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# Inclusion Probability Proportional to Size Sampling Plan Excluding Adjacent Units in Circular Order

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***Abstract***
*This paper suggests an inclusion probability proportional to size sampling plan excluding adjacent units (IPPSEA) separated by at most a distance of $m$ $(\geq 1)$ in circular order using linear programming on Microsoft. This makes computation easier and efficient than the proposed method by Mandel (2008).*
***Keywords:*** *IPPSEA, circular order, linear programming, Microsoft*

## I. Introduction

The purpose of sampling theory is to develop methods of sample selection and of estimation that best estimates the population parameter as well as precises our purpose. The most basic sample selection procedure is simple random sampling (SRS), providing an equal chance of selection to all the units in the sample space. When the sample selection of units varies according to size, such a sampling scheme is called the probability proportional to size (PPS) sampling. This scheme is coined by Hartley and Rao (1962)[1]. Inclusion probability proportional to size sampling schemes (IPPS) are the sampling schemes in which the first order inclusion probabilities are proportional to size measures. Nigam et al. (1985)[2] discussed IPPS sampling and Gabler et al. (1987)[3] coined 'nearest proportional to size sampling designs'. Samford (1967)[4] describes the IPPS sampling scheme in his literature. The IPPS schemes was available for $n = 2$ but he worked for $n \geq 2$. The plan given by him ensures $\pi_{ij} > 0 \ \forall \ i \neq j = 1, 2, \dots, N$ and $\pi_{ij} < \pi_i \pi_j \ \forall \ i \neq j = 1, 2, \dots, N$ which is the sufficient condition for non-negativity of variance estimator. IPPS sampling procedures uses Horvitz-Thompson (1952)[5] estimator for the estimation of variance. Sahoo et al. (2006)[6] discussed IPPS sampling scheme and Sahoo et al. (2010)[7] introduced a general class of IPPS sampling schemes. Sahoo et al. (2011)[8] constructed a new IPPS sampling scheme of two units for estimating the total of a finite population. Tiwari and Chilwal (2013)[9] used a simplified selection scheme for unequal probability sampling without replacement. Ozturk (2020)[10] constructed probability proportional to size ranked set sampling from a stratified population. Woong (2005)[11] suggested an optimal scheme of IPPS. Tiwari et al. (2007)[12] proposed a one-dimensional optimal controlled IPPS sampling design ensuring zero probability to non-preferred samples. Deshpande and Ajgaonkar (2008)[13] discussed IPPS sampling scheme. Mandal et al. (2008)[14] proposed inclusion probability proportional to size sampling plans excluding adjacent units (IPPSEA plans). IPPSEA plans may be obtained by trial-and-error methods using combinatorial properties of block designs. Mandel et al. (2008)[15] also suggested linear programming approach to obtain IPPSEA plans based on SAS coding for circular as well as linear arrangement of the population units. In this paper, we have proposed the linear programming approach to obtain IPPSEA plans based on Microsoft 2019 for circular and linear arrangements of the units. This approach has been discussed in the section 2, some examples described in section 3, followed by conclusion in section 4.

## II. Linear programming Approach to IPPSEA Plans

In this section, we discuss a linear programming approach of obtaining an IPPSEA plan. A sample of n units to be drawn from population size N with varying probability without replacement for estimating the population mean $\bar{Y} = N^{-1} \sum_{i=1}^{N} y_i$ . The sampling design is represented by *(S, $p_0(s)$|$s \in S$)*, where S, the sample space, is the set of all possible $^{N}C_{n}$ samples s and $p_0(s)$ is the probability of selecting the sample s. *($p_0(s)$|$s \in S$)* is the sampling plan and is also termed as the sampling design. Moreover, $\sum_{s \in S} p_0(s) = 1$.

### 2.1. Circular IPPS Plans Excluding Adjacent Units

We first consider the case of circular structure of the population units. The set of non-preferred samples is $S_1$.

