International Journal of Mathematics and Statistics Invention (IJMSI)
E-ISSN: 2321- 4767 P-ISSN: 2321-4759
www.ijmsi.org Volume 8 Issue 2 || Feb, 2020 || PP-15-21

Modeling of Infinite Objects Based on the Moment Scheme of Finite Elements Method

N.I.–V.Manko[1], O.G.Spytsia[2], S.M.Grebenyuk[3],S.I.Homenyuk[1]
[1]*Department of Software Engineering,Zaporizhzhia National University, Zaporizhzhia, Ukraine*
[2]*Department of General Mathematics, Zaporizhzhia National University, Zaporizhzhia, Ukraine*
[3]*Department of Fundamental Mathematics, Zaporizhzhia National University, Zaporizhzhia,Ukraine*
Corresponding Author: N.I.-V Manko.

***ABSTRACT**:In the following article researches the problem of mathematical modelling of the infinite and half-infinite areas in the mechanics of deformable solid. Adding describing implementation of method of finite elements due to minding tasks of mechanics of deformable solid. Global stiffness matrix construction methodic includes mathematical modelling composited with implementation both traditional countable elements, which use in areas of strain and movement problems and where the defined function changes with high-frequency and infinite finite elements. For modelling of infinity in one of direction was offered half-infinite finite element. Problem concern the fact that with help of countable element with nodes, which have defined coordinates create the infinite model. For coordinates approximation in undefined direction was implemented special form functions, which in the nodes infinite model approaches to infinite. For counting weak compressibility and excluding negative characteristics of traditional method of finite elements implementing moment scheme of finite element, which consists of triple approximation of components of displacements vector, deformation tensor and function of volume changes. Also was conducted digital research of stress-strain state of basis made of weakly elastic material.*
***KEYWORDS**: Basis, Half-Infinite finite element, Moment scheme, Deformations, Displacements, Function of volume changes, Form function.*

Date of Submission: 19-04-2020 Date of Acceptance: 03-05-2020

I. INTRODUCTION

A large amount of problems on mechanic defined by researches of infinite objects. Analytical accomplishment is possible only for simple problems. Numerical analysis including construction of discrete model at first, which contains certain amount of node points to investigate objects.

Usually, part of the body, which consist infinity, tests negligible impact of strains. In the defined part of searchable functions characterized by continuity and monotonicity. There is no need to generate dense networks? in this part with the usage of calculation method. To add the fact, generating even the sparse matrix. On the infinite space is a very hard task to do. It leads to increasing of dimensionality of tasks and unreasonable spending of computer time and capacities.

Another problem appears in dealing with problems for weak elastic materials. In this case Poisson's ratio leading to 0,5 while certain elastic coefficients to infinity. With the calculating by traditional methods could be find inaccurate results.

Research of infinite areas by the finite element method represented in one-dimensional and two-dimensional formulations. Application of half-infinite finite element in solving static problems of mechanic in two-dimensional formulation is described in the monography [1]. Numerical simulation of waves propagation with usage of ANSYS in one- and two-dimensional formulations by half-infinite finite elements showed in the article [2]. In article [3] infinite elements were used additional researches of previously compressed infinite habitats in underground studies. Solving of static tasks for unlimited areas with help of infinite elements written in article [4]. Modification of half-infinite finite element for improving accuracy of calculations by increasing the amount of nodes in unlimited direction and options of special approximating functions written in [5].

In the article [6] was received basic correlations with the stiffness matrix of half-infinite finite element in the hexagon form. Offered stiffness matrix allow to solve the problems of infinity and weak elastic materials.

In following research was reviewed usage of half-infinite finite elements due to solving the tasks of mechanic.

