International Journal of Mathematics and Statistics Invention (IJMSI)
E-ISSN: 2321- 4767 P-ISSN: 2321-4759
www.ijmsi.org Volume 8 Issue 6 || June, 2020 || PP 14-23

Application of Markov Analysis to Consumers Preference

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ABSTRACT
In real world today, the changes in consumers preferences is a basis for competitive advantage. This paper is an attempt to understand these changes in consumer's preference. The work investigates the changes in preference of a population after a period of time. The state of equilibrium of the market was determined. Transition Matrices are used to perform a Markov analysis on the preferred transport services of students in Ebonyi State University. A survey was conducted on 54 undergraduate Students of the school in particular, students of the Department of Industrial Mathematics and Applied Statistics. The data extracted from this survey were cast into a Markov Matrix and was analyzed. The result was used to forecast the preferences of consumers in the market environment in the future. Also the result showed that consumers exhibit different preferences with respect to some attribute such as; comfort, speed, availability and price.
KEYWORDS*: Brand loyalty, Consumer preference, Markov chain, Market equilibrium, Steady State*

Date of Submission: 13-06-2020 Date of Acceptance: 29-06-2020

I. INTRODUCTION

Markov Analysis is a stochastic process that is used to examine randomly changing system. It is probabilistic information about a decision situation that can help the decision markers in making decisions. It is used to forecast the variable whose future is variable to its current state. In market today building a competitive advantage takes a brand/company that understands the preference of the consumers that patronize that market and also the understanding of why consumers become loyal or disloyal to a brand. Markov analysis helps to understand the movement of consumers from one brand to another in a market. In 1970, the Russian Mathematician and Author Andreyevich introduced the basic concept of Markov Chain method. Since that time many authors has been working on Markov chain and have helped it to develop. Researchers have used Markov chain method intensively for research conducted on social topics as the brand selection of customers, income distribution, occupational mobility and immigration as a geographical structure.

In marketing, Markov Analysis has been widely studied by many researchers such as, [1] applied it in brand switching studies and market share forecasting and observed that Markov brand switching model studies customer loyalty and forecasts the brands, Products, or services that a customer is likely to purchase next, [2] tries to model Brand switching in consumers products with Markov analysis and transition matrix, using different brands of soft drinks, [3] studied the Application of Markovian models and transition Matrix to analyze the workforce movement in Jordanian productivity company's statistics. Michael in [4] studied the applications of finite Markov chain models to management. In [5], Aparna and Sarat researched on analysis of brand loyalty using homogeneous Markov models.

In this study we applied Markov chain analysis to understand the uncertainties of student's preference of transport services, investigate the changes in preference of a population after a period of time and determined the state of equilibrium of the market. Here we considered three transport services; Bus, Bike and tricycle (Keke Napep) and tried to find out reasons for discontinuity of students preference for transport services.

II. SOME DEFINITIONS

Definition 1.1 Markov chain is a mathematical and scientific method used to analyze the behavior of different phenomena (system/state) during the current period in order to predict the behavior of these phenomena's in the future in any later periods according to Touama (2015). It can also be defined Godwin et al (2013), as a dynamic quantitative model with markov property; the probability of moving to a state (X) at time (n+1) depends only on current state (immediate past purchase behavior) and not on the sequence of the entire previous states.

Definition 1.2 Brand loyalty refers to consumer's behavior of repeatedly purchasing/patronizing a specific brand over a certain period of time. According to [2], brand loyalty is consumer behavior of choosing to stick

with a particular brand, despite the availability of other closely related and similar brands in the market.

Definition 1.3

Brand Switching is choosing to switch from routine use of one product or brand to steady use of different but similar product. When consumers switch from one brand to another, building a picture of likely brand switching behavior occurs. It is also the behavior of consumers to rotate between use of different, but similar products over a period of time.

III. TRANSITION/STOCHASTIC MATRIX

Definition 1.4 A transition matrix is a square matrix whose entries are probability vectors, which is used to describe the transitions of a Markov chain. It is also called a probability matrix or Markov matrix.