The similar approach has been used which is described by Rao and Nigam (1990, 1992)[15,16], we provide the solution to the problem of obtaining an IPPS sampling plan with exclusion of adjacent units separated by a distance upto m unit. The optimal solution to the IPPS sampling plan excluding adjacent units using linear programming problem follows as:

Minimize the objective function $\phi = \sum_{s\epsilon S_1} p(s)$ with respect to the variables *{p(s) | s ϵ S)* subject to the linear constraints:

i. $\sum_{s\ni i} p(s) = nP_i \quad \forall\, i = 1, 2, \dots, N$

ii. $\sum_{\substack{s\ni i,j \\ \delta(i,j)\le m}} p(s) = 0, \quad i < j = 1,2,\dots,N$

(i.e., for all the pairs of units belonging to Ω)

iii. $p(s) \geq 0$ for all $s \ni S$

iv. $\sum_{s\in S} p(s) = 1$

Now, we discuss some examples to demonstrate the Linear Programming Approach for constructing circular and linear IPPSEA plan.

## III. Empirical examples

*Example 1*. Consider a population with *N=7, n=2* and *m=1* with initial probability of selection units as given below:

| **Units (i)** | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| **Initial probability of selection** | 0.10 | 0.12 | 0.14 | 0.15 | 0.15 | 0.16 | 0.18 |

The sample space S consists of all possible $\binom{7}{2}$ = 21 samples of size 2 and $S_1$ consists of the samples which contains the pair $(i, j)$ for $\delta(i, j) = 1, i \neq j = 1,2,\dots,7$.

| **Sample No.** | **Samples** | **Sample No.** | **Samples** |
|---|---|---|---|
| 1 | 1,2 | 12 | 3,4 |
| 2 | 1,3 | 13 | 3,5 |
| 3 | 1,4 | 14 | 3,6 |
| 4 | 1,5 | 15 | 3,7 |
| 5 | 1,6 | 16 | 4,5 |
| 6 | 1,7 | 17 | 4,6 |
| 7 | 2,3 | 18 | 4,7 |
| 8 | 2,4 | 19 | 5,6 |
| 9 | 2,5 | 20 | 5,7 |
| 10 | 2,6 | 21 | 6,7 |
| 11 | 2,7 | | |

These 21 possible samples are denoted by $S_1, S_2, \dots, S_{21}$ and their probabilities are $p(S_1), p(S_2), \dots, p(S_{21})$. Let us denote these probabilities by $p_1, p_2, \dots, p_{21}$ respectively.

The preferred samples are the samples without contiguous units which as follows:

| **Sample No** | **Probability** | **Samples** |
|---|---|---|
| $S_2$ | $p_2$ | 1,3 |
| $S_3$ | $p_3$ | 1,4 |
| $S_4$ | $p_4$ | 1,5 |
| $S_5$ | $p_5$ | 1,6 |
| $S_8$ | $p_8$ | 2,4 |
| $S_9$ | $p_9$ | 2,5 |
| $S_{10}$ | $p_{10}$ | 2,6 |
| $S_{11}$ | $p_{11}$ | 2,7 |
| $S_{13}$ | $p_{13}$ | 3,5 |
| $S_{14}$ | $p_{14}$ | 3,6 |
| $S_{15}$ | $p_{15}$ | 3, 7 |
| $S_{17}$ | $p_{17}$ | 4,6 |
| $S_{18}$ | $p_{18}$ | 4,7 |
| $S_{20}$ | $p_{20}$ | 5,7 |

When samples are arranged then it is observed that $S_1$ consists of sample numbers 1, 6, 7, 12, 16, 19, 21. Hence, objective function is

$\phi = p(s_1) + p(s_6) + p(s_7) + p(s_{12}) + p(s_{16}) + p(s_{19}) + p(s_{21})$
$= p(s_1) + 0.ps_2) + 0.p(s_3) + 0.p(s_4) + 0.p(s_5) + p(s_6) + p(s_7) + 0.p(s_8) + 0.p(s_9) + 0.p(s_{10}) + 0.p(s_{11}) + p(s_{12}) + 0.p(s_{13}) + 0.p(s_{14}) + 0.p(s_{15}) + p(s_{16}) + 0.p(s_{17}) + 0.p(s_{18}) + p(s_{19}) + 0.p(s_{20}) + p(s_{21})$

