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Contra Pairwise Continuity in Bitopological Spaces

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ABSTRACT: Erdal Ekici in a paper [6] has been introduced a new class of functions, called Contra R-map and proved various results related with Contra R-maps and nearly compact Spaces, S-closed spaces, contra R-graphs, connected spaces, hyper connected spaces etc. By introducing the notion of contra pairwise continuity in bitopological spaces we obtain generalization of various results of Erdal Ekici.

KEY WORDS AND PHRASES : Bitopological space, contra pairwise continuity, pairwise super continuity, contra pairwise R-map, pairwise almost s-continuous functions.
1990 AMS SUBJECT CLASSIFICATION: 54 E 55.

1. INTRODUCTION :

Noiri [10] introduced the concepts of δ-continuity and strong θ-continuity in topological spaces. "Recall a function $f:(X, \tau) \rightarrow (Y, r)$ is δ-continuous if for each $x \in X$ and $V \in r$ containing $f(x)$ then there is a $U \in \tau$ containing x such that $f(\text{int}(\text{Cl}U)) \subset \text{intCl}(V)$. Munsi and Bassan introduced a strong form of continuity, called super continuous mappings, which implies δ-continuity and is implied by strong θ-continuity. In (1983a) Reilly and Vamanamurty [14] have investigated super continuous functions by relating these functions with continuous functions. Continuing work in the same direction Arya and Gupta [1] introduced the class of completely continuous functions and obtained several preservation results for topological properties with respect to these functions.

Velicko [17] introduced the notion of δ-open and δ-closed sets in a space (X, τ).

In a topological space (X, τ) a set A is called regular open if $A = int(Cl(A))$ and regular closed if $A= Cl(int(A))$. Since the inter-section of two regular open sets is regular open, the family of regular open sets forms a base for a smaller topology τ_s on X, called the semi regularization of τ. The space (X, τ) is said to be semi-regular if $\tau_s = \tau$. Any semi regular space is semi regular, but the converse is not true.

Remark 2.3 of [11] is : for a continuous function $f:X\rightarrow Y$ gives the following implications:

δ-Closedness $\rightarrow$ Star-Closedness $\rightarrow$ almost-closedness and

Remark 2.6 of [10] is : for a function $f:X\rightarrow Y$ gives the following implications :-

$$\begin{tikzcd}
 & \text{Continuity} \arrow[dr] & \\
\text{SC} \arrow[ur] \arrow[dr] & & \text{almost continuity} \\
 & \delta\text{-Continuity} \arrow[ur] &
\end{tikzcd}$$

"Recall a function $f:X\rightarrow Y$ is said to be super continuous (Munshi and Bassan 1982] (resp. δ-continuous (Noiri 1980)), almost continuous (Singal and Singal 1968) if for each $x\in X$ and each open nbd V of f(x), there exists an open nbd U of x such that $f\{int(Cl(U))\} \subset V$ [resp. $f\{Int(Cl(U))\} \subset Int(Cl(V)$, $f(U) \subset Int(Cl(V))$ "Super Continuous" will be briefly denoted by "SC".

"Recall a function f: X→Y is called a R-map (Carnhan 1973) if $f^{-1}(V)$ is regular open in X for each regular open set V of Y."

Example : Let X = {a, b, c} and τ ={ϕ, {a}, {b},{a,b}, X}. Let f: (X, τ) → (X, τ) be the identity function. Then f is super continous (12) but it is not strongly θ-continuous by proposition (3) of Reilly and Vamanmurty (14). Moreover, f is an R-map, but it is not completely continuous since {a, b} is not regular open in (X, τ).

Remark : By theorem 4.11 of T. Noiri [11] gives the following implications:

Strong Continuity → Perfect Continuity → Complete Continuity → R-map → δ-Continuity.

$$\begin{tikzcd}
 & \text{CoC} \arrow[r] & \text{S.Q.C.} \arrow[rr] & & \text{SC} \arrow[r] \arrow[dll] \arrow[d] & \text{C} \arrow[d] \\
\text{SC} \arrow[r, dash] \arrow[ur] & \text{PC} \arrow[r] \arrow[u] \arrow[ur] & \text{C C} \arrow[r] & \text{R} \arrow[r] & \delta\text{-C} \arrow[r] & \text{aC}
\end{tikzcd}$$

where CoC = Cl-open Continuous, SθC = Strongly θ-Continuous, C = Continuous, SC = Strongly Continuous, PC=Perfectly Continuous, CC=Completely Continuous, R = R-map, δC = δ-Continuous ac = almost continuous

