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# On Nano Forms Of Weakly Open Sets

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***ABSTRACT :*** *The purpose of this paper is to define and study certain weak forms of nano-open sets namely, nano $\alpha$ -open sets, nano semi-open sets and nano pre-open sets. Various forms of nano $\alpha$ -open sets and nano semi-open sets corresponding to different cases of approximations are also derived.*

***KEYWORDS:*** *Nanotopology,nano-open sets,nano interior, nano closure, nano $\alpha$ -open sets, nano semi-open sets, nano pre-open sets, nano regular open sets.*
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## I. INTRODUCTION

Njastad [5], Levine [2] and Mashhour *et al* [3] respectively introduced the notions of $\alpha$ - open, semi-open and pre-open sets.Since then these concepts have been widelyinvestigated. It was made clear that each $\alpha$ -open set is semi-open and pre-open but the converse of each is not true. Njastad has shown that the family $\tau^{\alpha}$ of $\alpha$ - open sets is a topology on X satisfying $\tau \subseteq \tau^{\alpha}$ . The families SO(X, $\tau$ ) of all semi–open sets and PO(X, $\tau$ ) of all preopen sets in (X, $\tau$ )are not topologies. It was proved that both SO(X, $\tau$ ) and PO(X, $\tau$ ) are closed under arbitrary unions but not under finite intersection. Lellis Thivagar *et al* [1] introduced a nano topological space with respect to a subset X of an universe which is defined in terms of lower and upper approximations of X. The elements of a nano topological space are called the nano-open sets. He has also studied nano closure and nano interior of a set. In this paper certain weak forms of nano-open sets such as nano $\alpha$ -open sets, nano semi-open sets and nano pre-open sets are established. Various forms of nano $\alpha$ -open sets and nano semi-open sets under various cases of approximations sre also derived. A brief study of nano regular open sets is also made.

## II. PRELIMINARIES

**Definition 2.1** *A subset $A$ of a space $(X, \tau)$ is called*

(i) semi-open [2] if $A \subseteq Cl(Int(A))$ .

(ii) pre open [3] if $A \subseteq Int(Cl(A))$ .

(iii) $\alpha$ -open [4] if $A \subseteq Int(Cl(Int(A)))$ .

(iv) regular open [4] if $A = Int(Cl(A))$ .

**Definition 2.2** *[6] Let $\cup$ be a non-empty finite set of objects called the universe and R be an equivalence relation on $\cup$ named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair $(\cup, R)$ is said to be the approximation space. Let $X \subseteq \cup$ .*

*(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and its is denoted by $L_R(X)$ . That is, $L_R(X) = \bigcup_{x \in \cup} \{R(x) : R(x) \subseteq X\}$ , where R(x) denotes the equivalence class determined by x.*

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$ . That is, $U_R(X) = \bigcup_{x \in \cup} \{R(x) : R(x) \cap X \neq \phi\}$

(iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

**Property 2.3** [6] *If (U , R) is an approximation space and X, Y ⊆ U , then*

(i) $L_R(X) \subseteq X \subseteq U_R(X)$.

(ii) $L_R(\phi) = U_R(\phi) = \phi$ and $L_R(U) = U_R(U) = U$

(iii) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$

(iv) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$

(v) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$

(vi) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$

(vii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$

(viii) $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$

(ix) $U_R U_R(X) = L_R U_R(X) = U_R(X)$

(x) $L_R L_R(X) = U_R L_R(X) = L_R(X)$

**Definition 2.4** [1] *: Let U be the universe, R be an equivalence relation on U and* $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ *where* $X \subseteq U$. *Then by propetry 2.3,* $\tau_R(X)$ *satisfies the following axioms:*

(i) $U$ and $\phi \in \tau_R(X)$.

(ii) The union of the elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the nanotopology on U with respect to X. We call $(U, \tau_R(X))$ as the nanotopological space. The elements of $\tau_R(X)$ are called as nano-open sets.

**Remark 2.5** [1] *If* $\tau_R(X)$ *is the nano topology on U with respect to X, then the set B =* $\{U, L_R(X), B_R(X)\}$ *is the basis for* $\tau_R(X)$.