Unit 1 appears in 4 samples. For *i* = 1, the constraint (i) is
$\pi_1 = p_2 + p_3 + p_4 + p_5$
Similarly, other constraints for *i* = 2, 3, …., 7 are set.
As per constraint of equation $p_1 + p_2 +, \dots, + p_{21}$ = 1
Then the first order inclusion probabilities are:
$\pi_1 = p_2 + p_3 + p_4 + p_5$
$\pi_2 = p_8 + p_9 + p_{10} + p_{11}$
$\pi_3 = p_2 + p_{13} + p_{14} + p_{15}$
$\pi_4 = p_3 + p_8 + p_{17} + p_{18}$
$\pi_5 = p_4 + p_9 + p_{13} + p_{20}$
$\pi_6 = p_5 + p_{10} + p_{14} + p_{17}$
$\pi_7 = p_{11} + p_{15} + p_{18} + p_{20}$
From equation for *N=7, n=2* and *m=1*, the first order inclusion probabilities are:
$\pi_i = nP_i \quad \forall \quad i = 1, 2, .., 7$

| | |
|---|---|
| $\pi_1$ | 0.20 |
| $\pi_2$ | 0.24 |
| $\pi_3$ | 0.28 |
| $\pi_4$ | 0.30 |
| $\pi_5$ | 0.30 |
| $\pi_6$ | 0.32 |
| $\pi_7$ | 0.36 |

And second order inclusion probabilities are:
$\pi_{ij} = 0 \quad ; \quad \delta(i, j) \geq m \quad , \quad i \neq j = 1, 2, \dots, N$
The constraints arranged according to example 1 and minimizing the objective function gives the following optimal solution with $\phi = 0$.
Now, giving the first order inclusion probabilities and solving by Microsoft 2019 Package and the above formulation of linear programming problem gives the following sampling plan given in Table 1.

Table 1. Circular IPPSEA plan for *N=7*, *n=2* and *m=1*.

| $S_i$ | p(s) | $S_i$ | p(s) |
|---|---|---|---|
| $S_1$ | 0 | $S_{12}$ | 0 |
| $S_2$ | 0.004634 | $S_{13}$ | 0.086266 |
| $S_3$ | 0.075733 | $S_{14}$ | 0.093567 |
| $S_4$ | 0.058201 | $S_{15}$ | 0.095533 |
| $S_5$ | 0.061433 | $S_{16}$ | 0 |
| $S_6$ | 0 | $S_{17}$ | 0.093567 |
| $S_7$ | 0 | $S_{18}$ | 0.095533 |
| $S_8$ | 0.035167 | $S_{19}$ | 0 |
| $S_9$ | 0.06 | $S_{20}$ | 0.095533 |
| $S_{10}$ | 0.071434 | $S_{21}$ | 0 |
| $S_{11}$ | 0.0734 | | |

It is observed that the inclusion probabilities are proportional to initial probability of selection of the units and $\pi_{ij} = 0$ for $\delta(i, j) = 1, i \neq j = 1, 2, \dots, 7$. Therefore, the above sampling plan is IPPSEA plan.

*Example 2*. Consider a population with *N=9*, *n=3* and *m=1* with initial probability of selection units as given below:

| **Units (i)** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| **Initial probability of selection** | 0.137 | 0.046 | 0.172 | 0.08 | 0.073 | 0.146 | 0.082 | 0.237 | 0.027 |

The sample space S consists of all possible $\binom{9}{3}$ = 84 samples of size 3 and $S_1$ consists of the samples which contains the pair $(i,j)$ for $\delta(i,j) = 1, i \neq j = 1,2,\dots,9$.

