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N-Homeomorphism and N*-Homeomorphism in supra Topological spaces

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ABSTRACT: In this paper, we introduce the concept of strongly supra N-continuous function and perfectly supra N-continuous function and studied its basic properties. Also we introduce the concept of supra N-Homeomorphism and supra N*- Homeomorphism. We obtain the basic properties and their relationship with supra N-closed maps, supra N-continuous maps and supra N-irresolute maps in supra topological spaces.

KEYWORDS: supra N-Homeomorphism, supra N*- Homeomorphism, strongly supra N-continuous function, perfectly supra N-continuous functions.

I. INTRODUCTION

In 1983, A.S.Mashhour et al [6] introduced the supra topological spaces and studied, continuous functions and s* continuous functions. R.Devi[2] have studied generalization of homeomorphisms and also have introduced α-homeomorphisms in topological spaces.

In this paper, we introduce the concept of strongly supra N-continuous function and perfectly supra N-continuous function and studied its basic properties. Also we introduce the concept of supra N-homeomorphism and supra N*-Homeomorphism in supra topological spaces.

II. PRELIMINARIES

Definition 2.1[6]
A subfamily μ of X is said to be supra topology on X if
i) $X, \phi \in \mu$
ii)If $A_i \in \mu \ \forall i \in j$ then $\cup A_i \in \mu$. (X,μ) is called supra topological space.
The element of μ are called supra open sets in (X, μ) and the complement of supra open set is called supra closed sets and it is denoted by μ^c.

Definition 2.2[6]

The supra closure of a set A is denoted by $cl^{\mu}(A)$, and is defined as supra cl(A) = ∩{B : B is supra closed and A ⊆ B}.

The supra interior of a set A is denoted by $int^{\mu}(A)$, and is defined as supra int(A) = ∪{B: B is supra open and A ⊇ B}.

Definition 2.3[6]
Let (X, τ) be a topological space and μ be a supra topology on X. We call μ a supra topology associated with τ, if τ ⊆ μ.

Definition 2.4[5]
Let (X, μ) be a supra topological space. A set A of X is called supra semi- open set, if $A \subseteq cl^{\mu}(int^{\mu}(A))$. The complement of supra semi-open set is supra semi-closed set.

Definition 2.5[4]
Let (X, μ) be a supra topological space. A set A of X is called supra α -open set, if $A \subseteq int^{\mu}(cl^{\mu}(int^{\mu}(A)))$. The complement of supra α -open set is supra α-closed set.

Definition 2.6[7]
Let (X, μ) be a supra topological space. A set A of X is called supra Ω closed set, if $scl^{\mu}(A) \subseteq int^{\mu}(U)$,whenever $A \subseteq U$, U is supra open set. The complement of the supra Ω closed set is supra Ω open set.

Definition 2.7[7]

The supra Ω closure of a set A is denoted by $\Omega cl^{\mu}(A)$, and defined as $\Omega cl^{\mu}(A) = \cap\{B: B$ is supra Ω closed and $A \subseteq B\}$.

The supra Ω interior of a set A is denoted by $\Omega int^{\mu}(A)$, and defined as $\Omega int^{\mu}(A) = \cup\{B: B$ is supra Ω open and $A \supseteq B\}$.

Definition 2.8

Let (X, μ) be a supra topological space . A set A of X is called supra regular open if $A = int^{\mu}(cl^{\mu}(A))$ and supra regular closed if $A = cl^{\mu}(int^{\mu}(A))$.

Definition 2.9[9]

Let (X, μ) be a supra topological space . A set A of X is called supra N-closed set if $\Omega cl^{\mu}(A) \subseteq U$, whenever $A \subseteq U$, U is supra α open set. The complement of supra N-closed set is supra N-open set.

Definition 2.10[9]

The supra N closure of a set A is denoted by $Ncl^{\mu}(A)$, and defined as $Ncl^{\mu}(A) = \cap\{B: B$ is supra N-closed and $A \subseteq B\}$.

Definition 2.11[9]

Let (X, τ) and (Y, σ) be two topological spaces and μ be an associated supra topology with τ. A function $f:(X, \tau) \rightarrow (Y, \sigma)$ is called supra N-continuous function if $f^{-1}(V)$ is supra N-closed in (X, τ) for every supra closed set V of (Y, σ).