**Definition 2.6** [1] *If* $(U, \tau_R(X))$ *is a nano topological space with respect to X where* $X \subseteq U$ *and if* $A \subseteq U$*, then the nano interior of A is defined as the union of all nano-open subsets of A and it is denoted by N Int(A). That is, N Int(A) is the largest nano-open subset of A. The nano closure of A is defined as the intersection of all nano closed sets containing A and it is denoted by N Cl(A). That is, N Cl(A) is the smallest nano closed set containing A.*

***Definition 2.7*** [1] *A nano topological space* $(U, \tau_R(X))$ *is said to be extremally disconnected, if the nano closure of each nano-open set is nano-open.*

## III. NANO $\alpha$ – OPEN SETS

Throughout this paper $(U, \tau_R(X))$ is a nano topological space with respect to X where $X \subseteq U$, R is an equivalence relation on U , $U/R$ denotes the family of equivalence classes of U by R.

**Definition 3.1** *Let* $(U, \tau_R(X))$ *be a nano topological space and* $A \subseteq U$. *Then A is said to be*

(i) nano semi-open if $A \subseteq NCl(NInt(A))$

(ii) nano pre-open if $A \subseteq NInt(NCl(A))$

(iii) nano $\alpha$ -open if $A \subseteq NInt(NCl(NInt(A))$

NSO(U, X ), *NPO* (U ,X) and $\tau_R^\alpha$ (X) respectively denote the families of all nano semi-open, nano pre-open and nano $\alpha$ -open subsets of $\mathcal{U}$ .

**Definition 3.2** *Let* $(\mathcal{U}, \tau_R(X))$ *be a nanotopological space and* $A \subseteq \mathcal{U}$ *. A is said to be nano* $\alpha$ *-closed (respectively, nano semi- closed, nano pre-closed), if its complement is nano* $\alpha$ *-open (nano semi-open, nano pre-open).*

**Example 3.3** *Let* $\mathcal{U} = \{a, b, c, d\}$ *with* $\mathcal{U}/R = \{\{a\}, \{c\}, \{b, d\}\}$ *and* $X = \{a, b\}$ *. Then the nano topology,* $\tau_R(X) = \{\mathcal{U}, \phi, \{a\}, \{a, b, d\}, \{b, d\}\}$ *. The nano closed sets are* $\mathcal{U}, \phi, \{b, c, d\}, \{c\}$ *and* $\{a, c\}$ *. Then,* $\mathcal{N}SO(\mathcal{U}, X) = \{\mathcal{U}, \phi, \{a\}, \{a, c\}, \{a, b, d\}, \{b, c, d\}\}$ , $\mathcal{N}PO(\mathcal{U}, X) = \{\mathcal{U}, \phi, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$ *and* $\tau_R^\alpha(X) = \{\mathcal{U}, \phi, \{a\}, \{b, d\}, \{a, b, d\}\}$ *. We note that,* $\mathcal{N}SO(\mathcal{U}, X)$ *does not form a topology on* $\mathcal{U}$ *, since* $\{a, c\}$ *and* $\{b, c, d\} \in \mathcal{N}SO(\mathcal{U}, X)$ *but* $\{a, c\} \cap \{b, c, d\} = \{c\} \notin \mathcal{N}SO(\mathcal{U}, X)$ *. Similarly,* $\mathcal{N}PO(\mathcal{U}, X)$ *is not a topology on* $\mathcal{U}$ *, since* $\{a, b, c\} \cap \{a, c, d\} = \{a, c\} \notin \mathcal{N}PO(\mathcal{U}, X)$ *, even though* $\{a, b, c\}$ *and* $\{a, c, d\} \in \mathcal{N}PO(\mathcal{U}, X)$ *. But the sets of* $\tau_R^\alpha(X)$ *form a topology on* $\mathcal{U}$ *. Also, we note that* $\{a, c\} \in \mathcal{N}SO(\mathcal{U}, X)$ *but is not in* $\mathcal{N}PO(\mathcal{U}, X)$ *and* $\{a, b\} \in \mathcal{N}PO(\mathcal{U}, X)$ *but does not belong to* $\mathcal{N}SO(\mathcal{U}, X)$ *. That is,* $\mathcal{N}SO(\mathcal{U}, X)$ *and* $\mathcal{N}PO(\mathcal{U}, X)$ *are independent.*

**Theorem 3.4** *If A is nano-open in* $(\mathcal{U}, \tau_R(X))$ *, then it is nano* $\alpha$ *-open in* $\mathcal{U}$ *.*

**Proof:** Since A is nano-open in $\mathcal{U}$ , $\mathcal{N}IntA = A$ . Then $\mathcal{N}Cl(\mathcal{N}IntA) = \mathcal{N}Cl(A) \supseteq A$ . That is $A \subset \mathcal{N}Cl(\mathcal{N}IntA)$ . Therefore, $\mathcal{N}Int(A) \subseteq \mathcal{N}Int(\mathcal{N}Cl(\mathcal{N}Int(A)))$ . That is, $A \subseteq \mathcal{N}Int(\mathcal{N}Cl(\mathcal{N}Int(A)))$ . Thus, A is nano $\alpha$ -open.

