International Journal of Mathematics and Statistics Invention (IJMSI)
E-ISSN: 2321 – 4767 P-ISSN: 2321 - 4759
www.ijmsi.org Volume 5 Issue 3 || March. 2017 || PP-25-30

Poisson-Mishra Distribution

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Abstract: *A one parameter Poisson-Mishra distribution has been obtained by compounding Poisson distribution with Mishra distribution of B.K.Sah(2015). The first four moments about origin have been obtained. The maximum likelihood method and the method of moments have been discussed for estimating its parameter. The distribution has been fitted to some data-sets to test its goodness of fit. It has been found that this distribution gives better fit to all the discrete data sets which are used by Sankaran (1970) and others to test goodness of fit of Poisson-Lindley distribution.*
Keywords: *Mishra distribution, Poisson-Lindley distribution, mixture, moments, estimation of parameters, goodness of fit.*

I. INTRODUCTION

Lindley (1958) introduced a one-parameter continuous distribution given by its probability density function,

$$f(x,\phi) = \frac{\phi^2}{\phi+1}(1+x)\,e^{-\phi x} \quad ;x>0,\ \phi>0 \qquad \ldots \quad \ldots \quad \ldots \qquad (1.1)$$

This distribution is known as Lindley distribution and its cumulative distribution function has been obtained as

$$F(x,\phi) = 1-\left(1-\frac{\phi x}{\phi+1}\right)e^{-\phi x}; \quad x>0,\ \phi>0 \qquad \ldots \quad \ldots \quad \ldots \qquad (1.2)$$

Sankaran (1970) obtained a Poisson-Lindley distribution given by its probability mass function

$$P(x) = \frac{\phi^2\,(\phi+2+x)}{(\phi+1)^{x+3}} \quad ; x = 0, 1, 2, 3 \ldots \qquad \ldots \quad \ldots \quad \ldots \qquad (1.3)$$

assuming that the parameter of a Poisson distribution has Lindley distribution which can symbolically be expressed as

$$\text{Poisson}(\lambda) \underset{\lambda}{\wedge} \text{Lindley}(\phi). \qquad \ldots \quad \ldots \quad \ldots \qquad (1.4)$$

He discussed that this distribution has applications in errors and accidents.
The first four moments about origin of this distribution have been obtained as

$$\mu'_1 = \frac{(\phi+2)}{\phi(\phi+1)} \qquad \ldots \quad \ldots \quad \ldots \qquad (1.5)$$

$$\mu'_2 = \frac{(\phi+2)}{\phi(\phi+1)} + \frac{2(\phi+3)}{\phi^2(\phi+1)} \qquad \ldots \quad \ldots \quad \ldots \qquad (1.6)$$

$$\mu'_3 = \frac{(\phi+2)}{\phi(\phi+2)} + \frac{6(\phi+3)}{\phi^2(\phi+1)} + \frac{6(\phi+4)}{\phi^3(\phi+1)} \qquad \ldots \quad \ldots \quad \ldots \qquad (1.7)$$

$$\mu'_4 = \frac{(\phi+2)}{\phi(\phi+1)} + \frac{14(\phi+3)}{\phi^2(\phi+1)} + \frac{36(\phi+4)}{\phi^3(\phi+1)} + \frac{36(\phi+5)}{\phi^4(\phi+1)} \qquad \ldots \quad \ldots \quad \ldots \qquad (1.8)$$

And its variance as $\mu_2 = \left(\phi^3+4\phi^2+6\phi+2\right)\Big/\phi^2(\phi+1)^2$ $\qquad \ldots \quad \ldots \quad \ldots \qquad$ (1.9)

Ghitany et al (2009) discussed the estimation methods for the one parameter Poisson-Lindley distribution (1.3) and its applications.
In this paper, a one parameter Poisson-Mishra distribution has been obtained by compounding Poisson distribution with Mishra distribution

II. MISHRA DISTRIBUTION

A continuous random variable X is said to follow Mishra distribution with parameter ϕ if it assumes only non-negative value and its probability density function (pdf) is given by

$$f(x;\phi)=\frac{\phi^3(1+x+x^2)e^{-\phi x}}{(\phi^2+\phi+2)}\quad ;\phi>0,x>0 \qquad \ldots \quad \ldots \quad \ldots \qquad (2.1)$$

