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Magnetopolar Stratified Flow through A Porous Medium with Heat Source Parameter in Slip Flow Regime

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ABSTRACT : *Micropolar fluid behaviour on MHD steady viscous stratified flow through a porous medium in slip flow regime has been studied numerically. The exact solutions for velocity, angular velocity and temperature are obtained and are shown graphically for different parameters entered in to the problem. Moreover, expressions for skin friction and Nusselt number are also obtained. It is observed that increase in rotational parameter (α) increases the skin friction while increase in couple stress parameter (β) decreases the skin friction.*

KEYWORDS: *Magnetic field, polar fluid, porous medium, rotational velocity, slip velocity, stratified.*

I. INTRODUCTION

The study of the flow through a porous medium is of great importance to geophysicists and fluid dynamicists. Such flow are very much prevalent in nature and have applications in many engineering problems such as oil exploration, chemical catalytic rectors, thermal insulation and geothermal energy extractions. Brinkman [1],Yamamto and Yashida [2] have discussed fluid flow through porous medium considering generalized Darcy's law which contains the terms of viscous stress and convection acceleration. A comprehensive review on porous flow is given in the book of Nield and Bejan [3]. In all above works, generalized Darcy's law is derived without taking into account the angular velocity of the fluid particles. Magyari et al. [4] have discussed analytic solution for unsteady free convection in porous media.

In the recent years, considerable attention has been given to the stratified fluid. Fluid motion influenced by the density and viscosity variations in the fluid is characterized as stratified flow. Channabasappa and Ranganna [5] studied the flow of a viscous stratified fluid on variable viscosity fluid past a porous bed with the idea that of stratification may provide a technique for studying the pore size in a porous medium. Unsteady stratified flow using laplace transform technique has been considered by Gupta and Goyal [6], Kumar et al. [7], they have also discussed the problems of viscous stratified MHD flow along an infinite flat plate. Das et al. [8] have studied the effect of heat source and magnetic field on a viscous stratified fluid past a porous moving plate. Khandelwal and Jain [9] have considered unsteady MHD heat transfer problems of stratified fluid.

The dynamics of polar fluid has attracted considerable attention during the last few decades because traditional Newtorian fluids cannot precisely describe the characteristics of fluid flow with suspended particles. Aero et al. [10] derived and solved equations of the fluid in which angular velocity of the fluid particles was taken into account. These fluids are known as polar fluids in the literature and are more general than ordinary fluids (Lukaszewicz[11]. Raptis et al. [12], Raptis and Takhar [13] have discussed polar flow through porous medium. Jain and Gupta [14], considered magnetic effects on polar flow. Polar fluids belong to a class of fluids with microstructure and have asymmetrical stress tensor. Physically, they represent fluids consisting of randomly oriented particles suspended in a viscous medium. Recently Khandelwal [15] has studied magnetopolar stratified flow through a porous medium in slip flow regime.

In above referred works on polar fluids, no attempt has been made for temperature field with stratification effects which needs much attention. In the present paper, an attempt has been made with stratification effects for study the effects of heat source, couple stresses and stratification parameter on steady flow through a magnetopolar stratified fluid in slip flow regime. The aim of this paper is to make a numerical calculation which have been of interest to the engineering community and to the investigators dealing with the problem in geophysics, astrophysics, plasma studies, nuclear reactors etc. In general, the study of magnetopolar stratified flow through a porous medium is complicated. It is necessary to consider in detail the distribution of velocity, angular velocity and temperature across the boundary layer.

Representative results for the velocity, angular velocity and temperature profiles are displayed graphically showing the effects of several governing parameter entering into the problem. Moreover, expression for skin friction and Nusselt number are also obtained and shown graphically. It is observed that increase in rotational parameter (α) increases the skin friction, while increase in couple stress parameters (β) decreased the skin friction.

II. MATHEMATICAL FORMULATION OF THE PROBLEM

Let us consider a two dimensional steady MHD flow of stratified polar fluid through a porous medium over a porous plate with slip boundary conditions. The constant suction velocity ($-v_0$) is taken normal to the plate. The x-axis is taken along the plate and y-axis normal to it. The flow is assumed to be at small magnetic Reynolds number which enables us to neglect the induced magnetic field.
We assume

$$\rho = \rho_0\, \bar{e}^{\delta y}, \qquad \mu = \mu_0\, \bar{e}^{\delta y}, \qquad \mu = \mu_{0r}\, \bar{e}^{\delta y},$$
$$k = k_0\, \bar{e}^{\delta y}, \qquad S = S_0\, \bar{e}^{\delta y}, \qquad B = B_0\, \bar{e}^{\delta y/2}, \tag{2.1}$$