Now it is clear that there are many types of continuities introduced by several authors. A new class of function, called contra R-map has been defined and studied in[6]. It is shown that contra R-map is strictly weaker than regular set connectedness. Erdal Ekici [6] investigated the relationships among contra R-map and nearly compact spaces, S-closed spaces, contra R-graphs connected spaces hyper connected spaces etc. and discuss various properties of such spaces.In this paper we introduced a new class of functions called pairwise contra R-map in bitopological space and have got many theorems giving a generalization to Erdal's theorems by using the tools of pairwise perfectly continuous functions, pairwise almost S-continuous functions and pairwise regular set-connected functions etc.

"A bitopological space is a triple (X, τ_1,τ_2), where X endowed with two topologies τ_1 and τ_2. The study of bitopological was initiated by J.C. Kelly and was further pursued by various authors including T. Birsan, H.B. Hoyle, C.W. Patty, W.J. Patty, I.L. Reilly, J. Swart etc."

Throughout by a space we mean a bitopological space. Symbols X and Y are used for spaces and f is used for maps between bitopological spaces. For terms and notation not explained here we refer the reader to [6,7, 8,12,15].

II. PRELIMINARIES :

Throughout this paper (X, τ_1, τ_2) and (Y,r_1,r_2) represent bitopological spaces. Let A be a subset of a space (X, τ_1, τ_2), then

τ_1-Cl(A) represents τ_1-closure of A

τ_2-Cl(A) represents τ_2-closure of A

τ_1-Int(A) represents τ_1-interior of A

τ_2-Int(A) represents τ_2-interior of A

$A\subset \tau_1 cl\ (\tau_1\ int(A))$ represents τ_1-semi open set

$A \subset \tau_2 cl\ (\tau_2\ int(A))$ represents τ_2-semi open set

"The complement of a τ_1-semi open set (respectively τ_2–semi open) is called τ_1-semi closed (respectively τ_2-semi closed)."

"The intersection of all τ_1-semi closed sets (respectively τ_2-semi closed) containing A is called the τ_1-semi-closure (respectively τ_2-semi closure) of A and is denoted by τ_1s-Cl(A)(respectively s-τ_2Cl(A))."

"The τ_1-semi interior (respectively τ_2- semi interior) of A is defined by the union of all τ_1-semi open sets (respect. τ_2-semi open) contained in A and is denoted by (s-τ_1-int(A) (s-τ_2-int(A))."

"A subset A is said to be τ_1-regular open (resp. τ_2-regular open) if A = (τ_1-int(τ_1Cl(A)), resp. A = (τ_2-int(τ_2Cl(A))."

"A is said to be τ_1-regular closed (resp. τ_2-regular closed) is A= τ_1-Cl(τ_1-int(A)). (resp. A = τ_2- Cl(τ_2-int.(A))."

"The τ_1-δ-interior (resp. τ_2-δ-interior) of subset A is the union of all regular τ_1-open sets (resp. pairwise regular τ_2-open sets) of (X, τ_1) (respect (X, τ_2)) contained in A is denoted by τ_1-δ-int(A) (resp. τ_2-δ-int(A)).

"A subset A is said to be τ_1-δ-open (resp. τ_2-δ-open if A = τ_1-δ-int(A) (resp. A = τ_2-δ-int(A)) i.e. a set is τ_1-δ-open (resp. τ_2-δ-open) if it is the union of pairwise regular τ_1-open (resp. pairwise regular τ_2-open) sets. The complement of τ_1-δ-open (resp. τ_2-δ-open) set is called τ_1-δ-closed (res. τ_2-δ-closed).

Alternatively, a set A of (X, τ_1, τ_2) is said to be τ_1-δ-closed if A = τ_1-δ-Cl(A), where τ_1-δ-Cl(A) = $\{x\in(X, \tau_1): A\cap\tau_1\text{-int}(Cl(U)) \neq \phi,\ U\in\tau_1$ and $x\in U\}$ or τ_2-δ-closed if A = τ_2-δ-Cl(A), where τ_2-δ- Cl(A) = $\{x\in(X, \tau_2): A\cap\tau_2\text{-int}(Cl(V) \neq \phi,\ V\in \tau_2$ and $x\in V\}$.