The r^{th} moment about origin of this distribution is obtained as

$$\mu_r'=\frac{r!\left[\phi^2+(r+1)\phi+(r+1)(r+2)\right]}{\phi^r(\phi^2+\phi+2)} \qquad \ldots \quad \ldots \quad \ldots \qquad (2.2)$$

Moment Generating Function $[M_x(t)]$

Moment generating function of Mishra distribution can be obtained as

$$[M_x(t)]=\int_0^\infty e^{tx}f(x)dx=\frac{\phi^3}{(\phi^2+\phi+2)}\cdot\frac{\left\{(\phi-t)^2+(\phi-t)+2\right\}}{(\phi-t)^3} \qquad \ldots \quad \ldots \quad \ldots \qquad (2.3)$$

Distribution Function

Distribution function of this distribution function can be obtained as

$$F(x)=\frac{\phi^3}{(\phi^2+\phi+2)}\int_0^\infty(1+x+x^2)e^{-\phi x}dx=1-\left[1+\frac{\phi^2(x^2+3x)}{(\phi^2+\phi+2)}\right]e^{-\phi x} \qquad \ldots \quad \ldots \quad \ldots \qquad (2.4)$$

III. POISSON-MISHRA DISTRIBUTION

A Poisson-Mishra distribution can be obtained by mixing Poisson distribution with the Mishra distribution (2.1). Suppose that the parameter λ of Poisson distribution follows Mishra distribution (2.1).Thus, the one parameter Mishra mixture of Poisson distribution can be obtained as

$$\text{PMD }(x;\phi)=\int_0^\infty\left[\sum_{x=0}^\infty\frac{e^{-\lambda}\lambda^x}{x!}\right]\frac{\phi^3(1+\lambda+\lambda^2)e^{-\phi\lambda}}{(\phi^2+\phi+2)}d\lambda \qquad \ldots \quad \ldots \quad \ldots \qquad (3.1)$$

$$=\frac{\phi^3}{(\phi^2+\phi+2)}\cdot\frac{\left[(1+\phi)(2+\phi+x)+(1+x)(2+x)\right]}{(1+\phi)^{x+3}};\ x=0,1,2,\ldots\ ;\phi>0 \qquad \ldots \quad \ldots \quad \ldots \qquad (3.2)$$

We name this distribution as' Poisson-Mishra distribution (PMD)'. The expression (3.2) is the probability mass function of PMD.

IV. MOMENTS

The r^{th} moment about origin of the PMD (3.2) can be obtained as

$$\mu_r'=E\left[E(X^r/\lambda)\right]=\frac{\phi^3}{(\phi^2+\phi+2)}\int_0^\infty\left[\sum_{x=0}^\infty x^r\frac{e^{-\lambda}\lambda^x}{x!}\right](1+\lambda+\lambda^2)d\lambda \qquad \ldots \quad \ldots \quad \ldots \qquad (4.1)$$

Obviously, the expression under bracket is the r^{th} moment about origin of the Poisson distribution. So, the first four moments about origin of the PMD can be obtained as

$$\mu_1'=\frac{\phi^3}{(\phi^2+\phi+2)}\int_0^\infty\lambda(1+\lambda+\lambda^2)e^{-\phi x}d\lambda=\frac{\left[\phi(\phi+2)+6\right]}{\phi(\phi^2+\phi+2)} \qquad \ldots \quad \ldots \quad \ldots \qquad (4.2)$$

Taking r=2 in (4.1) and using the second moment about origin of the Poisson distribution, the second moment about origin of the PMD is obtained

$$\mu_2'=\frac{\phi^3}{(\phi^2+\phi+2)}\int_0^\infty(\lambda+\lambda^2)(1+\lambda+\lambda^2)e^{-\phi x}d\lambda$$

$$\mu_2'=\frac{\left[\phi(\phi+2)+6\right]}{\phi(\phi^2+\phi+2)}+\frac{\left[2\phi(\phi+3)+24\right]}{\phi^2(\phi^2+\phi+2)} \qquad \ldots \quad \ldots \quad \ldots \qquad (4.3)$$