For $y \geq 0$

where $\delta>0$ represents the stratification paramweter δ being assumed to be small positive number, ρ the density of the fluid, μ the viscosity of the fluid, μ_r the rotational viscosity, k the thermal conductivity, S coefficient of heat source, B the strength of magnetic field and ρ_0, μ_0, μ_{0r}, k_0, S_0 and B_0 are the density, viscosity, rotational viscosity, thermal conductivity, coefficient of heat source and strength of magnetic field respectively at y = 0. Under these assumptions the equations which govern the flow are :

$$\frac{\partial v}{\partial y} = 0 \tag{2.2}$$

$$\rho\left(v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left[(\mu+\mu_r)\frac{\partial u}{\partial y}\right] + 2\mu_r\frac{\partial \omega}{\partial y} - \frac{\mu u}{K} - \sigma\mu_e B^2 u \tag{2.3}$$

$$v\frac{\partial \omega}{\partial y} = \frac{\gamma}{I}\frac{\partial^2 \omega}{\partial y^2} \tag{2.4}$$

$$v\frac{\partial T}{\partial y} = \frac{1}{\rho C_p}\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{S}{\rho C_p}(T - T_\infty) \tag{2.5}$$

where μ_r is the rotational viscosity, ω the mean angular velocity of rotation of the particles, K the permeability of the porous medium, I a scalar constant of dimension equal to that of moment of inertia of unit mass and remaining symbols have their usual meanings and $\gamma = C_a + C_d$.
where C_a and C_d are coefficient of couple stress viscosities.
The boundary conditions are :

$$u = L_1\frac{\partial u}{\partial y}, \qquad \frac{\partial \omega}{\partial y} = -\frac{\partial^2 u}{\partial y^2}, \quad T = T_w, \text{ at } y = 0$$
$$u \to U_\infty, \qquad \omega \to 0, \quad T \to T_\infty, \text{ as } y \to \infty. \tag{2.6}$$

where $L_1 = \left(\frac{2-m_1}{m_1}\right)L$, L being mean free path and m_1 the Maxwell's reflection coefficient.

Integration of equation (2.2) for constant suction gives

$$v = -v_0. \tag{2.7}$$

For free stream velocity, we have from (2.3)

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial x} - \left(\frac{v}{K} + \frac{\sigma\mu_e B^2}{\rho}\right)U_\infty. \tag{2.8}$$

Now introducing the following non-dimensional quantities :

$$u^* = \frac{u}{U_\infty}, \quad y^* = \frac{y\nu_0}{\nu_0}, \quad \omega^* = \frac{\upsilon_0\omega}{\nu_0 U_\infty}, \quad K^* = \frac{K\nu_0^2}{\upsilon_0^2}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}$$

$\alpha = \dfrac{\upsilon_{0r}}{\upsilon_0}$ (Rotational parameter), $\beta = \dfrac{I\upsilon_0}{\gamma}$ (Couple stress parameter)

$M^2 = \dfrac{\sigma\mu_e B_0^2 \upsilon_0}{\rho_0 \nu_0^2}$ (Magnetic field parameter), $h_1 = \dfrac{\nu_0 L_1}{\upsilon_0}$ (Velocity slip parameter)

$Pr = \dfrac{\mu_0 C_p}{k_0}$ (Prandtl number), $S^* = \dfrac{\upsilon_0^2 S_0}{k_0 \nu_0^2}$ (Heat source parameter)

$\delta^* = \dfrac{\delta\upsilon_0}{\nu_0}$ (Stratification parameter)

In view of equations (2.1), (2.7) and for free stream velocity, the equations of motion in non-dimensional form after dropping the asterisks over them, reduced to :

$$(1+\alpha)\frac{d^2u}{dy^2} + [1 - \delta(1+\alpha)]\frac{du}{dy} + 2\alpha\frac{d\omega}{dy} + N^2(1-u) = 0 \tag{2.9}$$

$$\frac{d^2\omega}{dy^2} + \beta\frac{d\omega}{dy} = 0 \tag{2.10}$$

$$\frac{d^2\theta}{dy^2} + Pr\frac{d\theta}{dy} + S\theta = 0 \tag{2.11}$$

with corresponding boundary conditions as :

