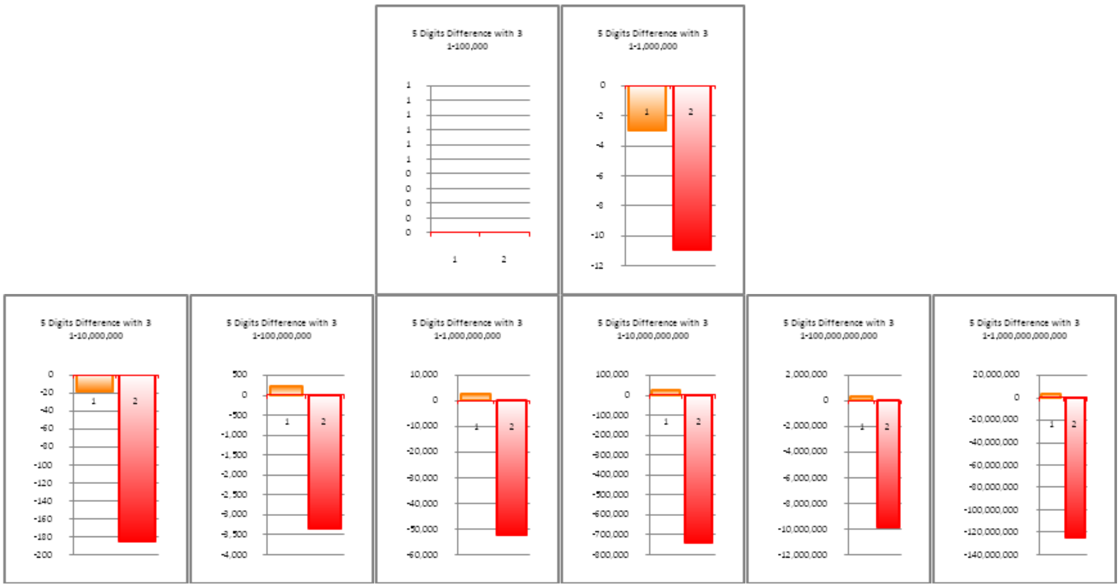

Fig. 6 : Differences of Number of Primes having FiveNon-successive 1's and FiveNon-successive 2's in their Digits with those having FiveNon-successive 3's in them in Ranges of 1 – 10^n.

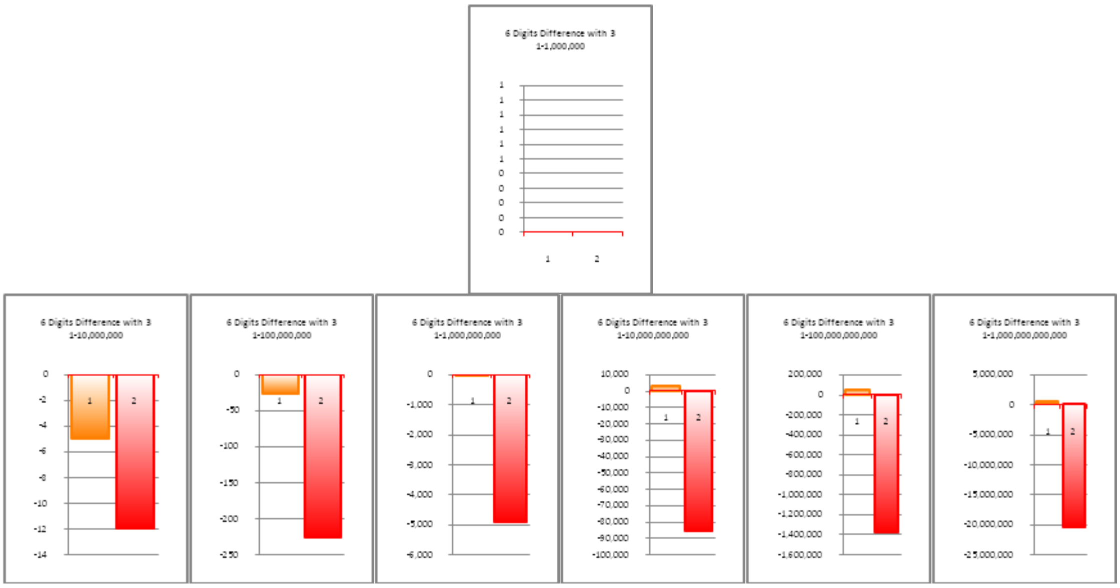

Fig. 7 : Differences of Number of Primes having SixNon-successive 1's and SixNon-successive 2's in their Digits with those having SixNon-successive 3's in them in Ranges of $1 - 10^n$.