II. APPLICATION OF THE METHOD

Basic relation of stiffness matrix defined by variable in two coordinate systems: global Descartes $(O' z_1' z_2' z_3')$ and local $(O' z_1' z_2' z_3')$, which combined with finite element. In global coordinate system defined geometry of construction, terms of consolidation and applied strain. In local coordinate system "half-infinite" finite element shaped like cube with length of rib equal 2 and origin of coordinates in the center of the cube with coordinates (0,0,0). Axis directions collinear with directions of cube ribs. In global coordinate system finite element displayed as a hexagon (Fig. 1). Nodes 5*, 6*, 7*, 8* are displayed in nodes with infinite coordinate by third direction. Operations with endlessness measures in calculation operations are uncomfortable procedure and requires additional algorithms during realization.

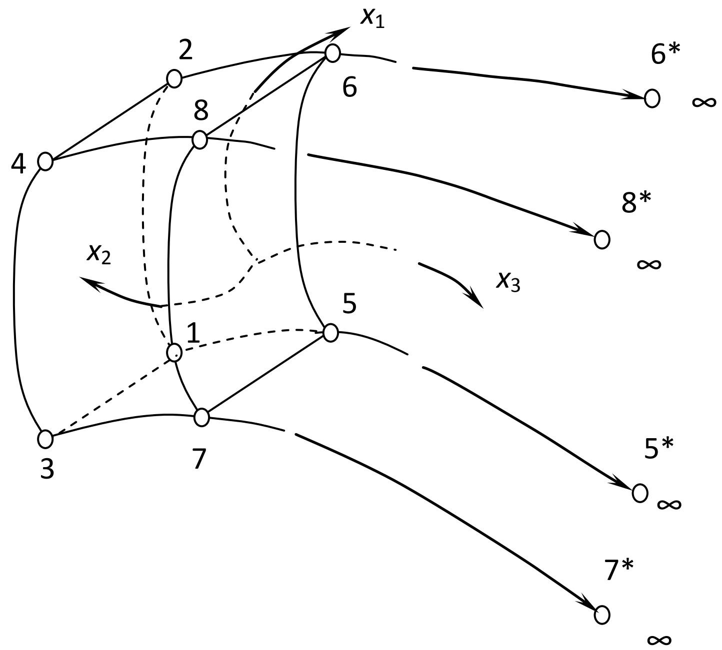


Figure 1. «Half-infinite» finite element

According to method of finite elements solving linear tasks of mechanics convergence to solving the system of linear algebraic equations:

$$Ku = P, \tag{1}$$

where $K = K^{fin} + K^{inf}$ – global stiffness matrix of infinite body, which consists of stiffness matrix finite K^{fin} and infinite K^{inf} parts, u – unknown movement vector, P – applied strain vector.

According to article [6] stiffness matrix l-numbered half-infinite element looks like:

$$\left[K_{(l)}^{k'm'}\right] = \left[K_{ij}^{k'm'}\right] + \left[K_{\theta}^{k'm'}\right]. \tag{2}$$

Components of stiffness matrix describes, equally, moveable and volume deformations of finite element:

$$\left[K_{ij}^{k'm'}\right] = \iiint\limits_V 2\mu g^{ik} g^{jl} [A]^T \left\{F_{ij}^{k'}\right\}^T \{\psi\}\{\psi\}^T \{F_{kl}^{m'}\}[A] dV \ , \tag{3}$$

$$\left[K_{\theta}^{k'm'}\right] = \iiint\limits_V \lambda g^{ij} [A]^T \left\{F_{\theta}^{k'}\right\}^T \{\psi\}\{\psi\}^T \{F_{\theta}^{m'}\}[A] dV \ . \tag{4}$$

where g^{ij} –metric tensor components,μ, λ – Lame constants, ε_{ij} – deformation vector components, θ – volume change function, V – finite element volume.

