

## **The contextual factors and efficiency;Data envelopment analysis approach**

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**ABSTRACT:** Data envelopment analysis (DEA) is a mathematical linear programming approach, based upon the technical efficiency concept, and it can measure and analyze the technical efficiency of different entities, productive and non-productive, public and private, profit and non-profit seeking firms. This non-parametric approach calculates the efficiency level by performing linear calculations for each unit in the sample. Contextual factors impact on efficiency level of organization that isn't under control of management. In this paper we define context factors and investigate the various methods of DEA that consider the role of contextual factors.

**KEYWORDS:** Data envelopment analysis; Efficiency; Contextual factors

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### **I. INTRODUCTION**

Evaluation of performance and productivity are among fundamental concepts in economy and management. It is very clear to calculate the firm efficiency on the basis of their output and input. Efficient organizations try to have maximized output production with given input or minimize input usage in the production of given outputs, relative to the performance of other producers in some comparison set.

Data envelopment analysis (DEA) is applied for estimating efficiency, and it is a helpful and popular nonparametric modeling approach. The DEA technique has been implemented to appraise efficiency of company across a variety of organizations, including industrial, commercial, educational, and financial services [2]. Since 1978 when Charnes, Cooper and Rhodes (CCR) offered a mathematical programming formulation for the empirical evaluation of relative efficiency of the observed quantities of input and output for a group of similar referent DMUs until now, DEA has kept improving. Primary input-oriented DEA efficiency scores have estimated the relative reduction in input while the level of output is maintained. It is very important for researchers that they know what the inefficiency factors are, and there is a desire to separate the component of inefficiency that is under the control of management from the component that is out of management's control. To answer such a subject, the standard DEA model has been adjusted to distribute non-controllable or environmental factors [12].

There are three distinctive factors that impact on efficiency level of organization. The first one is the role of manager who directs production activities. The second one is environmental factors that surround an organization. Finally, the role of luck, related variables and omitted factors that have influence in the process of regression assessment [3]. The managerial factors have internal source and the other two have external source. It is apparent that understanding the impact of three factors is so desirable in efficiency evaluation. To achieve this, we would have information about environmental features, input and output, and improve the standard DEA that consider environmental characteristics. Furthermore, in order to eliminate the impact of luck on efficiency of firm, the model must be stochastic. However, most DEA models and virtually all operational DEA models are deterministic.

### **II. PROBLEM STATEMENT**

Traditionally, the efficiency is the ratio of output to input, so recognition of input and output is so important to account efficiency. On the other hand, the contextual factors impact on input as a result of impact on efficiency. Hence, in the first place, to estimate efficiency we are required to know what contextual factors are and in the second place what the method is to calculate efficiency with contextual factors. We will review the previous researches that used different methods in solving this problem and we will discuss contextual factors.

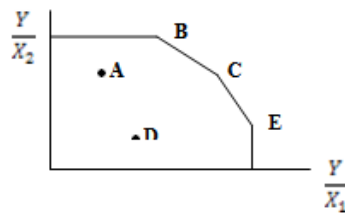
**2.1 Data Envelopment Analysis**

DEA is a mathematical linear programming approach based upon the technical efficiency concept and it can measure and analyze the technical efficiency of different entities, public and private, productive and non-productive, profit and non-profit seeking firms. This non-parametric approach calculates efficiency level by performing linear calculations for each unit in the sample. In this model, the efficiency of decision-making units is estimated by comparing them with the best unit in the statistical sample.

In measuring relative units, Farrell focused on weighted sum of input in order to make a virtual unit. The following equation was proposed as normal tool to measure technical efficiency [9].

$$effic \tag{1}$$

DEA is able to estimate the operational efficiency in consistent units with comparing them through a number of sampled units, which form together a curve of efficiency frontier that envelope all observations [9]. This approach is called DEA because it envelopes all observations [9]. Therefore, all efficient decision-making units will be on the curve of efficient frontier. Other decision-making units that are not on the curve are certainly inefficient.