**Theorem 3.5** $\tau_R^\alpha(X) \subseteq \mathcal{N}SO(\mathcal{U}, X)$ *in a nano topological spce* $(\mathcal{U}, \tau_R(X))$ .

**Proof:** If $A \in \tau_R^\alpha(X)$ , $A \subseteq \mathcal{N}Int(\mathcal{N}Cl(\mathcal{N}Int(A))) \subseteq \mathcal{N}Cl(\mathcal{N}Int(A))$ and hence $A \in \mathcal{N}SO(\mathcal{U}, X)$ .

**Remark 3.6** *The converse of the above theorem is not true. In example 3.3, {a,c} and {b,c,d} and nano semi-open but are not nano* $\alpha$ *-open in* $\mathcal{U}$ *.*

**Theorem 3.7** $\tau_R^\alpha(X) \subseteq \mathcal{N}PO(\mathcal{U}, X)$ *in a nano topological space* $(\mathcal{U}, \tau_R(X))$ .

**Proof:** If $A \in \tau_R^\alpha(X)$ , $A \subseteq \mathcal{N}Int(\mathcal{N}Cl(\mathcal{N}Int(A)))$ . Since $\mathcal{N}Int(A) \subseteq A$ , $\mathcal{N}Int(\mathcal{N}Cl(\mathcal{N}Int(A))) \subseteq \mathcal{N}Int(\mathcal{N}Cl(A))$ . That is, $A \subseteq \mathcal{N}Int(\mathcal{N}Cl(A))$ . Therefore, $A \in \mathcal{N}PO(\mathcal{U}, X)$ . That is, $\tau_R^\alpha(X) \subseteq \mathcal{N}PO(\mathcal{U}, X)$ .

**Remark 3.8** *The converse of the above theorem is not true. In example 3.3, the set {b} is nano pre-open but is not nano* $\alpha$ *-open in* $\mathcal{U}$ *.*

**Theorem 3.9** $\tau_R^\alpha(X) = \mathcal{N}SO(\mathcal{U}, X) \cap \mathcal{N}PO(\mathcal{U}, X)$ .

**Proof:** If $A \in \tau_R^\alpha(X)$ , then $A \in \mathcal{N}SO(\mathcal{U}, X)$ and $A \in \mathcal{N}PO(\mathcal{U}, X)$ by theorems 3.5 and 3.7 and hence $A \in \mathcal{N}SO(\mathcal{U}, X) \cap \mathcal{N}PO(\mathcal{U}, X)$ . That is $\tau_R^\alpha(X) \subseteq \mathcal{N}SO(\mathcal{U}, X) \cap \mathcal{N}PO(\mathcal{U}, X)$ . Conversely, if $A \in \mathcal{N}SO(\mathcal{U}, X) \cap \mathcal{N}PO(\mathcal{U}, X)$ , then $A \subseteq \mathcal{N}Cl(\mathcal{N}Int(A))$ and $A \subseteq \mathcal{N}Int(\mathcal{N}Cl(A))$ . Therefore, $\mathcal{N}Int(\mathcal{N}Cl(A)) \subseteq \mathcal{N}Int(\mathcal{N}Cl(\mathcal{N}Cl(\mathcal{N}Int(A)))) = \mathcal{N}Int(\mathcal{N}Cl(\mathcal{N}Int(A)))$ . That is, $\mathcal{N}Int(\mathcal{N}Cl(A)) \subseteq \mathcal{N}Int(\mathcal{N}Cl(\mathcal{N}Int(A))))$ . Also $A \subseteq \mathcal{N}Int(\mathcal{N}Cl(A)) \subseteq \mathcal{N}Int(\mathcal{N}Cl(\mathcal{N}Int(A)))$ implies that $A \subseteq \mathcal{N}Int(\mathcal{N}Cl(\mathcal{N}Int(A)))$ . That is, $A \in \tau_R^\alpha(X)$ . Thus, $\mathcal{N}SO(\mathcal{U}, X) \cap \mathcal{N}PO(\mathcal{U}, X) \subseteq \tau_R^\alpha(X)$ . Therefore, $\tau_R^\alpha(X) = \mathcal{N}SO(\mathcal{U}, X) \cap \mathcal{N}PO(\mathcal{U}, X)$ .