| Sample No. | Samples | Sample No. | Samples |
|---|---|---|---|
| 1 | 1,2,3 | 43 | 2,5,9 |
| 2 | 1,2,4 | 44 | 2,6,7 |
| 3 | 1,2,5 | 45 | 2,6,8 |
| 4 | 1,2,6 | 46 | 2,6,9 |
| 5 | 1,2,7 | 47 | 2,7,8 |
| 6 | 1,2,8 | 48 | 2,7,9 |
| 7 | 1,2,9 | 49 | 2,8,9 |
| 8 | 1,3,4 | 50 | 3,4,5 |
| 9 | 1,3,5 | 51 | 3,4,6 |
| 10 | 1,3,6 | 52 | 3,4,7 |
| 11 | 1,3,7 | 53 | 3,4,8 |
| 12 | 1,3,8 | 54 | 3,4,9 |
| 13 | 1,3,9 | 55 | 3,5,6 |
| 14 | 1,4,5 | 56 | 3,5,7 |
| 15 | 1,4,6 | 57 | 3,5,8 |
| 16 | 1,4,7 | 58 | 3,5,9 |
| 17 | 1,4,8 | 59 | 3,6,7 |
| 18 | 1,4,9 | 60 | 3,6,8 |
| 19 | 1,5,6 | 61 | 3,6,9 |
| 20 | 1,5,7 | 62 | 3,7,8 |
| 21 | 1,5,8 | 63 | 3,7,9 |
| 22 | 1,5,9 | 64 | 3,8,9 |
| 23 | 1,6,7 | 65 | 4,5,6 |
| 24 | 1,6,8 | 66 | 4,5,7 |
| 25 | 1,6,9 | 67 | 4,5,8 |
| 26 | 1,7,8 | 68 | 4,5,9 |
| 27 | 1,7,9 | 69 | 4,6,7 |
| 28 | 1,8,9 | 70 | 4,6,8 |
| 29 | 2,3,4 | 71 | 4,6,9 |
| 30 | 2,3,5 | 72 | 4,7,8 |
| 31 | 2,3,6 | 73 | 4,7,9 |
| 32 | 2,3,7 | 74 | 4,8,9 |
| 33 | 2,3,8 | 75 | 5,6,7 |
| 34 | 2,3,9 | 76 | 5,6,8 |
| 35 | 2,4,5 | 77 | 5,6,9 |
| 36 | 2,4,6 | 78 | 5,7,8 |
| 37 | 2,4,7 | 79 | 5,7,9 |
| 38 | 2,4,8 | 80 | 5,8,9 |
| 39 | 2,4,9 | 81 | 6,7,8 |
| 40 | 2,5,6 | 82 | 6,7,9 |
| 41 | 2,5,7 | 83 | 6,8,9 |
| 42 | 2,5,8 | 84 | 7,8,9 |

These 84 possible samples are denoted by $S_1, S_2, \dots, S_{84}$ and their probabilities are $p(S_1), p(S_2), \dots, p(S_{84})$. Let us denote these probabilities by $p_1, p_2, \dots, p_{84}$ respectively.

The preferred samples are the samples without contiguous units which as follows:

| Sample No | Probability | Samples |
|---|---|---|
| $S_9$ | $p_9$ | 1,3,5 |
| $S_{10}$ | $p_{10}$ | 1,3,6 |
| $S_{11}$ | $p_{11}$ | 1,3,7 |
| $S_{12}$ | $p_{12}$ | 1,3,8 |
| $S_{15}$ | $p_{15}$ | 1,4,6 |

| | | |
|---|---|---|
| $S_{16}$ | $p_{16}$ | 1,4,7 |
| $S_{17}$ | $p_{17}$ | 1,4,8 |
| $S_{20}$ | $p_{20}$ | 1,5,7 |
| $S_{21}$ | $p_{21}$ | 1,5,8 |
| $S_{24}$ | $p_{24}$ | 1,6,8 |
| $S_{36}$ | $p_{36}$ | 2,4,6 |
| $S_{37}$ | $p_{37}$ | 2,4,7 |
| $S_{38}$ | $p_{38}$ | 2,4,8 |
| $S_{39}$ | $p_{39}$ | 2,4,9 |
| $S_{41}$ | $p_{41}$ | 2,5,7 |
| $S_{42}$ | $p_{42}$ | 2,5,8 |
| $S_{43}$ | $p_{43}$ | 2,5,9 |
| $S_{45}$ | $p_{45}$ | 2,6,8 |
| $S_{46}$ | $p_{46}$ | 2,6,9 |
| $S_{48}$ | $p_{48}$ | 2,7,9 |
| $S_{56}$ | $p_{56}$ | 3,5,7 |
| $S_{57}$ | $p_{57}$ | 3,5,8 |
| $S_{58}$ | $p_{58}$ | 3,5,9 |
| $S_{60}$ | $p_{60}$ | 3,6,8 |
| $S_{61}$ | $p_{61}$ | 3,6,9 |
| $S_{63}$ | $p_{63}$ | 3,7,9 |
| $S_{70}$ | $p_{70}$ | 4,6,8 |
| $S_{71}$ | $p_{71}$ | 4,6,9 |
| $S_{73}$ | $p_{73}$ | 4,7,9 |
| $S_{79}$ | $p_{79}$ | 5,7,9 |