$$\begin{aligned} &u = h_1\frac{\partial u}{\partial y}, \quad \frac{\partial\omega}{\partial y} = -\frac{\partial^2 u}{\partial y^2}, \quad \theta = 1 \quad \text{at } y = 0 \\ &u \to 1, \quad \omega \to 0, \quad \theta \to 0, \quad \text{as } y \to \infty \end{aligned} \tag{2.12}$$

where $N^2 = M^2 + \dfrac{1}{K}$

III. SOLUTIONS OF THE PROBLEM

Equations (2.9) to (2.11) are second order ordinary differential equations we get the following exact solutions after using corresponding boundary conditions :

$$u = 1 + C_2\, \bar{e}^{R_2 y} + \frac{2\alpha\beta C_1}{(\beta + R_1)(\beta - R_2)}\bar{e}^{\beta y} \tag{3.1}$$

$$\omega = C_1\, \bar{e}^{\beta y} \tag{3.2}$$

$$\theta = C_3\, \bar{e}^{R_3 y} \tag{3.3}$$

Where

$$R_1 = \frac{-a_2 + \sqrt{a_2^2 + 4a_1 N^2}}{2a_1}, \qquad R_2 = \frac{a_2 + \sqrt{a_2^2 + 4a_1 N^2}}{2a_1}$$

$$R_3 = \frac{Pr + \sqrt{Pr^2 - 4S}}{2}, \qquad C_1 = \frac{-R_2^2}{\left[\beta(1 - a_3\beta)(1 + h_1 R_2) + R_2^2 a_3 (1 + h_1\beta)\right]}$$

$$C_2 = \frac{\left[1 - a_3 C_1 (1 + h_1\beta)\right]}{(1 + h_1 R_2)}, \qquad C_3 = 1$$

$$a_1 = 1+\alpha, \qquad a_2 = 1-\delta a_1, \qquad a_3 = \frac{2\alpha\beta}{(\beta + R_1)(\beta - R_2)}$$

IV. SKIN FRICTION AND NUSSELT NUMBER

With the help of velocity and temperature profiles, the important parameters skin friction (ι) and Nusselt number (Nu) are given as :

$$\tau_w = (\mu + \mu_r)\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

$$\tau = \frac{\tau_w}{\rho U_\infty \nu_0} = (1+\alpha)\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

$$= (1+\alpha)\left[-R_2 C_2 \frac{2\alpha\beta^2 C_1}{(\beta + R_1)(\beta - R_2)}\right] \qquad (4.1)$$

and

$$Nu = \frac{-L_1}{(T_w - T_\infty)}\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

$$= C_3 h_1 R_3 \qquad (4.2)$$

V. DISCUSSION AND CONCLUSIONS

To explore the results numerical values of the velocity distribution (u), angular velocity distribution(ω), temperature (θ), skin-friction (τ) and Nusselt number (Nu) are computed for different parameters viz. permeability parameter (K), Magnetic field parameter (M), Velocity slip parameter (h_1), Rotational parameter(α), Couple stress parameter(β), Stratification parameter(δ), Prandtl number (Pr) and Heat source parameter(S).

Figure 1, shows the velocity distribution against y for different values of K, M, h_1, α, β and δ. In figure 1, it is interesting to note that as h_1 increases in velocity increases near the plate and after some distance from the plate it decreases. This is due to the fact that the slip at the plate has effect in the visnity of it. The same observation is noted for free flow also. It is also observed that increase in K, α, β and δ increases the velocity while increase in M decreases the velocity.

Figure 2, shows the angular velocity distribution against y for different values of K, M, h_1, α, β and δ. It is being observed that increasing values of K, M, h_1 and δ, increases the angular velocity distribution but increase in α and β is decreases.

Temperature distribution is plotted against y in Figures 3. It is observed that temperature of fluid is more in case of air (Pr=0.71) in comparison with Water (Pr=7.0). Physically this effect may be due the fact that air is lighter than water and hence take temperature more rapidly. Moreover, increase in heat source parameter leads to rise in temperature for both the basic fluids.

In Figure 4, important parameter namely skin friction is plotted against M for different values of K, h_1, α, β and δ. We notice that skin friction decreases with increase in K, h_1, β and δ but increases with increase in rotational parameter (α) through out the flow. It is interesting to note that in no slip condition (h_1=0) the skin friction decrease near the plate but increases as we move away from it.

In Figure 5, another important parameter rate of heat transfer is plotted against sink(-s) for different values of Pr and h_1. We notice that Nusselt number is more for water (Pr=7.0) in comparison with air (Pr=0.71). Moreover, for both the basic fluids air and water Nusselt number increases with increase in h_1.

