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Enhancing the Mean Ratio Estimator for Estimating Population Mean Using Conventional Parameters

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ABSTRACT: *Use of auxilliary information in survey sampling has its eminent role in estimating the population parameters with greater precision. The present paper concentrates on estimating the finite population mean by proposing the new generalised ratio type estimators in simple random sampling without replacement using coefficient of variation and population deciles. The expressions for mean square error and bias were calculated and compared with the classical and existing estimators. By this comparison it is conformed that our proposed class of new estimators is a class of efficient estimators under percent relative efficiency (PRE) criterion.*
KEYWORDS: *Coefficient of variation; deciles; ratio-type estimators; mean square error; bias; efficiency.*

I. INTRODUCTION

The classical ratio type estimator was first given by Cochran (1940), for estimating population mean based on some prior information of population of an auxiliary variable X. Rao (1991) introduced difference type ratio estimator that outperforms conventional ratio and linear regression estimators. Upadhyaya & Singh (1999) introduced modified ratio type estimators using coefficient of variation and coefficient of kurtosis of the auxiliary variable. Singh & Tailor (2003) proposed a family of estimators using known values of some parameters. Sisodia & Dwivedi (1981) and Singh et al. (2004) utilized coefficient of variation of the auxiliary variable. Further improvements are achieved by introducing a large number of modified ratio estimators (see Subramani and Kumarpandiyan, (2012 a, b and c)).

Consider a finite population $U = \{U_1, U_2, U_3, ..., U_N\}$ of N distinct and identifiable units. Let Y be the study variable with value Y_i measured of U_i, $i = 1,2,3,..., N$ giving a vector $Y = \{Y_1, Y_2, Y_3, ..., Y_N\}$. The objective is to estimate population mean $\overline{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$ on the basis of a random sample. The mean ratio estimator for estimating the population mean, $\overline{Y}$, of the study variable Y is defined as

$$\hat{\overline{Y}}_r = \frac{\overline{y}}{\overline{x}}\overline{X} \quad (1)$$

The bias, related constant and the mean squared error (MSE) of the ratio estimator are respectively given by

$$B(\hat{\overline{Y}}_r) = \frac{(1-f)}{n}\frac{1}{\overline{X}}(RS_x^2 - \rho S_x S_Y) \quad R = \frac{\overline{Y}}{\overline{X}} \quad MSE(\hat{\overline{Y}}_r) = \frac{1-f}{n}(S_y^2 + R^2 S_x^2 - 2R\rho S_x S_Y) \quad (2)$$

II. EXISTING RATIO ESTIMATORS

Kadilar and Cingi (2004, 2006) suggested ratio type estimators for the population mean in the simple random sampling using some known auxiliary information on coefficient of kurtosis, coefficient of variation and coefficient of correlation. They showed that their suggested estimators are more efficient than traditional ratio estimator in the estimation of the population mean.

$$\hat{\overline{Y}}_1 = \frac{\overline{y} + b(\overline{X} - \overline{x})}{\overline{x}}\overline{X}, \quad \hat{\overline{Y}}_2 = \frac{\overline{y} + b(\overline{X} - \overline{x})}{(\overline{x} + C_x)}(\overline{X} + C_x), \quad \hat{\overline{Y}}_3 = \frac{\overline{y} + b(\overline{X} - \overline{x})}{(\overline{x} + \beta_2)}(\overline{X} + \beta_2),$$

$$\hat{\overline{Y}}_4 = \frac{\overline{y} + b(\overline{X} - \overline{x})}{(\overline{x}\beta_2 + C_x)}(\overline{X}\beta_2 + C_x), \quad \hat{\overline{Y}}_5 = \frac{\overline{y} + b(\overline{X} - \overline{x})}{(\overline{x}C_x + \beta_2)}(\overline{X}C_x + \beta_2), \quad \hat{\overline{Y}}_6 = \frac{\overline{y} + b(\overline{X} - \overline{x})}{(\overline{x} + \rho)}(\overline{X} + \rho),$$

$$\hat{\overline{Y}}_7 = \frac{\overline{y} + b(\overline{X} - \overline{x})}{(\overline{x}C_x + \rho)}(\overline{X}C_x + \rho), \quad \hat{\overline{Y}}_8 = \frac{\overline{y} + b(\overline{X} - \overline{x})}{(\overline{x}\rho + C_x)}(\overline{X}\rho + C_x), \quad \hat{\overline{Y}}_9 = \frac{\overline{y} + b(\overline{X} - \overline{x})}{(\overline{x}\beta_2 + \rho)}(\overline{X}\beta_2 + \rho),$$

$$\hat{\bar{Y}}_{10} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\rho + \beta_2)}(\bar{X}\rho + \beta_2). \quad (3)$$