III. CONTRA-PAIRWISE-CONTINUITY

3.1 Definition : A function f:(X, τ_1, τ_2) $\rightarrow$ (Y, r_1, r_2) is said to be **pairwise super continuous** if f^{-1} (V_1) is τ_1-δ-open and f^{-1}(V_2) is τ_2-δ-open in (X, τ_1) and (X, τ_2) respectively, for every τ_1-open set V_1 of (Y, r_1) and τ_2-open set V_2 of (Y, r_2).

3.2 Definition : A function $f:(X, \tau_1, \tau_2) \rightarrow (Y, r_1, r_2)$ is said to be **pairwise δ-continuous** if $f^{-1}(V_1)$ is τ_1-δ-open in (X, τ_1) and $f^{-1}(V_2)$ is τ_2-δ-open in (X, τ_2) for every $V_1 \in PRO(Y, r_1)$ and $V_2 \in PRO(Y, r_2)$.

The family of all pairwise regular open (resp. pairwise regular closed, pairwise semi open) set of a space (X, τ_1) and (X, τ_2) is denoted by $PRO(X, \tau_1)$ and $PRO(X, \tau_2)$ respectively {resp. $PRC(X, \tau_1)$ and $PRC(X, \tau_2)$, $PSO(X, \tau_1)$ and $PSO(X, \tau_2)$}. The family of pairwise regular open (resp. pairwise regular closed, pairwise semi open) of (X, τ_1) and (X, τ_2) containing $x \in (X, \tau_1, \tau_2)$ is denoted by $PRO(X, \tau_1, x)$ and $PRO(X, \tau_2, x)$ respectively (resp. $PRC(X, \tau_1, x)$ and $PRC(X, \tau_2, x)$, $PSO(X, \tau_1, x)$ and $PSO(X, \tau_2, x)$).

3.3 Definition : Pairwise Perfectly Continuous : A function $f:(X, \tau_1, \tau_2) \rightarrow (Y, r_1, r_2)$ is said to be pairwise perfectly continuous if $f^{-1}(V_1)$ is τ_1-clopen in (X, τ_1) and $f^{-1}(V_2)$ is τ_2-clopen in (X, τ_2) for every r_1-open set V_1 of (Y, r_1) and r_2-open set V_2 of (Y, r_2) respectively.

3.4 Definition : A function $f:(X, \tau_1, \tau_2) \rightarrow (Y, r_1, r_2)$ is said to be **pairwise regular set connected** if $f^{-1}(V_1)$ is τ_1-clopen in (X, τ_1) and $f^{-1}(V_2)$ is τ_2-clopen in (X, τ_2) for every $V_1 \in PRO(Y, r_1)$ and $V_2 \in PRO(Y, r_2)$.

3.5 Definition : A function $f:(X, \tau_1, \tau_2) \rightarrow (Y, r_1, r_2)$ is said to be pairwise almost s-continuous if for each $x \in (X, \tau_1, \tau_2)$, each $V_1 \in PSO(Y, f(x), r_1)$ and $V_2 \in PSO(Y, f(x), r_2)$ then there exists an τ_1-open set U_1 in (X, τ_1) and τ_2-open set U_2 in (X, τ_2) respectively containing x such that $f(U_1) \subset s\text{-}\tau_1 Cl(V_1)$ and $f(U_2) \subset s\text{-}\tau_2 cl(V_2)$.

3.6 Definition : A function $f:(X, \tau_1, \tau_2) \rightarrow (Y, r_1, r_2)$ is said to be **contra pairwise R-map** if $f^{-1}(V_1)$ is pairwise regular τ_1-closed in

(X, τ_1) and $f^{-1}(V_2)$ is pairwise regular τ_2-closed in (X, τ_2) for every pairwise regular r_1-open set V_1 of (Y, r_1) and pairwise regular r_2 open set V_2 of (Y, r_2) respectively.

3.7 Remark : For a function $f:(X, \tau_1, \tau_2) \rightarrow (Y, r_1, r_2)$ we have the following implications:

$$\begin{array}{c} \text{Pairwise Perfectly Continuous} \\ \downarrow \\ \text{Pairwise regular set connected} \rightarrow \text{Contra Pairwise R-map} \\ \uparrow \\ \text{Pairwise almost s-Continuous} \end{array}$$

3.8 Theorem : For a function $f:(X, \tau_1, \tau_2) \rightarrow (Y, r_1, r_2)$, following are equivalent:

(1) f is contra pairwise R-map

(2) the inverse image of a pairwise regular r_1-closed set of (Y, r_1) and pairwise regular r_2-closed set of (Y, r_2) is pairwise regular τ_1-open and τ_2-open respectively.