Matrices$\{\psi\}$, $[A]$, $\{F_{ij}^{k'}\}$, $\{F_{\theta}^{k'}\}$ received due to moment scheme of finite element for weak elastic materials. Matrix $\{\psi\}$ is formed by power function of expansion of movement vector:

$$u_{k'} = \sum_{p=0}^{1}\sum_{q=0}^{1}\sum_{r=0}^{1} \omega_{k'}^{(pqr)} \psi^{(pqr)} =$$
$$= \omega_{k'}^{(000)} + \omega_{k'}^{(100)}\psi^{(100)} + \omega_{k'}^{(010)}\psi^{(010)} + \omega_{k'}^{(001)}\psi^{(001)} + \omega_{k'}^{(110)}\psi^{(110)} +$$
$$+\omega_{k'}^{(101)}\psi^{(101)} + \omega_{k'}^{(011)}\psi^{(011)} + \omega_{k'}^{(111)}\psi^{(111)}, \tag{5}$$

wherep, q, r– powers of approximating polynom due to coordinate directions, $\psi^{(pqr)}$ – set of power coordinate functions, $\omega_{k'}^{(pqr)}$ – coefficients of component expansion of movement vector in k'-numbered direction of global coordinate system.

Matrix $[A]$ displays connection between expansion coefficients $\omega_{k'}^{(pqr)}$ and movement values $u_{k'}^{L}$ in nodes of finite element:

$$\{\omega_{k'}\} = [A]\{u_{k'}^{L}\}. \tag{6}$$

This connection estimates with counting on imagining movement function through network value of movements $u_{k'}^{L}$ and form functions N_L half-infinite finite element:

$$\{u_{k'}\} = \{u_{k'}^{L}\}^{T}\{N_L\}. \tag{7}$$

Form functions $N_L(x_1, x_2, x_3)$ have specificities for nodes 5*, 6*, 7*, 8* which allows modelling infinite in it:

$$\begin{aligned}
N_1(x_1,x_2,x_3) &= -\frac{1}{4}(1-x_1)(1-x_2)\frac{2x_3}{1-x_3},\\
N_2(x_1,x_2,x_3) &= -\frac{1}{4}(1+x_1)(1-x_2)\frac{2x_3}{1-x_3},\\
N_3(x_1,x_2,x_3) &= -\frac{1}{4}(1-x_1)(1+x_2)\frac{2x_3}{1-x_3},\\
N_4(x_1,x_2,x_3) &= -\frac{1}{4}(1+x_1)(1+x_2)\frac{2x_3}{1-x_3},\\
N_5(x_1,x_2,x_3) &= \frac{1}{4}(1-x_1)(1-x_2)\left(1+\frac{2x_3}{1-x_3}\right),\\
N_6(x_1,x_2,x_3) &= \frac{1}{4}(1+x_1)(1-x_2)\left(1+\frac{2x_3}{1-x_3}\right),\\
N_7(x_1,x_2,x_3) &= \frac{1}{4}(1-x_1)(1+x_2)\left(1+\frac{2x_3}{1-x_3}\right),\\
N_8(x_1,x_2,x_3) &= \frac{1}{4}(1+x_1)(1+x_2)\left(1+\frac{2x_3}{1-x_3}\right).
\end{aligned} \tag{8}$$

Connection between local and global coordinates will seem like:

$$z_{m'} = \sum_{L=1}^{8} N_L\,(x_1,x_2,x_3) z_{m'}^{L}, \tag{9}$$

where $z_{m'}^{L}$ – m'-th coordinate L-th node in global coordinate system.

Matrices$\{F_{ij}^{k'}\}$, $\{F_{\theta}^{k'}\}$ contains expansion coefficients $f_{(\mu\nu\eta)}^{k'}$ deformation component and volume change functions, which due to moment scheme of finite element, seem like:

$$\begin{aligned}
\varepsilon_{11} &= e_{11}^{(000)} + e_{11}^{(010)}\psi^{(010)} + e_{11}^{(001)}\psi^{(001)} + e_{11}^{(011)}\psi^{(011)},\\
\varepsilon_{22} &= e_{22}^{(000)} + e_{22}^{(100)}\psi^{(100)} + e_{22}^{(001)}\psi^{(001)} + e_{22}^{(101)}\psi^{(101)},\\
\varepsilon_{33} &= e_{33}^{(000)} + e_{33}^{(100)}\psi^{(100)} + e_{33}^{(010)}\psi^{(010)} + e_{33}^{(110)}\psi^{(110)},\\
\varepsilon_{12} &= e_{12}^{(000)} + e_{12}^{(001)}\psi^{(001)},\\
\varepsilon_{13} &= e_{13}^{(000)} + e_{13}^{(010)}\psi^{(010)},\\
\varepsilon_{23} &= e_{23}^{(000)} + e_{23}^{(100)}\psi^{(100)};
\end{aligned} \tag{10}$$

$$\theta = e_{11}^{(\alpha\beta\gamma)} g^{11} + e_{22}^{(\alpha\beta\gamma)} g^{22} + e_{33}^{(\alpha\beta\gamma)} g^{33}. \tag{11}$$

On based to moment scheme of finite element for expansion deformation coefficients $e_{ij}^{(pqr)}$ on (9), (10) we will have [7]:

$$\begin{aligned}
e_{11}^{(pqr)} &= \sum_{\mu=0}^{p}\sum_{\nu=0}^{q}\sum_{\eta=0}^{r} \omega_{k'}^{(\mu+1\,\nu\eta)} f_{(p+1-\mu q-\nu r-\eta)}^{k'};\\
e_{22}^{(pqr)} &= \sum_{\mu=0}^{p}\sum_{\nu=0}^{q}\sum_{\eta=0}^{r} \omega_{k'}^{(\mu\nu+1\,\eta)} f_{(p-\mu q+1-\nu r-\eta)}^{k'};\\
e_{33}^{(pqr)} &= \sum_{\mu=0}^{p}\sum_{\nu=0}^{q}\sum_{\eta=0}^{r} \omega_{k'}^{(\mu\nu\eta+1)} f_{(p-\mu q-\nu r+1-\eta)}^{k'};\\
e_{12}^{(pqr)} &= \frac{1}{2}\sum_{\mu=0}^{p}\sum_{\nu=0}^{q}\sum_{\eta=0}^{r}\left(\omega_{k'}^{(\mu\nu+1\,\eta)} f_{(p+1-\mu q-\nu r-\eta)}^{k'} + \omega_{k'}^{(\mu+1\nu\eta)} f_{(p-\mu q+1-\nu r-\eta)}^{k'}\right);
\end{aligned}$$

$$e_{13}^{(pqr)}=\frac{1}{2}\sum_{\mu=0}^{p}\sum_{\nu=0}^{q}\sum_{\eta=0}^{r}\left(\omega_{k'}^{(\mu\nu\eta+1)}f_{(p+1-\mu q-\nu r-\eta)}^{k'}+\omega_{k'}^{(\mu+1\nu\eta)}f_{(p-\mu q-\nu r+1-\eta)}^{k'}\right);$$

$$e_{23}^{(pqr)}=\frac{1}{2}\sum_{\mu=0}^{p}\sum_{\nu=0}^{q}\sum_{\eta=0}^{r}\left(\omega_{k'}^{(\mu\nu\eta+1)}f_{(p-\mu q+1-\nu r-\eta)}^{k'}+\omega_{k'}^{(\mu\nu+1\,\eta)}f_{(p-\mu q-\nu r+1-\eta)}^{k'}\right), \quad (12)$$

With symbolling

$$f_{(\mu\nu\eta)}^{k'}=\left.\frac{\partial^{\mu+\nu+\eta}z_{k'}}{(\partial x_1)^{\mu}(\partial x_2)^{\nu}(\partial x_3)^{\eta}}\right|_{x_1=x_2=x_3=0}. \quad (13)$$