**Theorem 3.10** *: If, in a nano topological space* $(U, \tau_R(X))$ , $L_R(X) = U_R(X) = X$ *, then* $U$ , $\phi$ , $L_R(X)(= U_R(X))$ *and any set* $A \supset L_R(X)$ *are the only nano-*$\alpha$ *-open sets.in* $U$ .

**Proof:** Since $L_R(X) = U_R(X) = X$ , the nano topology, $\tau_R(X) = \{U, \phi, L_R(X)\}$ . Since any nano-open set is nano-$\alpha$ -open, $U, \phi$ and $L_R(X)$ are nano $\alpha$ -open in $U$ . If $A \subset L_R(X)$ , then $N\,Int(A) = \phi$ , since $\phi$ is the only nano-open subset of A. Therefore $N\,Cl(N\,Int(A)) = \phi$ and hence A is not nano $\alpha$ -open. If $A \supset L_R(X)$ , $L_R(X)$ is the largest nano-open subset of A and hence, $N\,Int(N\,Cl(N\,Int(A))) = N\,Int(N\,Cl(L_R(X))) = N\,Int(B_R(X)^C) = N\,Int(U)$ , since $B_R(X) = \phi$ . Therefore, $N\,Int(N\,Cl(N\,Int(A))) = U$ and hence, $A \subseteq N\,Int(N\,Cl(N\,Int(A)))$ . Therefore, A is nano $\alpha$ -open. Thus $U$ , $\phi$ , $L_R(X)$ and any set $A \supset L_R(X)$ are the only nano $\alpha$ -open sets in $U$ , if $L_R(X) = U_R(X)$ .

**Theorem 3.11** *:* $U$ , $\phi$ , $U_R(X)$ *and any set* $A \supset U_R(X)$ *are the only nano* $\alpha$ *-open sets in a nano-topological space* $(U, \tau_R(X))$ *if* $L_R(X) = \phi$ .

**Proof:** Since $L_R(X) = \phi$ , $B_R(X) = U_R(X)$ . Therefore, $\tau_R(X) = \{U, \phi, U_R(X)\}$ and the members of $\tau_R(X)$ are nano $\alpha$ -open in $U$ . Let $A \subset U_R(X)$ . Then $N\,Int(A) = \phi$ and hence $N\,Int(N\,Cl(N\,Int(A))) = \phi$ . Therefore A is not nano $\alpha - open$ in $U$ . If $A \supset U_R(X)$ , then $U_R(X)$ is the largest nano-open subsetof A (unless, $U_R(X) = U$ , in case of which $U$ and $\phi$ are the only nano-open sets in $U$ ). Therefore, $N\,Int(N\,Cl(N\,Int(A))) = N\,Int(N\,Cl(U_R(X))) = N\,Int(U)$ and hence $A \subseteq N\,Int(N\,Cl(N\,Int(A)))$ . Thus, any set $A \supset U_R(X)$ is nano $\alpha$ -open in $U$ . Hence, $U$ , $\phi$ , $U_R(X)$ and any superset of $U_R(X)$ are the only nano $\alpha$ -open sets in $U$

**Theorem 3.12** *: If* $U_R(X) = U$ *and* $L_R(X) \neq \phi$ *, in a nano topological space* $(U, \tau_R(X))$ *, then* $U, \phi, L_R(X)$ *and* $B_R(X)$ *are the only nano* $\alpha$ *-open sets in* $U$ .