(3) $f^{-1}(\tau_1\text{-int}(\tau_1 Cl(G_1))$ is pairwise regular τ_1-closed for every r_1-open subset G_1 of (Y, r_1) and f^{-1} $(\tau_2\text{-int}$ $(\tau_2\text{-cl}(G_2)))$ is pairwise regular τ_2-closed for every r_2-open subset G_2 of (Y, r_2) .

(4) $f^{-1}\{\tau_1 Cl(\tau_1\text{-int}(F_1))\}$ is pairwise regular τ_1-open for every r_1-closed subset F_1 of (Y, r_1) and $f^{-1}\{\tau_2 Cl(\tau_2\text{ -int}(F_2))\}$ is pairwise regular τ_2-open for every r_2-closed subset F_2 of (Y, r_2).

Proof : 1 ⇔ 2 : Let $f:(X, \tau_1, \tau_2) \rightarrow (Y, r_1, r_2)$ be a function. Let F_1 be any pairwise regular r_1-closed set of (Y, r_1) and F_2 be pairwise regular r_2-closed set of (Y, r_2). Then $(Y, r_1)\backslash F_1 \in PRO\ (Y, r_1)$ and $(Y, r_2)\backslash F_2 \in PRO\ (Y, r_2)$. By (1) $f^{-1}\{(Y, r_1)\backslash F_1\} = (X, \tau_1)/f^{-1}(F_1) \in PRC(Y, r_1)$

and we have $f^{-1}(F_1) \in PRO(Y, r_1)$ and $f_{-1}(Y, r_2)\backslash F_2) = (X, \tau_2)\backslash f^{-1}(F_2) \in PRC(Y, r_2)$, we have $f^{-1}(F_2) \in PRO(Y, r_2)$.

(1) ⇔ (3) Let G_1 be an r_1-open set (Y, r_1) and G_2 be r_2-open set (Y, r_2). Since r_1-Int$\{r_1Cl(G_1)\}$ and r_2-Int$\{r_2cl(G_2)\}$ are pairwise regular r_1-open and pairwise regular r_2-open, then by (1), it follows that $f^{-1}\{\tau_1\text{-int}(\tau_1Cl(G_1)\}$ is pairwise regular τ_1-closed in (X, τ_1) and $f^{-1}\{\tau_2$ int$(\tau_2Cl(G_2)\}$ is pairwise regular τ_2-closed in (X, τ_2).

(2) ⇔(4) : it can be obtained similar as (1) ⇔ (3).

For two topological space X and Y and any function $f:X \rightarrow Y$, the subset $\{x,f(x)$ of the product space $[X\times Y]$ is called the graph of f.

For topological spaces Long and Herrington [8] defined a function $f:X \rightarrow Y$ to have a strongly closed graph is for each $(x, y) \notin G(f)$ there exists open sets U and V containing x and y, respectively, such that $[U\times V] \cap G(f) = \phi$.

According to Cammaroto and Noiri this graph G(f) is γ-closed with respect to Y is for every $(x, y) \notin G(f)$ there exists $U \in u_x$ and $V \in u$ (u_y) such that $[U\times V] \cap G(f) = \phi$.

In [16] we investigated the conditions for G(f) to be closed when the concerning spaces are bitopological spaces. We take the product $X\times Y$ as the bitopological space $(X\times Y, \tau_1\times r_2, \tau_2\times r_1)$ and define a function $f:(X, \tau_1, \tau_2) \rightarrow (Y, r_1, r_2)$ to have a pairwise θ-closed graph if G(f) is either θ-closed in $(X\times Y, \tau_1\times r_2)$ or θ-closed in $(X\times Y, \tau_2\times r_1)$ i.e. for each $(x, y) \notin G(f)$. Either there exists τ_1-open set U containing x and r_2-open set V containing y such that $\{\tau_1Cl(U)\times r_2Cl(V)\cap G(f)\} = \phi$ or there exists τ_2-open set S containing x and r_1-open set T containing y such that $\tau_2Cl(S) \times r_1Cl(T)\cap G(f)\} = \phi$.