III. PROPOSED MODIFIED RATIO ESTIMATOR

Motivated by the mentioned estimators in Section 2, we propose new class of efficient ratio type estimators using the linear combination of population deciles and coefficient of variation.

$$\hat{\bar{Y}}_{p1} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}D_1 + C_x)}(\bar{X}D_1 + C_x). \qquad \hat{\bar{Y}}_{p2} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}D_2 + C_x)}(\bar{X}D_2 + C_x).$$

$$\hat{\bar{Y}}_{p3} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}D_3 + C_x)}(\bar{X}D_3 + C_x). \qquad \hat{\bar{Y}}_{p4} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}D_4 + C_x)}(\bar{X}D_4 + C_x).$$

$$\hat{\bar{Y}}_{p5} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}D_5 + C_x)}(\bar{X}D_5 + C_x). \qquad \hat{\bar{Y}}_{p6} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}D_6 + C_x)}(\bar{X}D_6 + C_x).$$

$$\hat{\bar{Y}}_{p7} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}D_7 + C_x)}(\bar{X}D_7 + C_x). \qquad \hat{\bar{Y}}_{p8} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}D_8 + C_x)}(\bar{X}D_8 + C_x).$$

$$\hat{\bar{Y}}_{p9} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}D_9 + C_x)}(\bar{X}D_9 + C_x). \qquad \hat{\bar{Y}}_{p10} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}D_{10} + C_x)}(\bar{X}D_{10} + C_x). \quad (4)$$

The bias, related constant and the MSE for proposed estimator can be obtained as follows:

$$B(\hat{\bar{Y}}_{pj}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_j^2, \qquad R_j = \frac{\bar{X}D_j}{\bar{X}D_j + C_x} \qquad MSE(\hat{\bar{Y}}_{pj}) = \frac{(1-f)}{n}(R_j^2 S_x^2 + S_y^2(1-\rho^2)).$$

$j = 1,2,....,10$ (5)

IV. EFFICIENCY COMPARISON

Comparisons with existing ratio estimators

From the expressions of the MSE of the proposed estimators and the existing estimators, we have derived the conditions for which the proposed estimators are more efficient than the existing modified ratio estimators as follows:

$$MSE(\hat{\bar{Y}}_{pj}) \le MSE(\hat{\bar{Y}}_i),$$

$$\frac{(1-f)}{n}(R_{pj}^2 S_x^2 + S_y^2(1-\rho^2)) \le \frac{(1-f)}{n}(R_i^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$R_{pj}^2 S_x^2 \le R_i^2 S_x^2,$$

$$R_{pj} \le R_i, \quad (6)$$

Where $j = 1,2,....,10$ and $i = 1,2,....,10$.

V. EMPIRICAL STUDY

The Population is taken from Singh and Chaudhary (1986).

$N = 34$	$n = 20$	$\bar{Y} = 856.4117$	$\bar{X} = 199.4412$	
$\rho = 0.4453$	$S_y = 733.1407$	$C_y = 0.8561$	$S_x = 150.2150$	$C_x = 0.7531$
$\beta_2 = 1.0445$	$D_1 = 60.6000$	$D_2 = 83.0000$		
$D_3 = 102.7000$	$D_4 = 111.2000$	$D_5 = 142.5000$		
$D_6 = 210.2000$	$D_7 = 264.5000$	$D_8 = 304.4000$		
$D_9 = 373.2000$	$D_{10} = 634.0000$			