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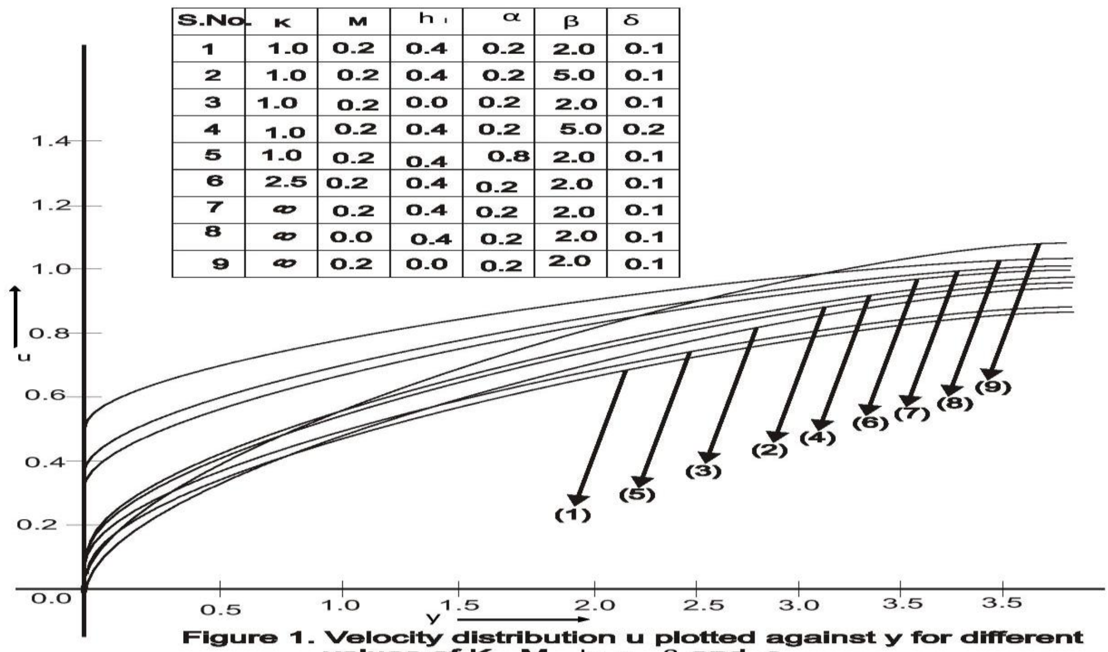
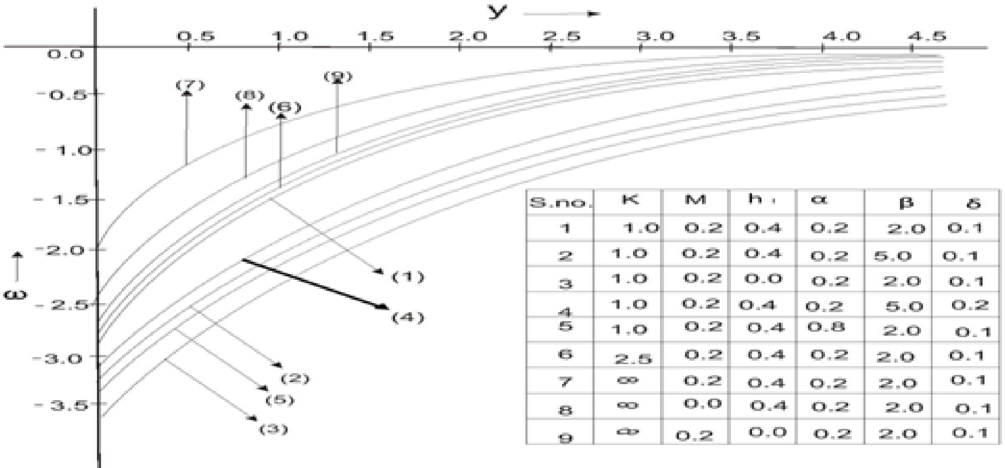