**Proof:** Since $U_R(X) = U$ and $L_R(X) \neq \phi$ , the nano-open sets in $U$ are $U, \phi, L_R(X)$ and $B_R(X)$ and hence they are nano $\alpha$ -open also. If $A = \phi$ ,then A is nano $\alpha$ -open. Therefore, let $A \neq \phi$ . When $A \subset L_R(X)$ , $N\,Int(A) = \phi$ , since the largest open subset of A is $\phi$ and hence $A \not\subset N\,Int(N\,Cl(N\,Int(A)))$ , unless A is $\phi$ . That is, A is not nano $\alpha$ -open in $U$ . When $L_R(X) \subset A$ , $N\,Int(A) = L_R(X)$ and therefore, $N\,Int(N\,Cl(N\,Int(A))) = N\,Int(N\,Cl(L_R(X))) = N\,Int(B_R(X)^C) = N\,Int(L_R(X)) = L_R(X) \subset A$ . That is, $A \not\subset N\,Int(N\,Cl(N\,Int(A)))$ . Therefore, A is not nano $\alpha$ -open in $U$ . Similarly, it can be shown that any set $A \subset B_R(X)$ and $A \supset B_R(X)$ are not nano $\alpha$ -open in $U$ . If A has atleast one element each of $L_R(X)$ and $B_R(X)$ , then $N\,Int(A) = \phi$ and hence A is not nano $\alpha$ -open in $U$ . Hence, $U$ , $\phi$ , $L_R(X)$ and $B_R(X)$ are the only nano $\alpha$ -open sets in $U$ when $U_R(X) = U$ and $L_R(X) \neq \phi$ .

**Corollary 3.13** *:* $\tau_R(X) = \tau_R^{\alpha}(X)$ *, if* $U_R(X) = U$ .

**Theorem 3.14** *: Let* $L_R(X) \neq U_R(X)$ *where* $L_R(X) \neq \phi$ *and* $U_R(X) \neq U$ *in a nano topological space* $(U, \tau_R(X))$ *. Then* $U$ , $\phi$ , $L_R(X)$ , $B_R(X)$ , $U_R(X)$ *and any set* $A \supset U_R(X)$ *are the only nano* $\alpha$ *-open sets in* $U$ .

**Proof:** The nano topology on $U$ is given by $\tau_R(X) = \{U, \phi, L_R(X), B_R(X), U_R(X)\}$ and hence $U$ , $\phi$ , $L_R(X)$ , $B_R(X)$ and $U_R(X)$ are nano $\alpha$ -open in $U$ . Let $A \subseteq U$ such that $A \supset U_R(X)$ . Then

$N Int(A) = U_R(X)$ and therefore, $N Int(N Cl(U_R(X))) = N Int(U) = U$. Hence, $A \subseteq N Int(N Cl(N Int(A))$. Therefore, any $A \supset U_R(X)$ is nano $\alpha$-open in $U$. When $A \subset L_R(X)$, $N Int(A) = \phi$ and hence $N(Int(N Cl(N Int(A))) = \phi$. Therefore, A is not nano $\alpha$-open in $U$. When $A \subset B_R(X)$, $N Int(A) = \phi$ and hence A is not nano $\alpha$-open in $U$. When $A \subset U_R(X)$ such that A is neither a subset of $L_R(X)$ nor a subset of $B_R(X)$, $N Int(A) = \phi$ and hence A is not nano $\alpha$-open in $U$. Thus, $U, \phi, L_R(X), B_R(X), U_R(X)$ and any set $A \supset U_R(X)$ are the only nano $\alpha$-open sets in $U$.

## IV. FORMS OF NANO SEMI-OPEN SETS AND NANO REGULAR OPEN SETS

In this section, we derive forms of nano semi-open sets and nano regular open sets depending on various combinations of approximations.

**Remark 4.1** $U$, $\phi$ *are obviously nano semi-open, since* $N Cl(N Int(U)) = U$ *and* $N Cl(N Int(\phi)) = \phi$

**Theorem 4.2** *If, in a nano topological space* $(U, \tau_R(X))$, $U_R(X) = L_R(X)$, *then* $\phi$ *and sets A such that* $A \supseteq L_R(X)$ *are the only nano semi-open subsets of* $U$

**Proof:** $\tau_R(X) = \{U, \phi, L_R(X)\}$. $\phi$ is obviously nano semi-open. If A is a non- empty subset of $U$ and $A \subset L_R(X)$, then $N Cl(N Int(A)) = N Cl(\phi) = \phi$. Therefore, A is not nano semi-open, if $A \subset L_R(X)$. If $A \supseteq L_R(X)$, then $N Cl(N Int(A)) = N Cl(L_R(X)) = U$, since $L_R(X) = U_R(X)$. Therefore, $A \subseteq N Cl(N Int(A))$ and hence A is nano semi-open. Thus $\phi$ and sets containing $L_R(X)$ are the only nano semi-open sets in $U$, if $L_R(X) = U_R(X)$.