There are several types of transition matrix namely:

Right Transition Matrix: A real squared matrix with each row summing to 1.

Left Transition Matrix: A real squared matrix with each column summing to 1

Double Stochastic Matrix: A squared matrix of non negative real numbers with each row and column summing to 1.

Because of the transition (changes) between different states, conditions or choices, markov analysis deals with transition probabilities. Transition matrix is used to exhibit/model these transition probabilities.

IV. ASSUMPTIONS:

In Markov Analysis for a Process (stochastic process) to be called a markov process, it must be characterized by some assumptions: An analysis of the markov method is based on the fundamental assumption that: any system dealt with in the first instance is in its initial state, in preparation for the transition to another state. This assumption is based on a certain probability law called the (Transition Probability), which are known as "transition Probabilities of a particular case to another case during a certain period of time".

For example, the probability of transition phenomenon of the case (i) in the current period (n) to another state (j) in the later period (n+1) Writes as follows according to Touama, (2015)

$$P\{X_{n+1} = j/X_n = i\} = P_{ij} \forall i, j$$

Where;

Xn: Value of the phenomenon (state) in the current period (n),

Xn+1: Value of the Phenomenon (state) in the subsequent period (n+1).

P_{ij} is the probability of transmission from the phenomenon of state (i) to the state (j).

Amare et al in [1] made the following assumptions before formulating a markov chain model:

- There is finite number of possible states.
- States are both collectively exhaustive and mutually exclusive.

- The transition probabilities depend only on the current state of the system, that is, if the current state is known, the conditional probability of the next state is independent of the states prior to the present state.

- The long-run probability of being in a particular state will be constant over time.

- The transition probabilities of moving to alternative states in the next time period, given a state in the current time period must sum to 1.

V. CLASSIFICATION OF THE STATES IN A MARKOV CHAIN

According to [7]; The states of a markov chain can be classified based on the transition probability (P_{ij}) of P;

ABSORBING: A state j is absorbing if it returns to itself with certainty in one transition. That is $P_{ij} = 1$.

TRANSIENT: A state j is transient if it can reach another state but cannot itself be reached back from another state. That is;

$$\lim_{n\to\infty} P_{ij}^n = 0, \forall\, i$$

RECURRENT: A state is recurrent if the probability of being revisited from other states is 1. This can happen if and only if the state is not TRANSIENT.

PERIODIC: A state is periodic with period $t \geq 1$ if a return is possible only in t, 2t, 3t, ... steps. This means that $P(n) = 0$ whenever n is not divisible by t.

Market Equilibrium: The Equilibrium situation is characterized by the equality, $P = P.A$. Obviously we have that;[4]

$$P_{1+}P_2 + \cdots + P_n = 1.$$

The state of the market equilibrium, is defined as "a situation" in which the market shares of the organization involved in the competition process in the market becomes STEADY-STATE without these shares unchanged in later periods [4].

VI. METHOD OF DATA COLLECTION

The purpose of this study is to examine the brand loyalty of customers of transport services using the Markov chains method. Around the main campus of EBSU, there are 3 transport services namely, Bus, Keke(tricycle) and Bike services. A consumer was defined as a Student who patronize these transport Services as a means of coming to the campus from their Hostels Frequently on daily basis. For this study, data of Brand loyalty has been collected from the students of Industrial Mathematics and Applied Statistics that frequented the Permanent site campus, of Ebonyi State University during the 2018/2019 Academic session. Participants were to fill a questionaire survey anonymously for which 54 consumers responded and all surveys were fully completed. The survey contained Multiple-Choice questions; Participants were asked to choose from the 3 transport services, the one they used on that day, select from the same 3 transport Services, their most preferred and lastly, they were asked to select from (Price, Speed/Time, Comfort and Availability) the Reason for their choice in the previous question.
Questionnaire survey was used for this study because they are very important in data collection, as they have major impact on data quality, reliability and validity.

Markov Chain Model has been used to analyze the dynamics (changes) in consumer's preference, predict the Change in preference of a population after a time period (n) and to forecast the state of equilibrium of the market (Steady-State) after a long-run. As the number of period increases, further changes in the state Probability become Smaller. Hence, POM-QM for windows software has been used to confirm how these proportions would Change after each day till they reach Steady-State.