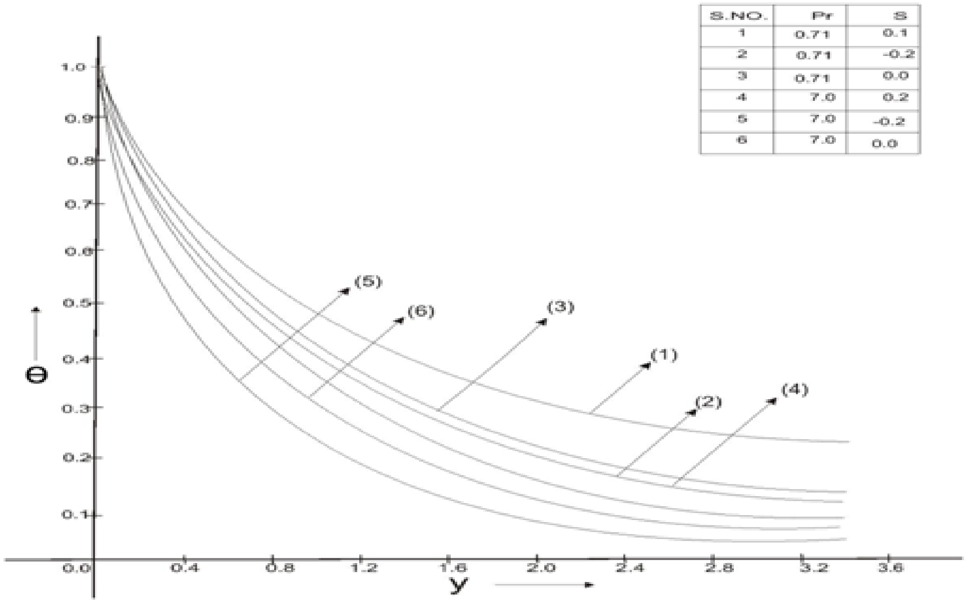
S.No.	K	M	h_1	α	β	δ
1	1.0	0.2	0.4	0.2	2.0	0.1
2	1.0	0.2	0.4	0.2	5.0	0.1
3	1.0	0.2	0.0	0.2	2.0	0.1
4	1.0	0.2	0.4	0.2	5.0	0.2
5	1.0	0.2	0.4	0.8	2.0	0.1
6	2.5	0.2	0.4	0.2	2.0	0.1
7	∞	0.2	0.4	0.2	2.0	0.1
8	∞	0.0	0.4	0.2	2.0	0.1
9	∞	0.2	0.0	0.2	2.0	0.1

Figure 1. Velocity distribution u plotted against y for different values of K , M , h_1, α , β and δ



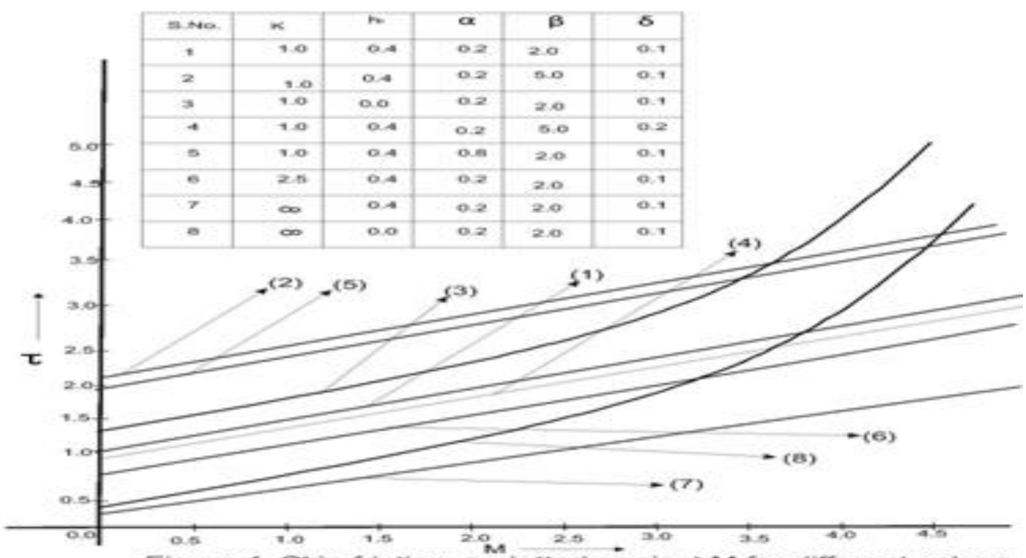
S.no.	K	M	h_1	α	β	δ
1	1.0	0.2	0.4	0.2	2.0	0.1
2	1.0	0.2	0.4	0.2	5.0	0.1
3	1.0	0.2	0.0	0.2	2.0	0.1
4	1.0	0.2	0.4	0.2	5.0	0.2
5	1.0	0.2	0.4	0.8	2.0	0.1
6	2.5	0.2	0.4	0.2	2.0	0.1
7	∞	0.2	0.4	0.2	2.0	0.1
8	∞	0.0	0.4	0.2	2.0	0.1
9	∞	0.2	0.0	0.2	2.0	0.1

Figure 2. Angular velocity distribution ω Plotted against y for Different values of K,M,h ,α ,β And δ



S.NO.	Pr	S
1	0.71	0.1
2	0.71	-0.2
3	0.71	0.0
4	7.0	0.2
5	7.0	-0.2
6	7.0	0.0

Figure 3. Temperature distribution Θ for different values of Pr and S.