So non-zero coefficients with counting on (8), (9) and (13) will seem like:

$$f_{(100)}^{k'}=-\frac{1}{4}z_{k'}^{5}+\frac{1}{4}z_{k'}^{6}-\frac{1}{4}z_{k'}^{7}+\frac{1}{4}z_{k'}^{8},$$

$$f_{(010)}^{k'}=-\frac{1}{4}z_{k'}^{5}-\frac{1}{4}z_{k'}^{6}+\frac{1}{4}z_{k'}^{7}+\frac{1}{4}z_{k'}^{8},$$

$$f_{(001)}^{k'}=-\frac{1}{2}z_{k'}^{1}-\frac{1}{2}z_{k'}^{2}-\frac{1}{2}z_{k'}^{3}-\frac{1}{2}z_{k'}^{4}+\frac{1}{2}z_{k'}^{5}+\frac{1}{2}z_{k'}^{6}+\frac{1}{2}z_{k'}^{7}+\frac{1}{2}z_{k'}^{8},$$

$$f_{(110)}^{k'}=\frac{1}{4}z_{k'}^{5}-\frac{1}{4}z_{k'}^{6}-\frac{1}{4}z_{k'}^{7}+\frac{1}{4}z_{k'}^{8},$$

$$f_{(101)}^{k'}=\frac{1}{2}z_{k'}^{1}-\frac{1}{2}z_{k'}^{2}+\frac{1}{2}z_{k'}^{3}-\frac{1}{2}z_{k'}^{4}-\frac{1}{2}z_{k'}^{5}+\frac{1}{2}z_{k'}^{6}-\frac{1}{2}z_{k'}^{7}+\frac{1}{2}z_{k'}^{8},$$

$$f_{(011)}^{k'}=\frac{1}{2}z_{k'}^{1}+\frac{1}{2}z_{k'}^{2}-\frac{1}{2}z_{k'}^{3}-\frac{1}{2}z_{k'}^{4}-\frac{1}{2}z_{k'}^{5}-\frac{1}{2}z_{k'}^{6}+\frac{1}{2}z_{k'}^{7}+\frac{1}{2}z_{k'}^{8},$$

$$f_{(111)}^{k'}=-\frac{1}{2}z_{k'}^{1}+\frac{1}{2}z_{k'}^{2}+\frac{1}{2}z_{k'}^{3}-\frac{1}{2}z_{k'}^{4}+\frac{1}{2}z_{k'}^{5}-\frac{1}{2}z_{k'}^{6}-\frac{1}{2}z_{k'}^{7}+\frac{1}{2}z_{k'}^{8}. \quad (14)$$

With help of this approach conducted the calculation of the basis of low elastic material.

III. NUMERICAL RESULTS

We consider the problem of indenting the few stamps into a multilayer environment. The stamps are arranged symmetrically about the axis of symmetry of the cylindrical circular stamp 1 (Fig. 2, 3). Two cases were investigated when the other two stamps 2 (one of them is shown, the other is symmetrical) are cylindrical with a parabolic section (Fig.2) and with a circular paraboloid section (Fig.3).