Definition 2.12[9]

Let (X, τ) and (Y, σ) be two topological spaces and μ be an associated supra topology with τ. A function $f:(X, \tau) \rightarrow (Y, \sigma)$ is called supra N-irresolute if $f^{-1}(V)$ is supra N-closed in (X, τ) for every supra N-closed set V of (Y, σ).

Definition 2.13[10]

A map $f:(X, \tau) \rightarrow (Y, \sigma)$ is called supra N-closed map(resp. supra N-open) if for every supra closed(resp. supra open) F of X, f(F) is supra N-closed(resp. supra N-open) in Y.

Definition 2.14[10]

A map $f:(X, \tau) \rightarrow (Y, \sigma)$ is said to be almost supra N-closed map if for every supra regular closed F of X, f(F) is supra N-closed in Y.

Definition 2.15[10]

A map $f:(X, \tau) \rightarrow (Y, \sigma)$ is said to be strongly supra N-closed map if for every supra N closed F of X, f(F) is supra N-closed in Y.

Definition: 2.16[10]

A supra topological space (X, τ) is $T_N^{\mu} - space$ if every supra N-closed set in it is supra closed.

III. SOME FORMS OF SUPRA N-CONTINUOUS FUNCTIONS

Definition 3.1

A map $f:(X, \tau) \rightarrow (Y, \sigma)$ is called strongly supra N-continuous function if the inverse image of every supra N-closed set in (Y, σ) is supra closed in (X, τ).

Definition 3.2

A map $f:(X, \tau) \rightarrow (Y, \sigma)$ is called perfectly supra N-continuous function if the inverse image of every supra N-closed set in (Y, σ) is both supra open and supra closed in (X, τ).

Theorem 3.3

Every perfectly supra N-continuous function is strongly supra N-continuous function.

Proof Let $f:(X, \tau) \rightarrow (Y, \sigma)$ be a perfectly N-continuous function. Let V be N-closed set in (Y, σ). Since f is perfectly N-continuous function $f^{-1}(V)$ is both supra open and supra closed in (X, τ). Therefore f is strongly supra N-continuous function.

The converse of the above theorem need not be true. It is shown by the following example.

Example 3.4

Let $X=Y=\{a, b, c\}$ and $\tau = \{X, \varphi, \{a\},\{b\},\{a,b\},\{b, c\}\}$, $\sigma = \{Y, \varphi, \{a,b\}\}$.

f:(X, τ) → (Y, σ) be the function defined by f(a)=b, f(b)=a, f(c)=c. Here f is strongly supra N-continuous but not perfectly supra continuous, since V={b,c} is supra N-closed in Y but f^{-1}({b,c}) = {a,c} is supra closed set but not supra open in X.

Theorem 3.5
Let f:(X, τ) → (Y, σ) be strongly supra N-continuous and g: (Y, σ) → (Z, υ) be strongly supra N-continuous then their composition gof:(X, τ) → (Z, υ) is a strongly supra N-continuous function.
Proof Let V be supra N-closed set in (Z, υ). Since g is strongly N-continuous, g^{-1}(V) is supra closed in (Y, σ). We know that every supra closed set is supra N-closed set, g^{-1}(V) is supra N-closed in (Y, σ). Since f is strongly N-continuous, $f^{-1}(g^{-1}(V))$ is supra closed in (X, τ), implies (gof)(V) is supra closed in (X, τ).Therefore gof is strongly N-continuous.

Example 3.6
Let X=Y=Z={a, b, c} and τ = { X, φ, {a},{b},{a,b},{b,c},{a,c}} , σ={Y,φ,{a},{b},{a,b},{b,c}}, υ={Z, φ,{a,b},{b,c} }.f:(X, τ) → (Y, σ) be the function defined by f(a)=b, f(b)=a,f(c)=c. g: (Y, σ) → (Z, υ) be a function defined by g(a)=c, g(b)=b, g(c)=a. Here f and g are strongly supra N-continuous and gof is also strongly supra N-continuous function.

IV. SUPRA N-HOMEOMORPHISM AND SUPRA N*-HOMEOMORPHISM

Definition 4.1
A bijection f:(X, τ) → (Y, σ) is called supra N-Homeomorphism if f is both supra N-continuous function and supra N-closed map(f^{-1} is N-continuous function).