3.9 Definition : A bitopological space (X, τ_1, τ_2) is said to be **pairwise weakly** T_2 if for each element of (X,τ_1,τ_2) is an intersection of pairwise regular closed sets.

3.10 Definition : A pairwise graph G(f) of function $f:(X, \tau_1, \tau_2) \rightarrow (Y, r_1, r_2)$ is said to be contra pairwise R-graph is for each $(x, y) \in \{X, \tau_1, \tau_2) \times (Y, r_1, r_2)\}\backslash G(f)$, there exists a pairwise regular τ_1-open set U_1 in (X, τ_1) and pairwise regular τ_2-open set U_2 in(X, τ_2) both containing x, there exists a pairwise regular r_1-closed set F_1 and pairwise regular r_2-closed set F_2, both containing y such that $(U_1\times F_2) \cap G(f) = \phi$ and $(U_2\times F_1) \cap G(f) = \phi$.

3.11 Theorem : The following properties are equivalent for a pairwise graph G(f) of a function $f:(X, \tau_1, \tau_2) \rightarrow (Y, r_1, r_2)$.

(1) G(f) is contra pairwise R-graph,

(2) for each $(x, y) \in (X\times Y, \tau_1\times r_2, \tau_2\times r_1)\backslash G(f)$, there exists a pairwise regular τ_1-open set U_1 and pairwise regular τ_2-open set U_2; both containing x and a pairwise regular r_1-closed set F_1 and pairwise regular r_2-closed set F_2 both containing y such $f(U_1) \cap F_2 = \phi$ and $f(U_2) \cap F_2 = \phi$.

Proof : The straight forward proof follows from definition itself is omitted.

3.12 Theorem : If $f:(X, \tau_1, \tau_2) \rightarrow (Y, r_1, r_2)$ is Contra pairwise R-map and (Y, r_1, r_2) is pairwise Urysohn, G(f) is Contra pairwise R-graph in $f:(X, \tau_1, \tau_2) \times (Y, r_1, r_2)$.

Proof : Suppose that (Y, r_1, r_2) is pairwise Urysohn. Let $(x, y) \in \{X, \tau_1, \tau_2) \times (Y, r_1, r_2)\backslash G(f)$. It follows that $f(x) \neq y$. Since Y is pairwise

Urysohn, there exists r_1-open sets V and r_2-open set W containing f(x) and y respectively, such that $r_2cl(V) \cap r_1Cl(W) = \phi$. Since f is contra pairwise R-map, there exists a pairwise τ_1-open set U_1 in (X, τ_1) and τ_2-open set U_2 in (X, τ_2) both containing x such that $f(U_1) \subset r_1Cl(W)$ and $f(U_2) \subset r_2cl(V)$.

Therefore, $f(U_1) \cap r_1Cl(W) = \phi$ and $f(U_2) \cap r_2Cl(W) = \phi$ and G(f) is contra pairwise R-graph in $(X, \tau_1, \tau_2) \times (Y, r_1, r_2)$.

3.13 Theorem : Let $f:(X, \tau_1, \tau_2) \rightarrow (Y, r_1, r_2)$ have a contra pairwise R-graph. If f is injective then (X, τ_1, τ_2) is pairwise T_1.

Proof : Let x and y be any two distinct points of (X, τ_1, τ_2). Then we have $(x, f(y)) \in \{(X, \tau_1, \tau_2) \times (Y, r_1, r_2)\} \setminus G(f)$. Then there exists $U_1 \in PRO(X,\tau_1,x)$, $U_2 \in PRO(X,\tau_2,x)$ and $F_1 \in PRC(Y, r_1, f(y))$, $F_2 \in PRC(Y, r_2, f(y))$ such that $f(U_1) \cap F_2 = \phi$ and $f(U_2) \cap F_1 = \phi$. Therefore, we get $y \notin U_1$ & U_2. This implies that (X, τ_1, τ_2) is pairwise T_1.

3.14 Theorem : Let $f:(X, \tau_1, \tau_2) \rightarrow (Y, r_1, r_2)$ have a contra pairwise R-graph. If f is surjective, then (Y, r_1, r_2) is pairwise weakly T_2.