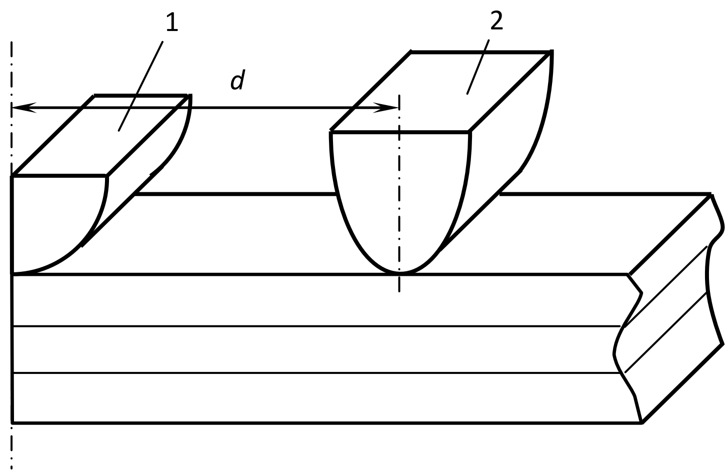


Figure 2. Contact interaction of parabolic stamps with multilayer environment

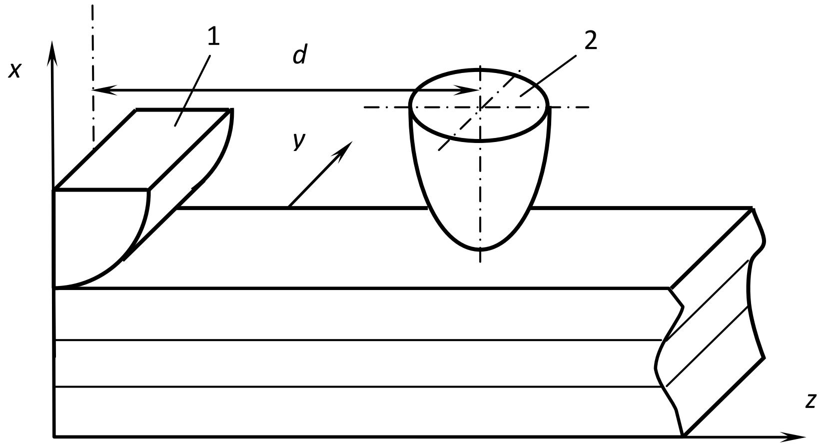


Figure 3. Contact interaction of paraboloid stamps with multilayer environment

Input: the width of environment is$\text{b} = 0.5$ m, the thickness of each of the three layers is 0.05 m, the total thickness is$\text{t} = 0.15$ m, the length is infinite. The stamps are absolutely rigid. The layers are made of different brands of rubber. The Poisson's ratio for all layers are the same and equal to$\nu = 0.49$. The shear modulus for the layers are: for the lower and upper$\text{G} = 2.8$ MPa, for the middle – $\text{G} = 0.78$ MPa.

The distance between the stamps is$\text{d} = 0.4$ m. The stamps immersion into the environment to a depth of 0.03 m. For the first case (Fig. 2) the profile of the stamp 1 is described by the equation$x^2 + z^2 = 0.22$, for the stamp 2 is$x = 2z^2$. For the second case (Fig. 3) the equation of the stamp 2 has the form$x = y^2 + z^2$.

The stressed deformed state of the three-layer environment was determined with the help of the software complex MIRELA+. The distribution of movements in the direction of the immersion of the stamp for case 2 is shown in Fig. 4.