**Theorem 4.3** *If* $L_R(X) = \phi$ *and* $U_R(X) \neq U$, *then only those sets contianing* $U_R(X)$ *are the nano semi-open sets in* $U$.

**Proof:** $\tau_R(X) = \{U, \phi, U_R(X)\}$. Let A be a non-empty subset of $U$. If $A \subset U_R(X)$, then $N Cl(N Int(A)) = N Cl(\phi) = \phi$ and hence $A \not\subset N Cl(N Int(A))$. Therefore, A is not nano semi-open in $U$. If $A \supseteq U_R(X)$, then $N Cl(N Int(A)) = N Cl(U_R(X)) = U$ and hence $A \subseteq N Cl(N Int(U))$. Therefore, A is nano semi-open in $U$. Thus, only the sets A such that $A \subseteq U_R(X)$ are the only nano semi-open sets in $U$.

**Theorem 4.4** *If* $U_R(X) = U$ *is a nano topological space, then* $U$, $\phi$, $L_R(X)$ *and* $B_R(X)$ *are the only nano semi-open sets in* $U$.

**Proof:** $\tau_R(X) = \{U, \phi, L_R(X), B_R(X)\}$. Let A be a non-empty subset of $U$. If $A \subset L_R(X)$, then $N Cl(N Int(A)) = \phi$ and hence A is not nano semi-open in $U$. If $A = L_R(X)$, then $N Cl(N Int(A)) = N Cl(L_R(X)) = L_R(X)$ and hence, $A \subseteq N Cl(N Int(A))$. Therefore, A is nano semi-open in $U$. If $A \supset L_R(X)$, then $N Cl(N Int(A)) = N Cl(L_R(X)) = L_R(X)$. Therefore, $A \not\subset N Cl(N Int(A))$ and hence A is not nano semi-open in $U$. If $A \subset B_R(X)$, $N Cl(N Int(A)) = \phi$ and hence A is not nano semi-open in $U$. If $A = B_R(X)$, then $N Cl(N Int(A)) = N Cl(B_R(X)) = B_R(X)$ and hence $A \subseteq N Cl(N Int(A))$. Therefore, A is nano semi-open in $U$. If $A \supset B_R(X)$, then $N Cl(N Int(A)) = N Cl(B_R(X)) = B_R(X) \subset A$ and hence A is not nano semi-open in $U$. If A has atleast one element of $L_R(X)$ and atleast one element of $B_R(X)$, then $N Cl(N Int(A)) = N Cl(\phi) = \phi$ and hence A is not nano semi-open in $U$. Thus, $U, \phi, L_R(X)$ and $B_R(X)$ are the only nano semi-open sets in $U$, if $U_R(X) = U$ and $L_R(X) \neq \phi$. If $L_R(X) = \phi$, $U$ and $\phi$ are the only nano semi-open sets in $U$, since $U$

and $\phi$ are the only sets in $\mathcal{U}$ which are nano-open and nano-closed.

**Theorem 4.5** *If $L_R(X) \neq U_R(X)$ where $L_R(X) \neq \phi$ and $U_R(X) \neq \mathcal{U}$, then $\mathcal{U}$, $\phi$, $L_R(X)$, $B_R(X)$, sets containing $U_R(X)$, $L_R(X) \cup B$ and $B_R(X) \cup B$ where $B \subseteq (U_R(X))^C$ are the only nano semi-open sets in $\mathcal{U}$.*