In order to meet our objectives using Markov Chain Method, some Procedures were followed;

Firstly, identify the possible outcomes $(i = 1,2, \ldots, m)$ of each of the sequence of events (Transport Services) in this case 3.Secondly, calculate P_{ij} that is the probability of being in State 'j' in the future given the current state 'i' , the outcome $S_1, S_2, \ldots S_m$ are called transition probabilities. When $P_{ij} = 0$, this means that no transition occurred from state i to state j and when $P_{ij} = 1$, this means that a system is in state i can move only to State j at the next transition. Thirdly, predicting future states involves knowing the system's probability of changing from one state to another. These probabilities were collected and placed in a transition matrix.

Since there are 3 States (transport Services) selected around permanent site campus, the state space is in the form; $S = \{Bus, Keke, Bike\}$. As a result of the survey conducted on the consumers, the data collected about the relationship between their existing preference and next transport service preferences are transformed into a markov matrix.

$$\begin{bmatrix} & INITIAL & BUS & KEKE & BIKE \\ BUS & X_{11} & a_{11} & a_{12} & a_{13} \\ KEKE & X_{21} & a_{21} & a_{22} & a_{23} \\ BIKE & X_{31} & a_{31} & a_{32} & a_{33} \end{bmatrix}$$

The Markov Matrix of the work

Where;
Column "INITIAL" represents the Present/initial situation (number of customers) of each state (transport services), and to it's right is our transition matrix containing the probability of consumers changing from one state to another in the future, given the initial state.

To get the first transition, the matrix above was split into matrix A and matrix B

where;

$$A = \begin{bmatrix} INITIAL \\ X_{11} \\ X_{21} \\ X_{31} \end{bmatrix}$$

$$B = \begin{bmatrix} BUS & KEKE & BIKE \\ a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}.$$

Then, the transpose of Matrix A was used to multiply matrix B;

$$A^T B = [X_{11} \quad X_{21} \quad X_{31}] \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Then the new initial will be;

$$\begin{bmatrix} BUS & KEKE & BIKE \\ (a_{11}X_{11} + a_{21}X_{21} + a_{31}X_{31}) & (a_{21}X_{11} + a_{22}X_{21} + a_{32}X_{31}) & (a_{31}X_{11} + a_{23}X_{21} + a_{33}X_{31}) \end{bmatrix}$$

This is the new situation (number of customers patronizing each transport service) of each State (future state) at t+1.
A new transition matrix will be gotten, by multiplying B by itself, to get Transition Matrix B(2) which satisfies Chapman-Kolmogorov Equation, as supported by Aypar
& Tuncay(2001). $_{ij}$

To reach Equilibrium (Steady State); There was a continual use of each new "initial" at (t+1) Where t = (1, 2, 3, ...), to multiply each new transition matrix, until we repeatedly get new initials of stationary values and new transition matrix of the same probability, at which point equilibrium was achieved.

VII. ANALYSIS AND RESULTS

From the survey conducted, out of 54 participants; 28 recently patronized Bus Service, 7 Patronized EBSU Keke Service and 19 patronized Bike service which amounts to 51.85%, 12.96%, and 35.19% of the population respectively for each transport service.

Among the 28 consumers who patronized Bus Service; 16 (57.14%) were Satisfied with Bus Services, 8 (28.57%) Preferred Keke Services, and 4 (14.29%) Preferred Bike Services.

Among the 7 consumers who patronized Keke Services; 4 (57.14%) Was Satisfied, only 1 (14.29%) Preferred Bus Services, 2 (28.57%) Preferred Bike Services.

Among the 19 consumers who patronized Bike Services; 14 (73.68%) were satisfied, 3 (15.79%) Preferred Bus services, 2 (10.53%) Preferred keke services.