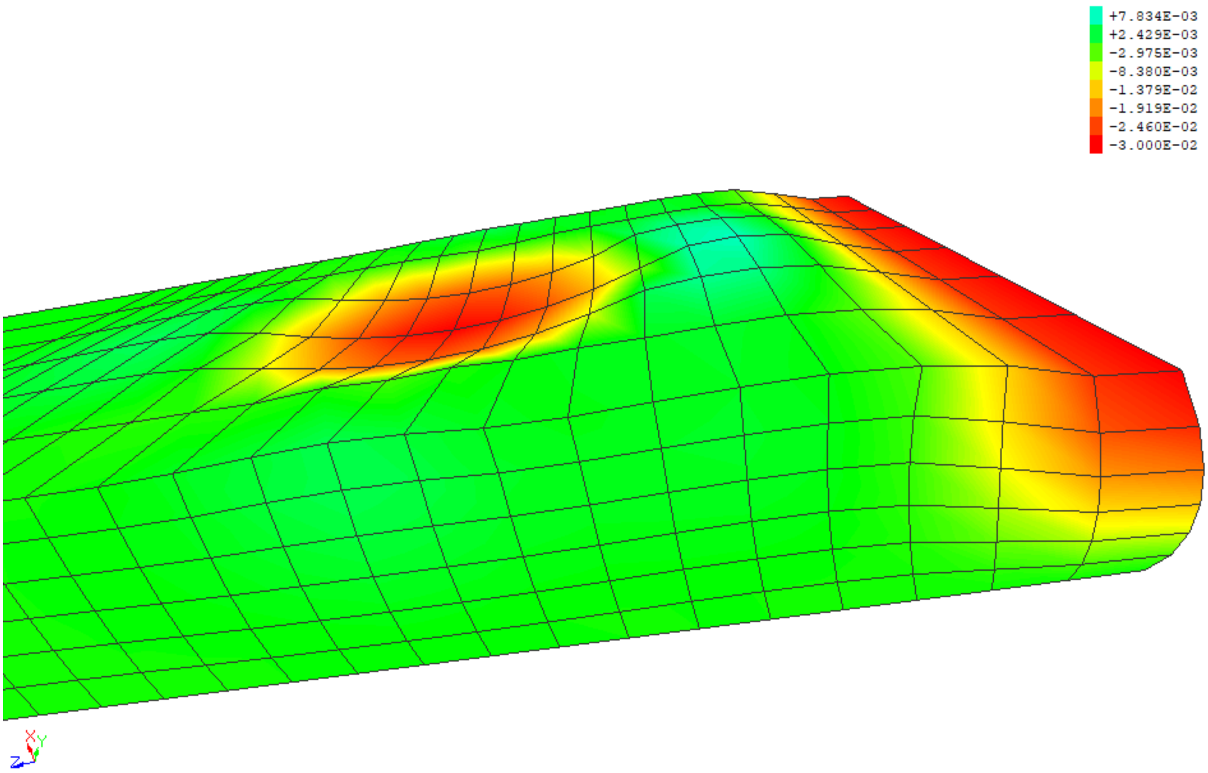
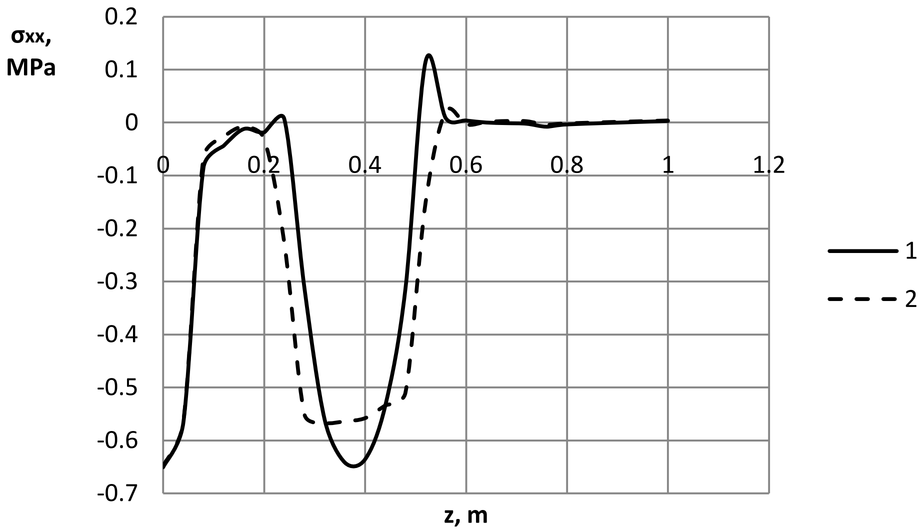


Figure 4. Movement distribution along the axis x

In Fig.5 shows the values of stresses σ_{xx} in the middle of the width of the environment at a depth of 0.0125 m from the upper surface.