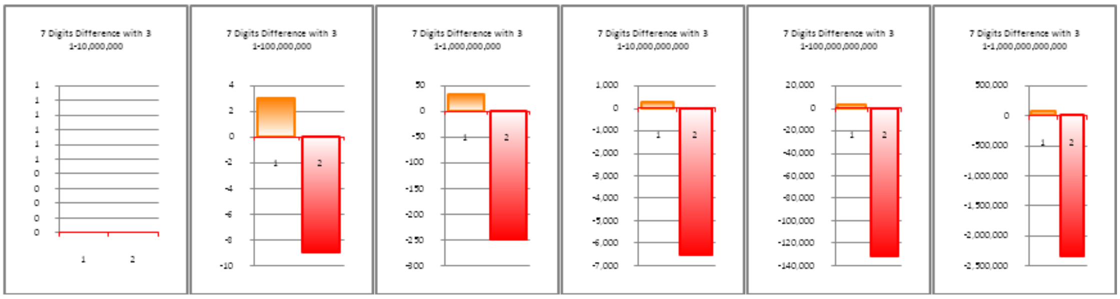

Fig. 8 : Differences of Number of Primes having SevenNon-successive 1's and SevenNon-successive 2's in their Digits with those having SevenNon-successive 3's in them in Ranges of $1 - 10^n$.

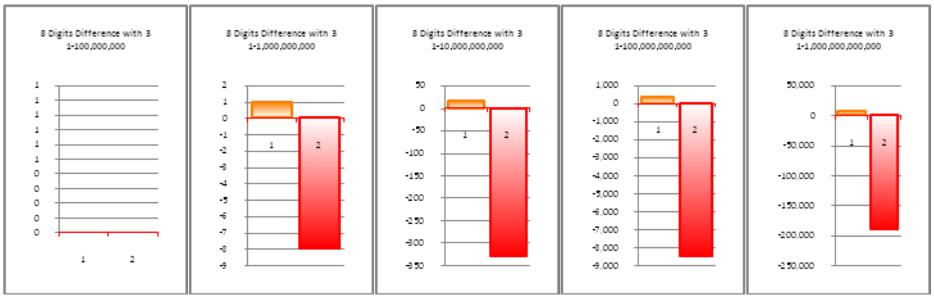

Fig. 9 : Differences of Number of Primes having EightNon-successive 1's and EightNon-successive 2's in their Digits with those having EightNon-successive 3's in them in Ranges of $1 - 10^n$.

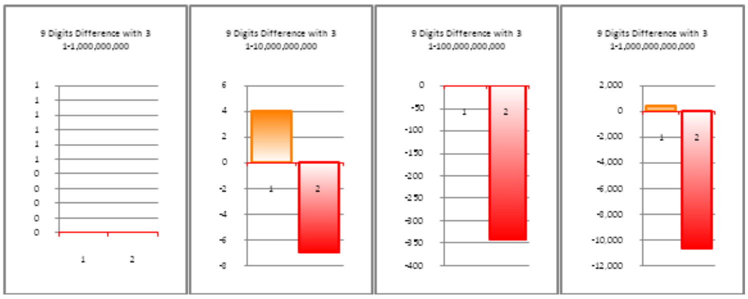

Fig. 10 : Differences of Number of Primes having NineNon-successive 1's and NineNon-successive 2's in their Digits with those having NineNon-successive 3's in them in Ranges of $1 - 10^n$.