Below is the Representation of the above data in a Transition Matrix form; Events - Choosing a Preferred transport service.

$$\begin{bmatrix} & INITIAL & BUS & KEKE & BIKE \\ BUS & 0.5185 & 0.5714 & 02857 & 0.1429 \\ KEKE & 0.1296 & 0.1429 & 0.5714 & 0.2857 \\ BIKE & 0.3519 & 0.1579 & 0.1053 & 0.7368 \end{bmatrix},$$

VIII. MARKET EQUIBRIUM (STEADY STATE)

As earlier stated, solving manually for the steady-state probabilities will involve many iterations. Hence the use of POM-QM for windows software will be useful; Where states 1, 2, 3 are Bus, Keke and Bike respectively and Initial contains the probabilities of being in each state at the beginning.

	Initial	State 1	State 2	State 3
State 1	.5185	.5714	.2857	.1429
State 2	.1296	.1429	.5714	.2857
State 3	.3519	.1579	.1053	0.7368

Table 1: Transition Matrix of the work IN POM-QM Format

	State 1	State 2	State 3
End of Period 1			
State 1	.5714	.2857	.1429
State 2	.1429	.5714	.2857
State 3	.1579	.1053	.7368
End prob (given initial)	.3704	.2592	.3704
End of Period 2			
State 1	.3899	.3415	.2686
State 2	.2084	.3974	.3942
State 3	.2216	.1829	.5955
End prob (given initial)	.3072	.2929	.3999

Table 2: Transition 1 and 2

End of Period 3			
State 1	.314	.3348	.3512
State 2	.2381	.3281	.4337
State 3	.2468	.2305	.5227
End prob (given initial)	.2805	.2973	.4222
End of Period 4			
State 1	.2827	.318	.3993
State 2	.2514	.3012	.4474
State 3	.2565	.2573	.4862
End prob (given initial)	.2694	.2945	.4361

Table3: Transition 3 and 4

	State 1	State 2	State 3
End of Period 5			
State 1	.27	.3045	.4254
State 2	.2574	.291	.4516
State 3	.2601	.2715	.4684
End prob (given initial)	.2649	.2912	.444
End of Period 6			
State 1	.265	.296	.4391
State 2	.2599	.2874	.4527
State 3	.2614	.2788	.4599
End prob (given initial)	.2631	.2888	.4481

Table 4: Transition 5 and 6

End of Period 7			
State 1	.263	.291	.4459
State 2	.2611	.2861	.4528
State 3	.2618	.2824	.4558
End prob (given initial)	.2623	.2874	.4503
End of Period 8			
State 1	.2623	.2884	.4493
State 2	.2616	.2858	.4527
State 3	.2619	.2841	.4539
End prob (given initial)	.2621	.2866	.4514

Table 5: Transition 7 and 8

	State 1	State 2	State 3
End of Period 9			
State 1	.262	.287	.4509
State 2	.2618	.2857	.4525
State 3	.2619	.285	.4531
End prob (given initial)	.262	.2861	.4519
End of Period 10			
State 1	.2619	.2864	.4517
State 2	.2619	.2857	.4525
State 3	.2619	.2854	.4527
End prob (given initial)	.2619	.2859	.4521

Table 6: Transition 9 and 10

End of Period 11			
State 1	.2619	.286	.4521
State 2	.2619	.2857	.4524
State 3	.2619	.2856	.4525
End prob (given initial)	.2619	.2858	.4523
End of Period 12			
State 1	.2619	.2859	.4522
State 2	.2619	.2857	.4524
State 3	.2619	.2857	.4524
End prob (given initial)	.2619	.2858	.4523

Table 7: Transition 11 and 12

	State 1	State 2	State 3
End of Period 13			
State 1	.2619	.2858	.4523
State 2	.2619	.2857	.4524
State 3	.2619	.2857	.4524
End prob (given initial)	.2619	.2857	.4523
End of Period 14			
State 1	.2619	.2858	.4523
State 2	.2619	.2857	.4524
State 3	.2619	.2857	.4524
End prob (given initial)	.2619	.2857	.4523

Table 8: Transition 13 and 14

End of Period 15			
State 1	.2619	.2857	.4523
State 2	.2619	.2857	.4524
State 3	.2619	.2857	.4524
End prob (given initial)	.2619	.2857	.4524