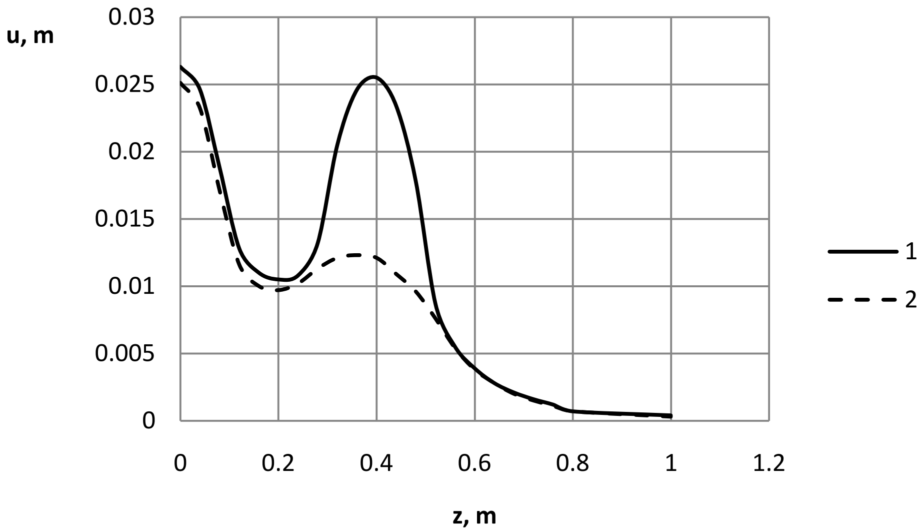


——— – case 1, - - - - - – case 2

Figure 5. Stress distribution σ_{xx} near the upper surface

As it can be noted, the stress distribution σ_{xx} is similar in both cases, some difference is observed in the contact zone with the stamp 2. Here, in the second case, the zone of maximum compressive stresses is greater than in the first case, but the magnitude of the stresses is somewhat smaller.

The maximum protrusion of the lateral surface is shown in Fig. 6.



——— – case 1, - - - - - – case 2

Figure 6. Protrusion of the lateral surface of the environment

IV. CONCLUSION

The deformation of the lateral surface (Fig. 6) also differs only in the zone of interaction of the stamp 2 with the surface of the three-layer environment. In the first case, this interaction is present across the entire width of the environment. In the second case, this deformation is much smaller, because the contact interaction with the stamp occurs in the depth of the array of three-layer environment.

The surface protrudes upwards approximately in the middle between the contact areas on the surface where the contact between the three-layer environment and the stamp occurs (the maximum protrusion for case 1 equal to 0.0093 m, for case 2 is 0.0078 m). This is due to its poor compressibility.

Thus, using a stiffness matrix with finite elements for a weakly compressible material, the stress-strain state of a three-layer elastomeric environment under the action of a system of stamps of various configurations is determined.

REFERENCES

[1]. O.C. Zienkiewiczand R. L. Taylor, The finite element method. Volume 1: The basis, Oxford: Butterworth-Heinemann, 2000, 689.

[2]. K.Edip, V.Sheshov, J.Bojadjieva, A.Demir and H.Ozturk, Development of infinite elements for simulation of unbounded media, Building materials and structures, 61, 2018, 3-13.

[3]. G.Beer, ‘Infinite domain’ elements in finite element analysis of underground excavations, International journal for numerical and analytical methods in geomechanics, 7, 1983, 1-7.

[4]. A.Curnier, A static infinite element, International journal for numerical methods in engineering, 19(10), 1983, 1479-1488.

[5]. F.Medina andR.L.Taylor, Finite element techniques for problems of unbounded domains, International journal for numerical methods in engineering, 19(8), 1983, 1209-1226.

[6]. С.В. Чопоров, Н.І.-В. Манько, О.Г. СпицятаС.М. Гребенюк,Матрицяжорсткості «напівнескінченного» скінченногоелементадляслабкостисливогоматеріалунаосновімоментноїсхеми,Вісник Запорізького-національного університету. Фізико-математичні науки, 1, 2019, 124-132.

[7]. В.В. Киричевский, Метод конечных элементов в механике эластомеров, Наук. думка, 2002, 655.

N.I.-V Manko."Modeling of Infinite Objects Based on the Moment Scheme of Finite Elements Method." *International Journal of Mathematics and Statistics Invention (IJMSI)*, vol. 08(02), 2020, pp.15-21.