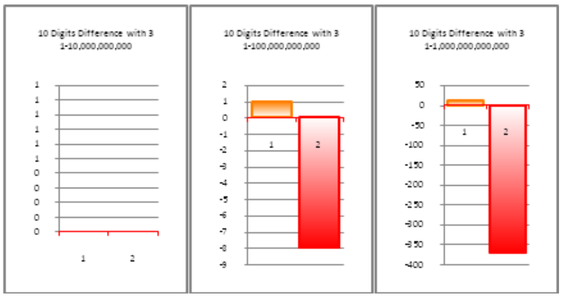

Fig. 11 : Differences of Number of Primes having TenNon-successive 1's and TenNon-successive 2's in their Digits with those having TenNon-successive 3's in them in Ranges of $1 - 10^n$.

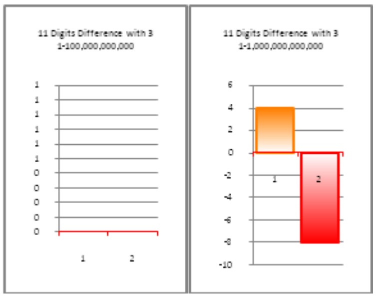

Fig. 12 : Differences of Number of Primes having ElevenNon-successive 1's and ElevenNon-successive 2's in their Digits with those having ElevenNon-successive 3's in them in Ranges of $1 - 10^n$.

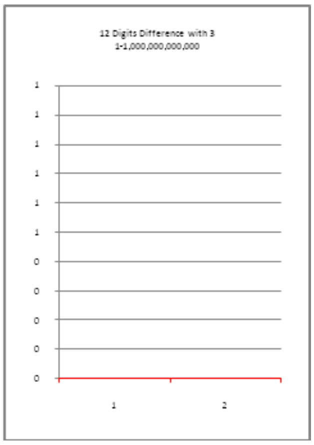

Fig. 13 : Differences of Number of Primes having TwelveNon-successive 1's and TwelveNon-successive 2's in their Digits with those having TwelveNon-successive 3's in them in Ranges of $1 - 10^n$.

IV. FIRST OCCURRENCE OF NON-SUCCESSIVE 3'S IN PRIME NUMBERS

Although the first positive integer containing one 3 is 3 itself, as is single, it's not considerable as non-successive. The first integer with 2 non-successive 3's is 303. The others are obtained by

Formula 1 [13] : If n and r are natural numbers, then the first occurrence of r number of non-successive 3's in numbers in range $1 \leq m < 10^n$ is

$$f = \begin{cases} - , & \text{if } r < 2 \text{ or } r \geq n \\ \sum\limits_{\substack{j=0 \\ j \neq r-1}}^{r} \left(3 \times 10^{j}\right), & \text{if } r \geq 2 \text{ and } r < n \end{cases}.$$

First prime numbers with multiple non-successive 3's don't seem to be in a simple pattern.

Table 2 : First Primes in Various Ranges with Multiple Non-successive 3's in Their Digits

Sr. No.	Range	First Prime Number in Range with					
		1 Non-successive 3	2 Non-successive 3's	3 Non-successive 3's	4 Non-successive 3's	5 Non-successive 3's	6 Non-successive 3's
1.	$1 - 10^{1}$	-	-	-	-	-	-
2.	$1 - 10^{2}$	-	-	-	-	-	-
3.	$1 - 10^{3}$	-	313	-	-	-	-
4.	$1 - 10^{4}$	-	313	3,313	-	-	-
5.	$1 - 10^{5}$	-	313	3,313	31,333	-	-
6.	$1 - 10^{6}$	-	313	3,313	31,333	313,333	-
7.	$1 - 10^{7}$	-	313	3,313	31,333	313,333	3,233,333
8.	$1 - 10^{8}$	-	313	3,313	31,333	313,333	3,233,333
9.	$1 - 10^{9}$	-	313	3,313	31,333	313,333	3,233,333
10.	$1 - 10^{10}$	-	313	3,313	31,333	313,333	3,233,333
11.	$1 - 10^{11}$	-	313	3,313	31,333	313,333	3,233,333
12.	$1 - 10^{12}$	-	313	3,313	31,333	313,333	3,233,333