Similarly, taking r=3 and 4 in (4.1) and using the respective moments of the Poisson distribution, we get finally, after a little simplification, the third and fourth moments about origin of the one parameter PMD

$$\mu_3' = \frac{\left[\phi(\phi+2)+6\right]}{\phi(\phi^2+\phi+2)} + \frac{\left[6\phi(\phi+3)+24\right]}{\phi^2(\phi^2+\phi+2)} + \frac{\left[6\phi(\phi+4)+120\right]}{\phi^3(\phi^2+\phi+2)} \quad \ldots \quad \ldots \quad \ldots \quad (4.5)$$

$$\mu_4' = \frac{\left[\phi(\phi+2)+6\right]}{\phi(\phi^2+\phi+2)} + \left[\frac{14(\phi(\phi+3)+12)}{\phi^2(\phi^2+\phi+2)}\right] + \frac{\left[36\left\{\phi(\phi+4)+20\right\}\right]}{\phi^3(\phi^2+\phi+2)} + \frac{\left[24\left\{\phi(\phi+5)+30\right\}\right]}{\phi^4(\phi^2+\phi+2)} \quad \ldots \quad \ldots \quad (4.6)$$

Probability Generating Function:

The probability generating function of the one parameter PMD (3.2) can be obtained as

$$P_X(t) = E(t^X) = \frac{\phi^3}{(\phi^2+\phi+2)}\int_0^\infty e^{-\lambda(1-t)}(1+\lambda+\lambda^2)e^{-\phi\lambda}\,d\lambda = \frac{\phi^3}{(\phi^2+\phi+2)}\frac{\left[(1+\phi-t)(2+\phi-t)+2\right]}{(1+\phi-t)^3} \quad \ldots \quad (4.7)$$

Moment Generating Function:

The moment generating function of the one parameter PMD (3.2) can be obtained as

$$M_X(t) = \frac{\phi^3}{(\phi^2+\phi+2)}\int_0^\infty e^{\lambda(e^t-1)}(1+\lambda+\lambda^2)e^{-\phi\lambda}\,d\lambda$$

$$= \frac{\phi^3\left[(1+\phi-e^t)(2+\phi-e^t)+2\right]}{(\phi^2+\phi+2)(1+\phi-e^t)^3} \quad \ldots \quad \ldots \quad \ldots \quad (4.8)$$

V. ESTIMATION OF PARAMETER

Here, at first, the method of moments has been used to estimate the parameter of one parameter PMD. An estimate of the parameter ϕ of this distribution can be obtained by solving the expression (4.2) of $\mu_1^{/}$ as

$$f(\phi) = \mu_1'\phi^3 + (\mu_1'-1)\phi^2 + 2(\mu_1'-1)\phi - 6 = 0 \quad \ldots \quad \ldots \quad \ldots \quad (5.1)$$

Replacing the population moment by the respective sample moment and solving the expression (5.1) by using Regula-Falsi method, we get an estimate of ϕ .

***The Maximum Likelihood Estimates**:*

Let $x_1, x_2, \ldots, x_n$ be the value of observations of a random sample of size n from the two-parameter PMD and let f_x be the observed frequency in the sample corresponding to X=x (x=1,2,...,k) such that $\sum_{x=1}^{k} f_x = n$,where k is the largest observed value having non-zero frequency. Let the likelihood function L of the two- parameter PMD is given by

$$L = \prod_{x=1}^{k}\left(\frac{\phi^3}{\phi^2+\phi+2}\right).\prod_{x=1}^{k}\frac{1}{(1+\phi)^{\sum_{x=1}^{k}(x+3)f_x}}\prod_{x=1}^{k}\left[(1+\phi)^2+(1+\phi)(1+x)+(1+x)(2+x)\right]^{f_x} \quad \ldots \quad \ldots \quad (5.2)$$

and the log likelihood function is obtained as

$$\log L = n\log\left(\frac{\phi^3}{\phi^2+\phi+2}\right) - \sum_{x=1}^{k} f_x(x+3)\log(1+\phi) + \sum_{x=1}^{k} f_x \log\left\{(1+\phi)^2+(1+x)(1+\phi)+(1+x)(2+x)\right\} \quad \ldots \quad \ldots \quad (5.3)$$

The likelihood equation is obtained as

$$\frac{\partial \log L}{\partial \phi} = \frac{3n}{\phi} - \frac{n(2\phi+1)}{(\phi^2+\phi+2)} - \frac{\sum_{x=1}^{k}(x+3)f_x}{(1+\phi)} + \sum_{x=1}^{k}\frac{(3+2\phi+x)}{(1+\phi^2)+(1+x)(1+\phi)+((1+x)(2+x)}f_x = 0 \quad \ldots \quad \ldots \quad (5.4)$$

Solving the expression (5.4), we get an estimate of ϕ .