Table 9: Transition 15

	(untitled) 15 ste		
	State 1	State 2	State 3
State 1	.2619	.2857	.4523
State 2	.2619	.2857	.4524
State 3	.2619	.2857	.4524
Ending probability (given	.2619	.2857	.4524
Steady State probability	.2619	.2857	.4524

Table10: Final Result Showing Steady-State Probability

IX. DISCUSSIONS

Table 10; The final result, shows the Probabilities at which the market will be in equilibrium (Steady-state probabilities) which are 0.2619, 0.2857 and 0.4524 for Bus, Keke and Bike respectively. 0.2619 is the probability of consumer preference changing in favor of Bus, 0.2857 is the probability of consumer preference changing in favor of keke, while 0.4524 is the probability of consumer preference changing in favor of Bike.

All our transition matrices are right transition matrices. At Steady-state

$$[0.2619 \quad 0.2857 \quad 0.4524]$$

The market is said to be in Equilibrium, this state was achieved after 15-iterations (15-days). At the state of equilibrium, the markov chain becomes absorbing, that is $P_{ij} = 1$ (it will continue to return to itself after any further iteration (iteration 16, 17, ..., t). This shows that the preference of the consumers will remain constant and unchanging after Transition 15. Hence, from the first row of the 15th transition matrix

$$\left[B_b^{(15)} \quad K_b^{(15)} \quad I_b^{(15)}\right] = \left[B_b^{(15+1)} \quad B_b^{(15+1)} \quad B_b^{(15+1)}\right]$$

Where,

$B_b^{(15)}$; The Probability of consumers of EBSU Bus Services remaining Satisfied after 15 iteration (15-days).
$K_b^{(15)}$; The probability of Bus Service consumers changing their preference to EBSU Keke Services after 15 iterations (15 days).
$I_b^{(15)}$; The probability of Bus Service consumers changing their preference to Bike Services after 15 iterations (15 days).
Since iteration 15 indicates 15 days, let days = t;

$$\left[B_b^{(t)} \quad K_b^{(t)} \quad I_b^{(t)}\right] = \left[B_b^{(t+1)} \quad B_b^{(t+1)} \quad B_b^{(t+1)}\right]$$

Where; t = (1, 2, 3, ...)

This implies that at equilibrium, the probability of consumer preference for each transport service at time (t), will remain unchanged in a future time (t+1). The number of customers that will prefer each of the transport service at Equilibrium (steady state) will be;
(Fraction of consumer preference changing in favour of each transport service)×(Total no. of Consumers)
BUS; $0.2619 \times 54 = 14.1426 \approx 14$ Consumers,
KEKE; $0.2857 \times 54 = 15.66 \approx 16$ Consumers.
BIKE; $0.4524 \times 54 = 24.4296 \approx 24$ Consumers.

In Clearer terms, at equilibrium (long-run);

14(26.19%) of consumers will patronize Bus Services.
16(28.57%) of consumers will patronize Keke Services.
24(45.24%) of consumers will patronize Bike Services.

To understand what brought about this change in consumer preference, that has made EBSU Bus Services fall from being the most patronized to become the least patronized (dynamics of consumer Preference), The following data were extracted from our initial survey in tabular form.

TRANSPORT SERVICE	NO. OF CONSUMERS	P	S	C	A	P∩A	S∩C	S∩A
BUS	16	11	1	2	1	1	0	0
KEKE	8	0	3	4	0	0	1	0
BIKE	4	0	3	1	0	0	0	0

Table 11 Initial Consumer Preference of EBSU Bus Service with Respect To some decision variable

TRANSPORT SERVICE	NO. OF CONSUMERS	P	S	C	A	P∩A	S∩C	S∩A
BUS	1	0	0	1	0	0	0	0
KEKE	4	0	1	1	0	0	1	0
BIKE	2	0	1	1	0	0	0	0

Table12: Initial Consumer Preference of EBSU Keke Service with Respect To some Decision variables