Proof : Let x and y be two distinct points of (X, τ_1, τ_2). Since f is a contra pairwise R-graph, then we have $(x, f(y) \in (x \times y, \tau_1 \times r_2, \tau_2 \times r_1) \setminus G(f)$. Then there exists $U_1 \in PRO(X, \tau_1, x)$, $U_2 \in PRO(X, \tau_2, x)$ and $F_1 \in PRC(Y, r_1, f(y))$, $F_2 \in PRC(Y, r_2, f(y))$ such that $f(U_1) \cap F_2 = \phi$ and $f(U_2) \cap F_1 = \phi$; therefore $U_1 \cap f^{-1}(F_2) = \phi$ and $U_2 \cap f^{-1}(F_1) = \phi$, then we have $y \notin U_1 \& U_2$. this shows that (X, τ_1, τ_2) is pairwise T_1.

3.15 Theorem : Let $f:(X, \tau_1, \tau_2) \rightarrow (Y, r_1, r_2)$ have contra pairwise R-graph. If f is surjective, then (Y, r_1, r_2) is pairwise weakly T_2.

Proof : Let y_1 and y_2 be any two distinct points of (Y, r_1, r_2). Since f is surjective, $f(x) = y_1$ for some $x\in(X, \tau_1, \tau_2)$ and $(x, y_2) \in (X\times Y, \tau_1\times r_2, \tau_2\times r_1)\backslash G(f)$. Then there exists $U_1\in PRO(X, \tau_1, x)$, $U_2\in PRO (X, \tau_2, x)$ and $F_1\in PRC (Y, r_1, y_2)$, $F_2\in PRC (Y, r_2, y_2)$ such that :
$f_2(U_1) \cap F_2 = \phi$ and $f(U_2) \cap F_1 = \phi$; (since f is contra pairwise R-graph) hence $y_1\notin F_1$ and F_2. Then it is clear that (Y, r_1, r_2) is pairwise weakly T_2.

3.16 Theorem : If $f:(X, \tau_1, \tau_2) \rightarrow (Y, r_1, r_2)$ and $g:(X, \tau_1, \tau_2) \rightarrow (Y, r_1, r_2)$ are contra pairwise R-map and (Y, r_1, r_2) is pairwise Urysohn, then $E_1 = \{x\in(X, \tau_1) : f(x) = g(x)\}$ and $E_2 = \{x\in(X, \tau_2) : f(x) = g(x)\}$ are τ_1-closed in (X, τ_1) and τ_2-closed in (X, τ_2) respectively.

Proof : Suppose $x\in(X, \tau_1)\backslash E_1$ and $x\in(X, \tau_2)\backslash E_2$ then it follows that $f(x) \neq g(x)$. Since Y is pairwise Urysohn, there exists r_1-open sets V_1 and r_2-open set V_2 containing f(x) and g(x), respectively, such that $r_2Cl(V_1)\cap r_1Cl(V_2) = \phi$. Since f and g are contra pairwise R-map there exists a pairwise regular τ_1-open set U_1 and pairwise regular τ_2-open set U_2 containing x such that $f(U_1) \subset r_1Cl(V_2)$ and $g(U_2) \subset r_2Cl(V_1)$. Set $\alpha = U_1\cap U_2$ then, α is pairwise regular τ_1-open in (X, τ_1) and τ_2-open in (X, τ_2). Hence $f(\alpha)\cap g(\alpha) = \phi$ and it follows that $x\notin\delta\text{-}\tau_1Cl(E_2)$ and $\delta\text{-}\tau_2Cl(E_1)$. This implies that E_1 is τ_1-δ-closed in (X, τ_1) and E_2 is τ_2-δ-closed in (X, τ_2).

3.17 Theorem : If f is contra pairwise R-map injection and (Y, r_1, r_2) is pairwise Urysohn then (X, τ_1, τ_2) is pairwise Hausdorff.

Proof : Suppose that (Y, r_1, r_2) is pairwise Urysohn, by the injectivity of f, it follows that $f(x) \neq f(y)$ for any distinct points x and y in (X, τ_1, τ_2). Since (Y, r_1, r_2) is pairwise Urysohn, there exists r_1-open sets V_1 and r_2-open set V_2 containing f(x) and f(y), respectively such that $r_2Cl(V_1) \cap r_1Cl(V_2) = \phi$. Since f is contra pairwise R-map, there exists pairwise regular τ_1-open sets U_1 in (X, τ_1) and pairwise regular τ_2-open sets U_2 in (X, τ_2) containing x and y respectively. Such that $f(U_1) \subset r_1Cl(V_2)$ and $f(U_2) \subset r_2Cl(V_1)$.