VI. GOODNESS OF FIT

The one parameter Poisson-Mishra distribution (PMD) has been fitted to a number of discrete data-sets to which earlier one parameter Poisson-Lindley distribution (PLD)has been fitted and it is found that to all these data-sets, the PMD provides better fits than the one parameter PLD of Sankaran (1970) and the two-parameter PLD of Rama Shanker and Mishra (2014). Here the fittings of the PMD to three data-sets have been presented in the following tables. The first data is the Student's historic data Hemocytometer counts of yeast cell, used by Borah (1984) for fitting the Gegenbauer distribution and the second is due to Kemp and Kemp (1965) regarding the distribution of mistakes in copying groups of random digits and the third is due to Beall(1940) regarding the distribution of Pyraustanublilalis in 1937.

Table –I

Hemocytometer Counts of Yeast Cell

Number of Yeast Cell per square	Observed frequency	Expected frequency of PLD	Expected frequency of two-parameter PLD	Expected frequency of PMD
0	213	234.4	227.6	234.3
1	128	99.3	101.5	99.3
2	37	40.4	43.6	40.6
3	18	16.0	17.9	16.0
4	3	6.2	6.8	6.1
5	1	2.4	2.2	2.3
6	0	1.3	0.6	1.4
Total	400	400	400	400
μ'_1	0.6825			
μ'_2	1.2775			
$\hat{\phi}$		1.9602	3.6728	2.3914
$\hat{\alpha}$		-	-0.0916	-
χ^2		14.3	12.25	14.23
d.f		4	3	4
P-value		0.0068	0.0069	0.007

Table-II

Distribution of mistakes in copying groups of random digits

Number of errors per group	Observed frequency	Expected frequency of PLD	Expected frequency of two-parameter PLD	Expected frequency of PMD
0	35	33.1	32.4	32.9
1	11	15.3	15.8	15.3
2	8	6.8	7.0	6.8
3	4	2.9	2.9	3.6
4	2	1.2	1.9	1.4
Total	60	59.3	60	60
μ'_1	0.7833			
μ'_2	1.8500			
$\hat{\phi}$		1.7434	1.9997	2.1654758
$\hat{\alpha}$		-	0.3829	-
χ^2		2.20	2.11	1.72
P-value		0.48	0.28	0.58

Table-III

Distribution of Pyraustanublilalis in 1937

Number of errors per group	Observed frequency	Expected frequency of PLD	Expected frequency of two-parameter PLD	Expected frequency of PMD
0	33	31.5	31.9	31.4
1	12	14.2	13.8	14.2
2	6	6.1	5.9	6.2
3	3	2.5	2.5	2.6
4	1	1.0	1.1	1.0
5	1	0.7	0.8	0.6
Total	56	56	56	56

μ'_1	0.75			
μ'_2	1.85714			
$\hat{\phi}$		1.8081	1.5257	2.234
$\hat{\alpha}$		-	0.3.8873	-
χ^2		0.53	0.36	0.47
P-value		0.83	0.798	0.85

The Kolmogorov - Smirnov Test (K-S Test):

The K-S test is a simple non parametric method for testing whether there is a significant difference between the observed frequency distribution and a theoretical frequency distribution. The K-S test is therefore another measure of the goodness of fit of a theoretical frequency distribution. For more validity of the Poisson-Mishra distribution, we may apply K-S test to test whether the PMD is a good fit to the above mentioned data-sets or not.

We have obtained the absolute deviation of expected relative cumulative frequency of the PMD and observed relative cumulative frequency of the above mentioned data-sets and are placed in the table 4, 5 and 6 respectively. Some notations used for applying K-S test are as follows.

F_e = Expected Relative Cumulative Frequency of the Poisson-Mishra distribution.