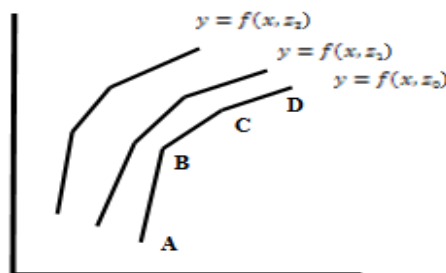
**Figure 1** The curve of efficient frontier

For example, in Figure 1, five DMUs (A, B, C, D, and E) were shown which each of them had two inputs X1 and X2 to produce Y quantity of output. The level of efficiency for each unit was determined by related data of input and output as shown in Figure 1. The B, C and E DMUs have the lie on frontier curve, as a result they are efficient, while A and D DMUs are not on efficient curve hence, they are inefficient.

It is remarkable we know that, based upon the theoretical concepts, to be on the efficient frontier curve is not the sign of optimal efficiency, but also represents a virtual performance. Accordingly, the actual style of the distribution process of resources and products is mirrored in this model. In fact, DEA helps to recognize inefficiency decision-making units and to improve themselves. To be efficient units and inefficient units near each other provide an opportunity to identify inefficient and trying to improve it [7].

**2.2 Contextual factors**

All organizations run in a competitive environment that has different and various dimensions. Some of them are controllable by manager while some of them are out of control. Although, some nondiscretionary factors are effective in the trend of running organization, they aren't observable. To the best of our knowledge from investigation on various articles, there are several words for the same concept in this case. As seen, the word contextual is replaced by other terms such as environmental [3], [4], external factors [8] and nondiscretionary factors [1]. In this respect, all firms compete in a context such as environment, position, rules and regulations, the power of unions and etc. Thus, the terms environmental, external and nondiscretionary are not as comprehensive as the term contextual, since it encompasses a broader range of the related concepts such as the ones mentioned above [1].



**Figure 2** Contextual factors and efficiency

Contrariwise, the concept of context can be a competent word to represent and express this concept. Characteristics of contextual factors are: unobservable, non controllable, able to influence and external resource.

### III. DEA MODELS AND CONTEXTUAL FACTORS

There are several models that the contextual factors take into consideration to estimate performance in the following sections:

#### 3.1 Banker and Morey

Banker and Morey want to improve the CCR and BCC models in order to account the extent to which the discretionary or manageable input can be reduced by the DMU manager while keeping the exogenously fixed input at their current. They used DEA standard model and modified that how can account exogenously fixed inputs. Banker and Morey classified exogenously fixed input from high until low, and estimated their impact on the efficiency of firm. Indeed, they succeeded to drive out one of the constraints in standard equation [10].

They tested their model on 60 fast food restaurants. In this example, they particularly served to illustrate the impact of fixed uncontrollable input. They compared the difference in controllable input targets that results from two different treatments of the non-discretionary input; one in which all input is indeed treated as discretionary, and the other in which only the truly controllable inputs are treated as discretionary. They measured data on 6 inputs and 3 outputs for 60 restaurants in the fast food chain. All output was controllable and only 4 inputs including supplies, materials, the age of the store, and the cost of advertising were clearly discretionary. Last 2 inputs were demographic and pointed out whether the store was located in an urban or rural area that they were contextual, uncontrollable and non-discretionary [10].

#### 3.2 Ray's model

To calculate efficiency in public schools, Ray offered a two-stage model for the first time. He classified endogenous factors that directly affect efficiency. He used standard DEA in the first stage and then in the second stage he used other socio-economic factors (contextual factors) as exogenous factors in a regression model [11]. This approach requires a priori functional form specification for the second-stage regression; mis-specification leads to distorted measurement [6].

#### 3.3 Ruggiero's Model

In the first place, Ruggiero started his argument with the main purpose of developing a modified DEA model that maintains consistency with known properties of public sector production. His model expanded the Banker and Morey model to enable or activate uninterrupted contextual (environmental) variables [5]. This model breaks the limits of categorizing the environment. In essence, the presence of non-discretionary input leads to different frontiers. To control these fixed factors, Ruggiero added constraints to exclude DMUs for more favorable production environment [6].