Hence $U_1 \cap U_2 = \phi$. This shows that (X, τ_1, τ_2) is pairwise Hausdorff.

3.18 Theorem : If f is a contra pairwise R-map injection and (Y, r_1, r_2) is pairwise weakly T_2, then (X, τ_1, τ_2) is pairwise T_1.

Proof : Suppose that (Y, r_1, r_2) is pairwise weakly T_2. For any distinct points x and y in (X, τ_1, τ_2), there exists $V_1 \in PRC\ (Y, r_1)$ and $V_2 \in PRC\ (Y, r_2)$, such that $f(x) \in V_1$, $f(y) \notin V_1$ $f(x) \notin V_2$ and $f(y) \in V_2$. Since f is contra pairwise R-map $f^{-1}(V_1)$ is pairwise regular τ_1-open subset in (X, τ_1) and $f^{-1}(V_2)$ is pairwise regular τ_2-open subset in (X, τ_2) such that $x \in f^{-1}(V_1)$, $y \in f^{-1}(V_2)$, $y \notin f^{-1}(V_1)$ and $x \in f^{-1}(V_2)$. This shows that (X, τ_1, τ_2) is pairwise T_1.

3.19 Definitions : A space (X, τ_1, τ_2) is said to be

(1) Pairwise S-closed if every pairwise regular, pairwise closed cover of (X, τ_1, τ_2) has a finite sub cover.

(2) Pairwise countably S-closed if every pairwise countable cover of (X, τ_1, τ_2) by pairwise regular closed sets (w.r.t. τ_1, τ_2) has a finite subcover.

(3) Pairwise S-Lindelof if every pairwise cover of (X, τ_1, τ_2) by pairwise regular closed sets (w.r.t. τ_1, τ_2) has a finite pairwise countable sub cover.

(4) Nearly pairwise compact if every pairwise regular open cover of (X, τ_1, τ_2) has a finite subcover.

(5) Pairwise countably nearly pairwise compact if every pairwise countable cover of (X, τ_1, τ_2) by pairwise regular open sets (w.r.t. τ_1, τ_2) has a finite subcover.

(6) Nearly pairwise Lindelof if every pairwise regular open cover of (X, τ_1, τ_2) has a pairwise countable subcover.

3.20 Theorem : Let f: (X, τ_1, τ_2) $\rightarrow$ (Y, r_1, r_2) be a contra pairwise R-map surjection. Then the following statements are true:

(1) if (X, τ_1, τ_2) is nearly pairwise compact, then (Y, r_1, r_2) is pairwise S-closed.

(2) if (X, τ_1, τ_2) is nearly pairwise Lindelof; then (Y, r_1, r_2) is pairwise S-Lindelof.

(3) if (X, τ_1, τ_2) is pairwise countably nearly pairwise compact, then (Y, r_1, r_2) is pairwise countably S-closed.

Proof : Let {V_i and S_i : $i\in I$} be any pairwise regular closed cover of (Y, r_1) and (Y, r_2) respectively. Since f is contra pairwise R-map, then {$f^{-1}(V_i)$: $i\in I$} is a pairwise regular open cover of (X, τ_1) and {$f^{-1}(S_i)$: $i\in I$} is a pairwise regular open cover of (X, τ_2). Hence there exists a finite subset I_0 of I such that (X, τ_1) = $\bigcup$ {f^{-1}(Vi) : $i\in I_0$} and (X, τ_2) = $\bigcup$ {$f^{-1}(S_i)$: $i\in I_0$}}. Then, we have

(Y, r_1) = $\bigcup$ {V_i : $i\in I_0$} and

(Y, r_2) = $\bigcup$ {S_i : $i\in I_0$} and (Y, r_1, r_2) is pairwise S-closed.

The proofs of (2) and (3) is left for the reader.

3.21 Corollary : Let f:(X, τ_1, τ_2) $\rightarrow$ (Y, r_1, r_2) be a contra pairwise R-map surjection. Then the following are true :

(1) if (X, τ_1, τ_2) is pairwise S-Closed then (Y, r_1, r_2) is nearly pairwise compact. (2)

(2) if (X, τ_1, τ_2) is pairwise S-Lindelof, then (Y, r_1, r_2) is nearly pairwise Lindelof.

(3) if (X, τ_1, τ_2) is pairwise countably S-Closed, then (Y, r_1, r_2) is pairwise countably nearly pairwise compact.

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