Table 2 : Continued …

Sr. No.	Range	First Prime Number in Range with				
		7 Non-successive 3's	8 Non-successive 3's	9 Non-successive 3's	10 Non-successive 3's	11 Non-successive 3's
1.	$1 - 10^{1}$	-	-	-	-	-
2.	$1 - 10^{2}$	-	-	-	-	-
3.	$1 - 10^{3}$	-	-	-	-	-
4.	$1 - 10^{4}$	-	-	-	-	-
5.	$1 - 10^{5}$	-	-	-	-	-
6.	$1 - 10^{6}$	-	-	-	-	-
7.	$1 - 10^{7}$	-	-	-	-	-
8.	$1 - 10^{8}$	31,333,333	-	-	-	-
9.	$1 - 10^{9}$	31,333,333	333,233,333	-	-	-
10.	$1 - 10^{10}$	31,333,333	333,233,333	3,233,333,333	-	-
11.	$1 - 10^{11}$	31,333,333	333,233,333	3,233,333,333	33,331,333,333	-
12.	$1 - 10^{12}$	31,333,333	333,233,333	3,233,333,333	33,331,333,333	333,313,333,333

All of these are not first occurrences of same number of general 3's. Some of them happen to be successive and few others non-successive.

V. LAST OCCURRENCE OF NON-SUCCESSIVE 3'S IN PRIME NUMBERS

The last natural number in ranges 1 - 10^{n} with r number of non-successive 3's is given by

Formula 2 [13] : If n and r are natural numbers, then the last occurrence of r number of non-successive 3's in numbers in range $1 \leq m < 10^{n}$ is

$$l = \begin{cases} - & , \text{ if } r < 2 \text{ or } r \geq n \\ \sum\limits_{\substack{j=0 \\ j \neq r-1}}^{r} \left(3 \times 10^{j}\right) + \sum\limits_{\substack{j=r-1 \\ j \neq r}}^{n-1} \left(9 \times 10^{j}\right), & \text{ if } r \geq 2 \text{ and } r < n \end{cases}.$$

The last prime number in such ranges with multiple non-successive 3's in their digits are determined to be as follows.

Table 3 : Last Primes in Various Ranges with Multiple Non-successive 3's in Their Digits

Sr. No.	Number of Non-Successive 3's	Last Prime Number in Range 1 –							
		10^{1}	10^{2}	10^{3}	10^{4}	10^{5}	10^{6}	10^{7}	10^{8}
1.	1	-	-	-	-	-	-	-	-
2.	2	-	-	383	9,343	98,323	998,353	9,998,393	99,999,373
3.	3	-	-	-	3,833	93,383	993,323	9,993,383	99,993,833
4.	4	-	-	-	-	38,333	933,433	9,933,373	99,933,313
5.	5	-	-	-	-	-	353,333	9,373,333	99,338,333
6.	6	-	-	-	-	-	-	3,733,333	93,733,333
7.	7	-	-	-	-	-	-	-	34,333,333
8.	8	-	-	-	-	-	-	-	-
9.	9	-	-	-	-	-	-	-	-
10.	10	-	-	-	-	-	-	-	-
11.	11	-	-	-	-	-	-	-	-

Table 3 : Continued …

Sr. No.	Number of Non-Successive 3's	Last Prime Number in Range 1 –		
		10^9	10^{10}	10^{11}
1.	1	-	-	-
2.	2	999,999,353	9,999,998,363	99,999,999,353
3.	3	999,993,833	9,999,993,833	99,999,993,833
4.	4	999,935,333	9,999,937,333	99,999,933,433
5.	5	999,343,333	9,999,383,333	99,999,335,333
6.	6	993,383,333	9,993,383,333	99,993,433,333
7.	7	933,833,333	9,933,333,833	99,934,333,333
8.	8	373,333,333	9,353,333,333	99,383,333,333
9.	9	-	3,334,333,333	93,433,333,333
10.	10	-	-	38,333,333,333
11.	11	-	-	-

Remark: The maximum number of non-successive digits 3's in any prime number in the range 1 - 10^n, for $n > 2$, is at most $n - 1$.

Various integers coming in all sections of this work form interesting sequences which merit independent treatment.

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REFERENCES

[1] Benjamin Fine, Gerhard Rosenberger, "Number Theory: An Introduction via the Distribution of Primes," Birkhauser, 2007.

[2] Neeraj Anant Pande, "Numeral Systems of Great Ancient Human Civilizations", Journal of Science and Arts, Year 10, No. 2 (13), 2010, 209-222.