**Proof:** $\tau_R(X) = \{\mathcal{U}, \phi, L_R(X), U_R(X), B_R(X)\}$. Let A be a non-empty, proper subset of $\mathcal{U}$. If $A \subset L_R(X)$, then $N\,int(A) = \phi$ and hence, $N\,cl(N\,int(A)) = \phi$. Therefore, A is not nano semi-open in $\mathcal{U}$. If $A = L_R(X)$, then $N\,cl(N\,int(A)) = N\,cl(L_R(X)) = L_R(X) \cup [U_R(X)]^C$ and hence $A \subseteq N\,cl(N\,int(A))$. Therefore, $L_R(X)$ is nano semi-open in $\mathcal{U}$. If $A \subset B_R(X)$, then $N\,int(A) = \phi$ and hence A is not nano semi-open in $\mathcal{U}$. If $A = B_R(X)$, then $N\,cl(N\,int(A)) = N\,cl(B_R(X)) = B_R(X) \cup [U_R(X)]^C$ and hence $A \subseteq N\,cl(N\,int(A))$. Therefore, $B_R(X)$ is nano semi-open in $\mathcal{U}$. Since $L_R(X)$ and $B_R(X)$ are nano semi-open, $L_R(X) \cup B_R(X) = U_R(X)$ is also nano semi-open in $\mathcal{U}$. Let $A \subset U_R(X)$ such that A has atleast one element each of $L_R(X)$ and $B_R(X)$. Then $N\,int(A) = \phi$ or $L_R(X)$ or $B_R(X)$ and consequently, $N\,cl(N\,int(A)) = \phi$ or $L_R(X) \cup [U_R(X)]^C$ or $B_R(X) \cup [U_R(X)]^C$ and hence $A \not\subseteq N\,cl(N\,int(A))$. Therefore, A is not nano semi-open in $\mathcal{U}$. If $A \supset U_R(X)$, then $N\,cl(N\,int(A)) = N\,cl(U_R(X)) = \mathcal{U}$ and hence $A \subseteq N\,cl(N\,int(A))$. Therefore, A is nano semi-open. If A has a single element each of $L_R(X)$ and $B_R(X)$ and atleast one element of $[U_R(X)]^C$, then $N\,int(A) = \phi$. Then, $N\,cl(N\,int(A)) = \phi$ and hence A is not nano semi-open in $\mathcal{U}$. Similarly, when A has a single element of $L_R(X)$ and atleast one element of $[U_R(X)]^C$, or a single element of $B_R(X)$ and atleast one element of $[U_R(X)]^C$ then $N\,cl(N\,int(A)) = \phi$ and hence A is not nano semi-open in $\mathcal{U}$. When $A = L_R(X) \cup B$ where $B \subseteq [U_R(X)]^C$, then $N\,cl(N\,int(A)) = N\,cl(L_R(X)) = L_R(X) \cup [U_R(X)]^C \supseteq A$. Therefore, A is nano semi-open in $\mathcal{U}$. Similarly, if $A = B_R(X) \cup B$ where $B \subseteq (U_R(X))^C$, then $N\,cl(N\,int(A)) = B_R(X) \cup [U_R(X)]^C \supseteq A$. Therefore, A is nano semi-open in $\mathcal{U}$. Thus, $\mathcal{U}$, $\phi$, $L_R(X)$, $U_R(X)$, $B_R(X)$, any set containing $U_R(X)$, $L_R(X) \cup B$ and $B_R(X) \cup B$ where $B \subseteq [U_R(X)]^C$ are the only nano semi-open sets in $\mathcal{U}$.

**Theorem 4.6** *If A and B are nano semi-open in $\mathcal{U}$, then $A \cup B$ is also nano semi-open in $\mathcal{U}$.* **Proof:** If A and B are nano semi-open in $\mathcal{U}$, then $A \subseteq N\,Cl(N\,Int(A))$ and $B \subseteq N\,Cl(N\,Int(A))$. Consider $A \cup B \subseteq N\,Cl(N\,Int(A)) \cup N\,Cl(N\,Int(B)) = N\,Cl(N\,Int(A) \cup N\,Int(B)) \subseteq N\,Cl(N\,Int(A \cup B))$ and hence $A \cup B$ is nano semi-open.

**Remark 4.7** *If A and B are nano semi-open in $\mathcal{U}$, then $A \cap B$ is not nano semi-open in $\mathcal{U}$. For example, let $\mathcal{U} = \{a, b, c, d\}$ with $\mathcal{U}/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\}$. Then $\tau_R(X) = \{\mathcal{U}, \phi, \{a\}, \{a, b, d\}, \{b, d\}\}$. The nano semi-open sets in $\mathcal{U}$ are $\mathcal{U}$, $\phi$, $\{a\}$, $\{a, c\}$, $\{b, d\}$, $\{a, b, d\}$, $\{b, c, d\}$. If $A = \{a, c\}$ and $B = \{b, c, d\}$, then A and B are nano semi-open but $A \cap B = \{c\}$ is not nano semi-open in $\mathcal{U}$.*