TRANSPORT SERVICE	NO. OF CONSUMERS	P	S	C	A	P∩A	S∩C	S∩A
BUS	3	1	0	2	0	0	0	0
KEKE	2	0	0	2	0	0	0	0
BIKE	14	0	4	4	3	0	2	1

Table 13: Initial Consumer Preference of Bike Service With Respect To some Decision Variables

TRANSPORT SERVICE	NO. OF CONSUMERS	P	S	C	A	P∩A	S∩C	S∩A
BUS	28	12	1	5	1	1	0	0
KEKE	7	0	5	7	0	0	0	2
BIKE	19	0	8	6	3	0	1	2

Table 14: Summation of Table 11, 12 And 13

The tables contain the following decision variables; Price (P), speed(S), comfort(C), availability(A), price and availability($P \cap A$), speed and comfort ($S \cap C$), speed and availability ($S \cap A$) which are defined below;

PRICE (P): This refers to how affordable and cheap a transport services is.

SPEED(S): This refers to how fast and less time-consuming a transport service is.

COMFORT(C): This refer to space(how less congested a transport service is).

AVAILABILITY (A): This refers to reliability (how easily can a transport service be found around, when needed urgently).

PRICE AND AVAILABILITY ($P \cap A$): This refers to how cheap and reliable a transport service is.

SPEED AND COMFORTABILITY($S \cap C$): These refer to how less time consuming and less congested a transport service is.

SPEED AND AVAILABILITY($S \cap A$): This refers to how Fast and Reliable a transport service is.

These Variables are what prompted the decision of preference in consumers. Each table shows the preference of consumers and how many were prompted by each decision variable. As supported by Noor-ui-Ain Nawaz and Ahmad usman (2016). From table 4 , which sums table 1, 2, and 3. We see that; 22.22%(12) of consumers Preference was influenced by price and 25.93%(14) of consumers preferences was influenced by speed. 33.33%(18) of consumers preference was influenced by comfort.7.41%(4) of

consumers preference was by influenced availability.

1.85%(1) of consumers preference was influenced by both speed and availability. From the above, it is obvious that consumer preference was mostly influenced by comfort (33.33%), followed by speed, then price (22.22%). This explains why EBSU Bus Services came out as least patronized at steady-state (Market Equilibrium), it's only advantage was it's cheap price, it was not providing enough comfort for its consumers (the number of consumers per seat (5), is congested and uncomfortable), and it does not have speed (as they waste a lot of time on the road for example, sorting out balance for consumers in monetary terms). If they can adjust their defaults, they stand to gain at least 64.81% of consumers (33.33% for speed, 7.41% for speed and comfort, and 1.85% for speed and availability).

Bike Services at equilibrium rose from second most patronized, to most patronized mostly because they are fast(less time consuming) and Available. If they maintain this and work on their price, they would remain at the top of consumer's preference list. Keke Services at equilibrium rose from being least patronized to second most patronized, as it is the most comfortable and second fastest after Bike Services in terms of Speed. There is room for improvement as they can work on their availability (numbers on the road).

X. CONCLSION

So far we have conducted a Survey of consumer's preference on transport services around Ebonyi State University main campus, the data retrieved was transformed into a Markov matrix and the change in preference of consumers over a long period of time was observed. After a number of successive transitions on the Markov matrix, it was possible to reach a balanced condition (Market Equilibrium). According to the mentioned balanced condition, it was observed that the preferences of consumers intensified on Bike Services and Keke Services, while that for Bus Services was small. It was also observed that in forming the tendency of changing preference among consumers, the decision Variables play an important role. In respect to the decision Variables, the ideal transport Service, will be the one that has the following qualities; comfort, speed, availability, and cheap price, we cannot generalize for the whole campus, but it is obvious that, to gain the preference of the whole campus, the transport services have to gain the preference of each department, hence our research paper is very useful on a micro-scale.

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T. E. Efor, et. al. "Application of Markov Analysis to Consumers Preference." *International Journal of Mathematics and Statistics Invention (IJMSI)*, vol. 08(06), 2020, pp. 14-23.