F_o = Observed Relative Cumulative Frequency

$D_{n\,(Max)}$ = the maximum absolute deviation of F_e and F_o

$D_{n\,(Tabulated)}$ = the critical value of the K-S test statistic at 1% level of significance

Calculation of the absolute deviation of F_e and F_o:

Table –IV

Hemocytometer counts of Yeast Cell

Number of Yeast Cell per square	Observed frequency	Expected frequency of PMD	F_e	F_o	$\lvert F_e - F_o \rvert$
0	213	234.3	0.586	0.533	0.053
1	128	99.3	0.834	0.853	0.019
2	37	40.6	0.935	0.945	0.001
3	18	16.0	0.975	0.990	0.015
4	3	6.1	0.991	0.997	0.006
5	1	2.3	0.996	1.000	0.004
6	0	1.4	1.000	1.000	0.000
	400	400			

The maximum absolute deviation of F_e and F_o= $D_{n(calculated)}$ =0.053 at x=0 .The Critical value for D_n at 1% level of significance can be computed by $D_{n\,(Tabulated)} = \frac{1.63}{\sqrt{n}} = 0.082$

Table-V

Distribution of mistakes in copying groups of random digits

Number of errors per group	Observed frequency	Expected frequency of PMD	F_e	F_o	$\lvert F_e - F_o \rvert$
0	35	32.9	0.548	0.583	0.035
1	11	15.3	0.803	0.767	0.036
2	8	6.8	0.917	0.900	0.017
3	4	3.6	0.977	0.967	0.010
4	2	1.4	1.000	1.000	0.000
Total	60	60			

The maximum absolute deviation of F_e and F_o= $D_{n(calculated)}$ =0.036 at x=1 .The Critical value for D_n at 1% level of significance can be computed by $D_{n(Tabulated)} = \frac{1.63}{\sqrt{n}} = 0.210$

Table-VI

Distribution of Pyraustanublilalis in 1937

Number of errors per group	Observed frequency	Expected frequency of PMD	F_e	F_o	$\lvert F_e - F_o \rvert$
0	33	31.4	0.561	0.589	0.028
1	12	14.2	0.814	0.800	0.014
2	6	6.2	0.925	0.911	0.014
3	3	2.6	0.971	0.964	0.007
4	1	1.0	0.989	0.982	0.007
5	1	0.6	1.000	1.000	0.000
Total	56	56			

The maximum absolute deviation of F_e and F_o= $D_{n(calculated)}$ =0.028 at x=0 .
The Critical value for D_n at 1% level of significance can be computed by

$$D_{n(Tabulated)} = \frac{1.63}{\sqrt{n}} = 0.218$$

By comparing the calculated and tabulated value of K-S test statistic, we may conclude that the Poisson-Mishra distribution is a good fit for the data-set.

VII. CONCLUSION

It has been observed that the Poisson-Mishra distribution gives better fit to all the discrete data-sets than the one parameter PLD of Sankaran and the two-parameter PLD of Rama Shanker and Mishra because of p-value. Hence, it provides a better alternative to the one parameter PLD of Sankaran (1970) and two-parameter PLD of Rama Shanker and Mishra (2014). Perhaps, it is the best alternative to all the existing Quasi Poisson-Lindley distributions.

ACKNOWLEDGEMENTS

The author expresses their gratitude to the referee for his valuable comments and suggestions which improved the quality of the research article. The hearty thank goes to Professor A. Mishra for his tremendous guidance and valuable suggestions.

REFERENCES

[1]. **Beall.G. (1940)**: The fit and significance of contagious distribution when applied to observations on larval insects, Ecology, 21,460-474.
[2]. **Borah, M. (1984):** The Gegenbauer distribution revisited: Some recurrence relation for moments, cumulants, etc., estimation of parameters and its goodness of fit, Journal of Indian Society of Agricultural Statistics, Vol. 36, pp.72-78.
[3]. **Ghitany, M. E, and Al-Mutairi, D.K. (2009):** Estimation methods for the discrete Poisson-Lindley distribution. J.Stat.Compt. Simul.79 (1); 1-9.
[4]. **Kemp, C.D. and Kemp, A.W. (1965):** Some properties of the Hermite distribution, Biometrika, Vol. 52, pp.381-394.
[5]. **Lindley, D.V. (1958):** Fiducial distribution and Bayes theorem, Journal of Royal Statistical Society, Ser.B, Vol.20, pp.102-107.
[6]. **Sah, Binod Kumar, (2015):** Mishra distribution, International Journal of Mathematics and Statistics Inventions(IJMSI), Vol.3, Issue 8, pp.14-17
[7]. **Sankaran, M. (1970):** The discrete Poisson –Lindley distribution, Biometrics, 26,145-149.
[8]. **Shanker, R. and Mishra, A. (2014):** A two-parameter Poisson-Lindley distribution, Statistics in Transition New Series, Vol. 14, No. 1, pp.45-56.