Definition 4.2
A bijection f:(X, τ) → (Y, σ) is called supra N^*-Homeomorphism if f and f^{-1} are supra N-irresolute.

Theorem 4.3

Let f:(X, τ) → (Y, σ) be a bijective supra N-continuous map. Then the following are equivalent
1) f is an N-open map
2) f is an N-homeomorphism
3) f is an N-closed map.

Proof (i) ⟹ (ii):If f is a bijective supra N-continuous function, suppose (i) holds. Let V be supra closed in (X, τ) then V^c is supra open in (X, τ). Since f is supra N-open map, f(V^c) is supra N-open in (Y, σ). Hence f(V) is supra N-closed in (Y, σ) implies f^{-1} is supra N-continuous. Therefore f is an supra N-homeomorphism.
(ii) ⟹ (iii):Suppose f is an supra N-Homeomorphism and f is bijective supra N-continuous function then from the definition4.1, f^{-1} is supra N-continuous, implies f is supra N-closed map.
(iii) ⟹ (i):Suppose f is supra N-closed map. Let V be supra open in (X, τ) then V^c is supra closed in (X, τ). Since f is supra N-closed map, f(V^c) is supra N-closed in (Y, σ). Hence f(V) is supra N-open in (Y, σ). Therefore f is an supra N-open map.

Remark 4.4
The composition of two supra N-Homeomorphism need not be an supra N-Homeomorphism. Since composition of two supra N-continuous function need be supra N-continuous and composition of two supra N-closed map need not be supra N-closed map. It is seen from the following example

Example 4.5
Let X=Y=Z={a, b, c} and τ = {X, φ, {a},{b,c}} , σ = {Y,φ, a}}. υ={ Z, φ,{a}, {b}, {a, b}, {b, c}}. f:(X, τ) → (Y, σ) be the function defined by f(a)=b, f(b)=c, f(c)=a. and g: (Y, σ) → (Z, υ) be the function defined by g(a)=b, g(b)=c, g(c)=a. Here f and g is supra N-closed map, but its composition is not supra N-closed map, since gof{b,c}={a,b} is not supra N-closed in Z. Therefore gof is not an supra N-Homeomorphism

Theorem 4.6
Every supra N-Homeomorphism is supra N-continuous.
Proof It is obvious from the definition 4.1
The converse of the above theorem need not be true. It is shown by the following example.

Example 4.7
Let X=Y={a, b, c} and τ = {X, φ, {a},{b, c}} , σ = {Y, φ, {a,b},{b,c}}.
f:(X, τ) → (Y, σ) be the function defined by f(a)=b, f(b)=c, f(c)=a. Here f is supra N-continuous but not supra N-Homeomorphism, since f^{-1} is not supra N-continuous.

Theorem 4.8
Every supra N^*-Homeomorphism is supra N-irresolute.
Proof It is obvious from the definition 4.2
The converse of the above theorem need not be true. It is shown by the following example.

Example 4.9
Let X=Y={a, b, c} and $\tau = \{X, \varphi, \{a\},\{b, c\}\}$, $\sigma = \{Y, \varphi, \{a,b\},\{b,c\}\}$. f:(X, τ) → (Y, σ) be the function defined by f(a)=b, f(b)=c, f(c)=a.Here f is supra N-irresolute but not supra N^*-Homeomorphism, since f^{-1} is not supra N-irresolute.

Theorem 4.10
If f:(X, τ) → (Y, σ) and g: (Y, σ) → (Z, υ) are supra N^*-Homeomorphism then the composition gof is also supra N^*-Homeomorphism
Proof Let V be a supra N-closed set in (Z, υ). Since g is supra N^*-Homeomorphism g and g^{-1} are supra N-irresolute, then $g^{-1}(V)$ is supra N-closed set in (Y, σ).Now $(gof)^{-1}(V) = f^{-1}(g^{-1}(V))$. Since f is N^*-Homeomorphism f and f^{-1} are supra N-irresolute, then $f^{-1}(g^{-1}(V))$ is supra N-closed set in (X, τ). Thus gof is supra N-irresolute.