He criticized the two-stage models in research and presented a three-stage model. This model was constructed on convexity assumption for contextual input. Ruggiero, in the first place, estimated efficiency by standard DEA regardless of environment variables. In the second stage, all contextual or environmental factors regressed with quantity of DEA obtained from stage one. Subsequently, he made indicator  $Z_j$  instead of intervening contextual effects from the following equation:

$$DEA_{1st,j} = \alpha \cdot \tag{2}$$

While, (3)

After that, the weights  $\lambda_h$  conditioned on indicator  $Z_j$  was calculated, So that any firm having a more favorable environment than that of firm  $j$  will not be included in the frontier. Solving the following problem was the third stage of his model:

$$j=1, \dots, J \tag{4}$$

s.t  $k=1, \dots, n$  (5)

$$\lambda_h = \begin{cases} \text{if } Z_h > \end{cases} \quad i=1, \dots, m \quad (6)$$

$$h=1, \dots, J \quad (7)$$

$$\text{if VRS} \quad (8)$$

Ruggiero believed that the three-stage model took preference over the two-stage model due to assuming that the second-stage regression produces unbiased estimates of the parameter weights the model maintains for the desirable properties of the Ruggiero model. Second, this model overcomes the identified weakness of identifying DMUs as efficient by default inherent in the Ruggiero model. Instead, this model is able to weigh the importance of the non-discretionary input. Furthermore, Ray's model uses the error term to measure efficiency, and hence, will be sensitive to mis-specification [6].

### 3.4 Four- stage Model

Fried et al (1999) offered a different approach for accounting contextual factors on DEA that involves four stages. Like other models, in the first stage they accounted the standard DEA by normal input and output while contextual variables were kept out. For each observation, they calculated scores of radial technical efficiency. Followed by, every dependent variable for each equation is identified as sum of non-radial and radial input for an input-oriented model or radial plus non-radial output surplus for an output-oriented model in the stage two. Characteristic of contextual factors was measured by independent variables in this stage. The variation in total by-variable measures of inefficiency chargeable to factors outside the control of management was identified by made equation system. Subsequently, they predicted the total output surplus or input from the second stage using the parameter estimates in the third stage. Eventually, the fourth step was to repeat the DEA model under the specification of initial input, using the set of adjusted data [4].

### 3.5 Multi-stage

Fried et al (1999) offered a different approach for accounting contextual factors on DEA that involves four stages. Like other models, in the first stage they accounted the standard DEA by normal input and output while contextual variables were kept out. For each observation, they calculated scores of radial technical efficiency. Followed by, every dependent variable for each equation is identified as sum of non-radial and radial input for an input-oriented model or radial plus non-radial output surplus for an output-oriented model in the stage two. Characteristic of contextual factors was measured by independent variables in this stage. The variation in total by-variable measures of inefficiency chargeable to factors outside the control of management was identified by made equation system. Subsequently, they predicted the total output surplus or input from the second stage using the parameter estimates in the third stage. Eventually, the fourth step was to repeat the DEA model under the specification of initial input, using the set of adjusted data [4].

$$\text{total input} \quad (9)$$

Then, a relationship between contextual input and slacks was investigated by SFA

$$\text{total input slack}_{ij} = \beta_0 + \beta_1 z_{1j} + \dots \quad i=1, \dots, m \quad (10)$$

$$\text{where } v_{ij} \sim \quad (11)$$

In order to regulate all discretionary inputs the following equation is used:

$$x_{ij,adj} = x_{ij} + [\max_j(z_j \hat{\beta}^i) - z_j \hat{\beta}^i], \quad i= 1, \dots, J \quad (12)$$

where  $\hat{\beta}^i$  is a parameter vector for discretionary input

In the third stage, DEA is reused to modify data and output with the aim of accounting each score of organization efficiency.

**IV. SUMMARY**

In this paper, the concept of contextual factors was defined and equivalents of this concept were investigated in other papers. Generally, the contextual factors in organization have three essential features as follows:

They are external factors;

They are out of control;

They impact on input while not observables;

Data envelopment analysis is a popular and flexible method to account efficiency of firms. As mentioned, contextual factors impact the efficiency of firms and there are several models to estimate efficiency in this situation that is classified in the table below:

row	Creator(s) model	year	consideration
1	Banke & Morey	1990	It is good when we can classify contextual factors in the specific ranked category
2	Roy	1991	Two-stage model ,use DEA and regress
3	Ruggiero	1996	Two-stage
4	Ruggiero	1997	Three-stage
5	Fried et al	1999	Four-stage
6	Fried et al	2002	Multi-stage , use DEA and SFA

**V. ACKNOWLEDGMENTS**

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