[3] Neeraj Anant Pande, "Improved Prime Generating Algorithms by Skipping Composite Divisors and Even Numbers (Other Than 2)", Journal of Science and Arts, Year 15, No.2(31), 2015, 135-142.

[4] Neeraj Anant Pande, "Analysis of Primes Less Than a Trillion", International Journal of Computer Science and Engineering Technology, Vol. 6, No 6, 2015, 332-341.

[5] Neeraj Anant Pande, "Analysis of Occurrence of Digit 0 in Natural Numbers Less Than 10^n", American International Journal of Research in Formal, Applied and Natural Sciences, Communicated, 2016.

[6] Neeraj Anant Pande, "Analysis of Successive Occurrence of Digit 0 in Natural Numbers Less Than 10^n", IOSR-Journal of Mathematics, Vol. 12, Issue 5, Ver. VIII, 2016, 70 - 74.

[7] Neeraj Anant Pande, "Analysis of Non-successive Occurrence of Digit 0 in Natural Numbers Less Than 10^n", International Journal of Emerging Technologies in Computational and Applied Sciences, Communicated, 2016.

[8] Neeraj Anant Pande, "Analysis of Occurrence of Digit 0 in Prime Numbers till 1 Trillion", Journal of Research in Applied Mathematics, Vol. 3, Issue 1, 2016, 26-30.

[9] Neeraj Anant Pande, "Analysis of Successive Occurrence of Digit 0 in Prime Numbers till 1 Trillion", International Journal of Mathematics And its Applications, Accepted, 2016.

[10] Neeraj Anant Pande, "Analysis of Non-successive Occurrence of Digit 0 in Prime Numbers till 1 Trillion", International Journal of Computational and Applied Mathematics,Accepted, 2016.

[11] Neeraj Anant Pande, "Analysis of Occurrence of Digit 1 in Natural Numbers Less Than 10^n", Advances in Theoretical and Applied Mathematics, 11(2), 2016, 99-104.

[12] Neeraj Anant Pande, "Analysis of Successive Occurrence of Digit 1 in Natural Numbers Less Than 10^n", American International Journal of Research in Science, Technology, Engineering and Mathematics, 16(1), 2016, 37-41.

[13] Neeraj Anant Pande, "Analysis of Non-successive Occurrence of Digit 1 in Natural Numbers Less Than 10^n", International Journal of Advances in Mathematics and Statistics, Accepted, 2016.

[14] Neeraj Anant Pande, "Analysis of Occurrence of Digit 1 in Prime Numbers till 1 Trillion", International Research Journal of Mathematics, Engineering and IT, Communicated, 2016.

[15] Neeraj Anant Pande, "Analysis of Successive Occurrence of Digit 1 in Prime Numbers till 1 Trillion", International Journal of Mathematics Trends and Technology, Vol. 38, No. 3, 2016, 125-129.

[16] Neeraj Anant Pande, "Analysis of Non-successive Occurrence of Digit 1 in Prime Numbers till 1 Trillion", Journal of Computer and Mathematical Sciences, Vol. 7 (10), 2016, 499-505.

[17] Neeraj Anant Pande, "Analysis of Occurrence of Digit 2 in Prime Numbers till 1 Trillion", International Journal of Recent Research in Mathematics Computer Science and Information Technology, Vol. 3, Issue 2, 2016, 1 - 7.

[18] Neeraj Anant Pande, "Analysis of Successive Occurrence of Digit 2 in Prime Numbers till 1 Trillion", Journal of Computer and Mathematical Sciences, Communicated, 2016.

[19] Neeraj Anant Pande, "Analysis of Non-successive Occurrence of Digit 2 in Prime Numbers till 1 Trillion", International Journal Of Advancement In Engineering Technology, Management and Applied Science, Communicated, 2016.

[20] Neeraj Anant Pande, "Analysis of Occurrence of Digit 3 in Prime Numbers till 1 Trillion", IOSR-Journal of Mathematics, Communicated, 2016.

[21] Neeraj Anant Pande, "Analysis of Successive Occurrence of Digit 3 in Prime Numbers till 1 Trillion", International Journal of Mathematics and Computer Research, Communicated, 2016.

[22] Nishit K Sinha, "Demystifying Number System", Pearson Education, New Delhi, 2010.