For an supra N-closed set V in (X, τ). (gof)(V)=g(f(V)).By Hypothesis f(V) is supra N-closed set in (Y, σ). Thus g(f(V)) is supra N-closed set in (Z, υ). Hence $(gof)^{-1}$ is supra N-irresolute. Therefore gof is supra N^*-Homeomorphism.

Theorem 4.11
If f:(X, τ) → (Y, σ) is a supra N^*-Homeomorphism then $Ncl(f^{-1}(B))=f^{-1}(Ncl(B))$,for every $B \subseteq Y$ is supra N-closed.
Proof Since f is supra N^*-Homeomorphism f and f^{-1} are supra N-irresolute. Let B be supra N-closed set in (Y, σ). Since f is supra N-irresolute $f^{-1}(B)$ is supra N-closed set in (X, τ). Since B is supra N-closed set, B=Ncl(B). Therefore $f^{-1}(Ncl(B))$ is supra N-closed set in (X, τ). Since $f^{-1}(B)$ is supra N-closed set, $Ncl(f^{-1}(B))=f^{-1}(B)$ is supra N-closed in (X, τ). Therefore Ncl $(f^{-1}(B))= f^{-1}(Ncl(B))$ is supra N-closed set in (X, τ).

Theorem 4.12
If f:(X, τ) → (Y, σ) is a supra N^*-Homeomorphism then Ncl(f(B))=f(Ncl(B)),for every $B \subseteq X$ is supra N-closed.
Proof Since f is supra N^*-Homeomorphism f and f^{-1} are supra N-irresolute. Let B be supra N-closed set in (X, τ). Since f^{-1} is supra N-irresolute f(B) is supra N-closed set in (Y, σ). Since B is supra N-closed set, B=Ncl(B). Therefore f(Ncl(B)) is supra N-closed set in (Y, σ). Since f(B) is supra N-closedset, Ncl(f(B))=f(B) is supra N-closed set in (Y, σ). Therefore Ncl(f(B))= f(Ncl(B)) is supra N-closed in (Y, σ).

Theorem 4.13
Every supra N^*-Homeomorphism is strongly supra N-closed map.
Proof Since f:(X, τ) → (Y, σ) is supra N^*-Homeomorphism f and f^{-1} are supra N-irresolute. f^{-1} is supra N- irresolute implies f is strongly supra N-closed map.
The converse of the above theorem need not be true. It is shown by the following example.

Example 4.14
Let X=Y={a, b, c} and $\tau = \{X, \varphi, \{a,b\},\{b, c\}\}$, $\sigma = \{Y, \varphi, \{a\},\{b,c\}\}$. f:(X, τ) → (Y, σ) be the function defined by f(a)=b, f(b)=c, f(c)=a. Here f is strongly supra N-closed but not supra N^*-Homeomorphism, since f^{-1} is supra N-irresolute(strongly supra N-closed map) but f is not supra N-irresolute.

Theorem 4.15
If (X, τ) and (Y, σ) is a $T_N^{\mu} - space$, and if f:(X, τ) → (Y, σ) is a supra N-continuous function then f is supra N-Homeomorphism.
Proof Let V be supra closed in (Y, σ). Since f is supra N-continuous $f^{-1}(V)$ is supra N-closed in (X, τ). Since (X, τ) and (Y, σ) is a $T_N^{\mu} - space$,then every supra N-closed set is supra closed set. Let B be supra closed set in (X, τ) , then f(B) is supra N-closed in (Y, σ), implies f is N-closed map(f^{-1} is N-continuous function). Hence f is N-Homeomorphism.

Theorem 4.16
The set N^*-h(X, τ) from (X, τ) on to itself is a group under the composition of maps.
ProofLet f,g ∈ N^*-h(X, τ), then by theorem4.10 gof ∈ N^*-h(X, τ), we know that the composition of mapping is associative and the identity I: (X, τ) →(X, τ) belonging to N^*-h(X, τ) serves as the Identity element.

If $f \in N^*\text{-}h(X, \tau)$ then $f^{-1} \in N^*\text{-}h(X, \tau)$ such that $fof^{-1} = f^{-1}of = I$. Therefore inverse exists for each element of $N^*\text{-}h(X, \tau)$. Hence $N^*\text{-}h(X, \tau)$ is a group under composition of maps.

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