Estimators	Constant	Bias	MSE
$\hat{Y}_r$	4.294	4.940	10960.76
$\hat{Y}_1$	4.294	10.002	17437.65
$\hat{Y}_2$	4.278	9.927	17373.31
$\hat{Y}_3$	4.272	9.898	17348.62
$\hat{Y}_4$	4.279	9.930	17376.04
$\hat{Y}_5$	4.264	9.865	17319.75
$\hat{Y}_6$	4.285	9.957	17399.52
$\hat{Y}_7$	4.281	9.943	17387.08
$\hat{Y}_8$	4.258	9.834	17294.19
$\hat{Y}_9$	4.285	9.960	17401.14
$\hat{Y}_{10}$	4.244	9.771	17239.66
$\hat{Y}_{p1}$	0.999937692	0.540062163	9296.257115
$\hat{Y}_{p2}$	0.999954507	0.540080327	9296.272671
$\hat{Y}_{p3}$	0.999963233	0.540089753	9296.280743
$\hat{Y}_{p4}$	0.999966043	0.540092788	9296.283343
$\hat{Y}_{p5}$	0.999973502	0.540100845	9296.290243
$\hat{Y}_{p6}$	0.999982036	0.540110064	9296.298138
$\hat{Y}_{p7}$	0.999985724	0.540114048	9296.301550
$\hat{Y}_{p8}$	0.999987595	0.540116069	9296.303281
$\hat{Y}_{p9}$	0.999989882	0.540118540	9296.305396
$\hat{Y}_{p10}$	0.999994044	0.540123036	9296.309247

Percentage relative efficiency (PRE) of proposed estimator with existing estimators

	$\hat{\bar{Y}}_{p1}$	$\hat{\bar{Y}}_{p2}$	$\hat{\bar{Y}}_{p3}$	$\hat{\bar{Y}}_{p4}$	$\hat{\bar{Y}}_{p5}$	$\hat{\bar{Y}}_{p6}$	$\hat{\bar{Y}}_{p7}$	$\hat{\bar{Y}}_{p8}$	$\hat{\bar{Y}}_{p9}$	$\hat{\bar{Y}}_{p10}$
$\hat{\bar{Y}}_r$	117.90508	117.90488	117.90478	117.90475	117.90466	117.90456	117.90452	117.90450	117.90447	117.90442
$\hat{\bar{Y}}_1$	187.57710	187.57679	187.57662	187.57657	187.57643	187.57627	187.57620	187.57617	187.57613	187.57605
$\hat{\bar{Y}}_2$	186.88499	188.88468	188.88452	188.88447	188.88433	188.88417	188.88410	188.88407	188.88402	188.88395
$\hat{\bar{Y}}_3$	186.61940	186.61909	186.61893	186.61888	186.61874	186.61858	186.61851	186.61848	186.61843	186.61836
$\hat{\bar{Y}}_4$	186.91436	186.91405	186.91389	186.91383	186.91369	186.91354	186.91347	186.91343	186.91339	186.91331
$\hat{\bar{Y}}_5$	186.30885	186.30854	186.30837	186.30832	186.30818	186.30803	186.30796	186.30792	186.30788	186.30780
$\hat{\bar{Y}}_6$	187.16694	187.16662	187.16646	187.16641	187.16627	187.16611	187.16604	187.16601	187.16596	187.16589`
$\hat{\bar{Y}}_7$	187.03312	187.03281	187.03264	187.03259	187.03245	187.03229	187.03222	187.03219	187.03215	187.03207
$\hat{\bar{Y}}_8$	186.03390	186.03359	186.03343	186.03337	186.03324	186.03308	186.03301	186.03297	186.03293	186.03286
$\hat{\bar{Y}}_9$	187.18436	187.18405	187.18389	187.18383	187.18369	187.18354	187.18347	187.18343	187.18339	187.18331
$\hat{\bar{Y}}_{10}$	185.44732	185.44701	185.44685	185.44680	185.44666	185.44650	185.44643	185.44640	185.44636	185.44628

VI. CONCLUSION

From the above empirical study we found that our proposed estimators are more efficient than the classical and existing estimators as their MSE and bias is lower than classical and existing estimators and also by percent relative efficiency (PRE) criterion. Hence, can be preferred for practical application.

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