**Definition 4.8** *A subset A of a nano topological space $(U, \tau_R(X))$ is nano-regular open in U, if $N\,Int(N\,Cl(A)) = A$.*

**Example 4.9** *Let $U = \{x, y, z\}$ and $U/R = \{\{x\}, \{y, z\}\}$. Let $X = \{x, z\}$. Then the nano topology on U with respect to X is given by $\tau_R(X) = \{U, \phi, \{x\}, \{y, z\}\}$. The nano closed sets are $U, \phi, \{y, z\}, \{x\}$. Also, $N\,Int(N\,Cl(A)) = A$ for $A = U, \phi, \{x\}$ and $\{y, z\}$ and hence these sets are nano regular open in U.*

**Theorem 4.10** *Any nano regular open set is nano-open.*

**Proof:** If A is nano regular open in $(U, \tau_R(X))$, $A = N Int(N Cl(A))$. Then $N Int(A) = N Int(N Int(N Cl(A))) = N Int(N Cl(A)) = A$. That is, A nano-open in U.

**Remark 4.11** *The converse of the above theorem is not true. For example, let* $U = \{a, b, c, d, e\}$ *with* $U/R = \{\{a, b\}, \{c, e\}, \{d\}\}$. *Let* $X = \{a, d\}$. *Then* $\tau_R(X) = \{U, \phi, \{d\}, \{a, b, d\}, \{a, b\}\}$ *and the nano closed sets are* $U, \phi, \{a, b, c, e\}, \{c, e\}, \{c, d, e\}$. *The nano regula open sets are* $U, \phi, \{d\}$ *and {a,b}. Thus, we note that {a,b,d} is nano-open but is not nano regular open. Also, we note that the nano regular open sets do not form a topology, since* $\{d\} \cup \{a, b\} = \{a, b, d\}$ *is not nano regular open, even though {d} and {a,b} are nano regular.*

**Theorem 4.12** *In a nano topological space* $(U, \tau_R(X))$ *,if* $L_R(X) \neq U_R(X)$, *then the only nano regular open sets are* $U, \phi, L_R(X)$ *and* $B_R(X)$.

**Proof:** The only nano-open sets $(U, \tau_R(X))$ are $U, \phi, L_R(X), U_R(X)$ and $B_R(X)$ and hence the only nano closed sets in $U$ are $U, \phi, [L_R(X)]^C, [U_R(X)]^C$ and $[B_R(X)]^C$ which are respectively $U, \phi, U_R(X^C)$, $L_R(X^C)$ and $L_R(X) \cup L_R(X^C)$.

**Case 1:** Let $A = L_R(X)$ .Then $N CL(A) = [B_R(X)]^C$. Therefore, $N Int(N Cl(A)) = N Int[B_R(X)]^C$ $=[N Cl(B_R(X))]^C = [(L_R(X))^C]^C = L_R(X) = A$. Therefore, $A = L_R(X)$ is nano-regular open.

**Case 2:** Let $A = B_R(X)$. Then $N Cl(A) = [L_R(X)]^C$, Then $N Int(N Cl(A)) = N Int[L_R(X)]^C =$ $[N Cl(L_R(X))]^C = [[B_R(X)]^C]^C = B_R(X) = A$. That is, $A = B_R(X)$ is nano regular open.

**Case 3:** If $A = U_R(X)$, then $N Cl(A) = U$. Therefore, $N Int(N Cl(A)) = N Int(U) = U \neq A$. That is, $A = U_R(X)$ is not nano regular open unless $U_R(X) = U$.

**Case 4:** Since $N Int(N Cl(U)) = U$ and $N Int(N Cl(\phi)) = \phi$, U and $\phi$ are nano regular open. Also any nano regular open set is nano-open. Thus, $U, \phi, L_R(X)$ and $B_R(X)$ are the only nano regular open sets .

**Theorem 4.13** *: In a nano topological space* $(U, \tau_R(X))$ *,if* $L_R(X) = U_R(X)$, *then the only nano regular open sets are* $U$ *and* $\phi$.

**Proof:** The nano- open sets in $U$ are $U, \phi$ and $L_R(X)$ And $N Int(N Cl(L_R(X))) = U \neq L_R(X)$ Therefore, $L_R(X)$ is not nano regular open. Thus, the only nano regular open sets are $U$ and $\phi$.

**Corollary:** If A and B are two nano regular open sets in a nano topological space, then $A \cap B$ is also nano regular open.

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