S.No.	K	h_1	α	β	δ
1	1.0	0.4	0.2	2.0	0.1
2	1.0	0.4	0.2	5.0	0.1
3	1.0	0.0	0.2	2.0	0.1
4	1.0	0.4	0.2	5.0	0.2
5	1.0	0.4	0.8	2.0	0.1
6	2.5	0.4	0.2	2.0	0.1
7	∞	0.4	0.2	2.0	0.1
8	∞	0.0	0.2	2.0	0.1

Figure 4. Skin friction τ plotted against M for different values of K, h_1, α, β and δ.

S.NO.	Pr	h_1
1	0.25	0.1
2	0.71	0.4
3	7.0	0.8
4	0.71	0.1
5	7.0	0.4

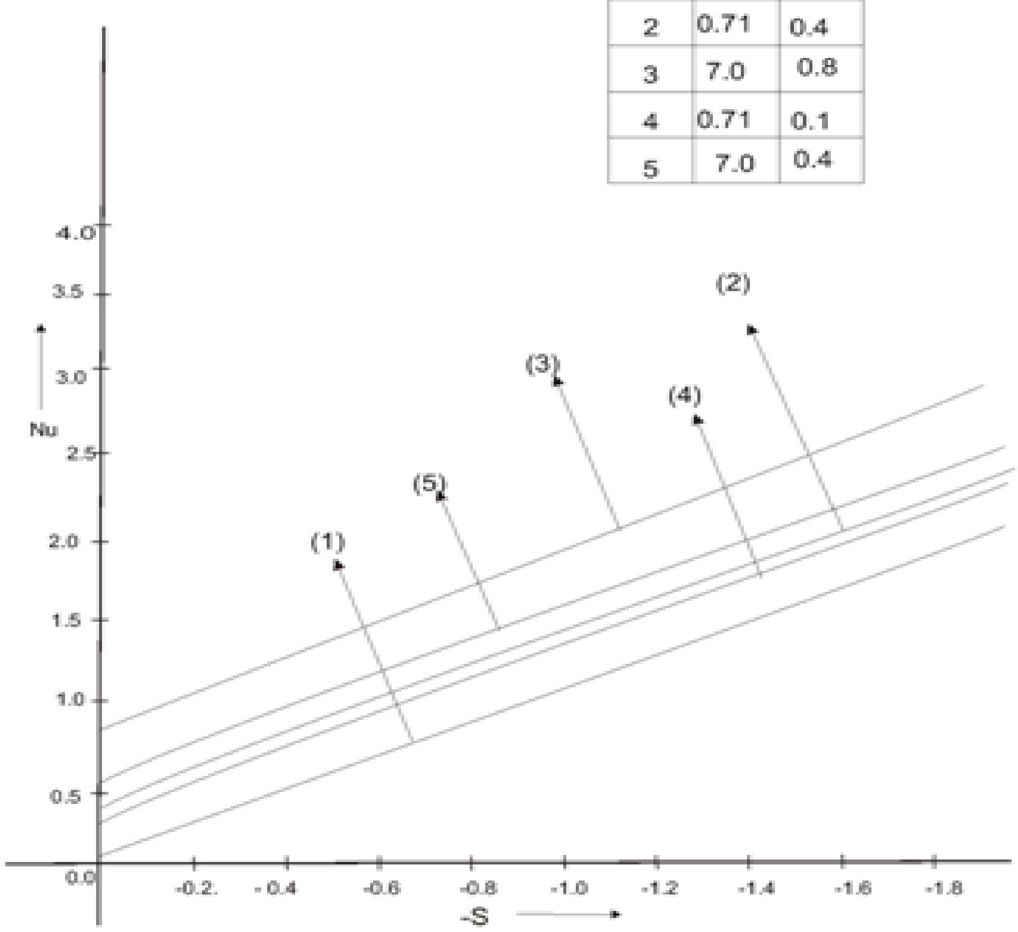


Figure 5. Nusselt number Nu plotted against -s (sink) for different values of Pr and h_1