When samples are arranged then it is observed that $S_1$ consists of sample numbers 1, 2, 3, 4, 5, 6, 7, 8, 13, 14, 18, 19, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 40, 44, 47, 49, 50, 51, 52, 53, 54, 55, 59, 62, 64, 65, 66, 67, 68, 69, 72, 74, 75, 76, 77, 78, 80, 81, 82, 83 and 84. Hence, objective function is

$$\phi = p(s_1) + p(s_2)+\ldots+p(s_8) + p(s_{13}) + p(s_{14}) + p(s_{18})+\ldots+p(s_{84})$$
$$= p(s_1) + ps_2) + \cdots + p(s_8) + 0.p(s_9) + 0.p(s_{10}) + 0.p(s_{11}) + 0.p(s_{12}) + p(s_{13}) + p(s_{14})+\ldots+p(s_{84})$$

Unit 1 appears in 10 samples. For *i* = 1, the constraint (i) is

$$\pi_1 = p_9 + p_{10} + p_{11} + p_{12} + p_{15} + p_{16} + p_{17} + p_{20} + p_{21} + p_{24}$$

Similarly, other constraints for *i* = 2, 3, …., 84 are set.

As per constraint of equation $p_1 + p_2 +, \ldots, + p_{84} = 1$

Then the first order inclusion probabilities are:

$$\pi_1 = p_9 + p_{10} + p_{11} + p_{12} + p_{15} + p_{16} + p_{17} + p_{20} + p_{21} + p_{24}$$
$$\pi_2 = p_{36} + p_{37} + p_{38} + p_{39} + p_{41} + p_{42} + p_{43} + p_{45} + p_{46} + p_{48}$$
$$\pi_3 = p_9 + p_{10} + p_{11} + p_{56} + p_{57} + p_{58} + p_{60} + p_{61} + p_{63}$$
$$\pi_4 = p_{15} + p_{16} + p_{17} + p_{36} + p_{37} + p_{38} + p_{39} + p_{70} + p_{71} + p_{73}$$
$$\pi_5 = p_{20} + p_{21} + p_{41} + p_{42} + p_{43} + p_{56} + p_{57} + p_{58} + p_{79}$$
$$\pi_6 = p_{10} + p_{15} + p_{36} + p_{45} + p_{46} + p_{60} + p_{61} + p_{70} + p_{71}$$
$$\pi_7 = p_{11} + p_{16} + p_{20} + p_{37} + p_{41} + p_{48} + p_{56} + p_{63} + p_{73} + p_{79}$$
$$\pi_8 = p_{12} + p_{17} + p_{21} + p_{38} + p_{42} + p_{45} + p_{57} + p_{60} + p_{70}$$
$$\pi_9 = p_{39} + p_{43} + p_{46} + p_{48} + p_{58} + p_{61} + p_{63} + p_{71} + p_{73} + p_{79}$$

From equation for *N=9*, *n=3* and *m=1*, the first order inclusion probabilities are:

$\pi_i = nP_i \quad \forall \quad i = 1, 2, .., 9$

| | |
|---|---|
| $\pi_1$ | 0.411 |
| $\pi_2$ | 0.138 |
| $\pi_3$ | 0.516 |
| $\pi_4$ | 0.24 |
| $\pi_5$ | 0.219 |
| $\pi_6$ | 0.438 |
| $\pi_7$ | 0.246 |

| | |
|---|---|
| $\pi_8$ | 0.711 |
| $\pi_9$ | 0.081 |

And second order inclusion probabilities are:
$\pi_{ij} = 0 \;\; ; \quad \delta(i,j) \geq m \;, \quad i \neq j = 1,2, \ldots, N$
The constraints arranged according to example 2 and minimizing the objective function gives the following optimal solution with $\phi = 0$.
Now, giving the first order inclusion probabilities and solving by Microsoft 2019 Package and the above formulation of linear programming problem gives the following sampling plan given in Table 2.

Table 2. Circular IPPSEA plan for *N=9*, *n=3* and *m=1*.

| **$S_i$** | **p(s)** | **$S_i$** | **p(s)** |
|---|---|---|---|
| $S_1$ | 0 | $S_{43}$ | 0 |
| $S_2$ | 0 | $S_{44}$ | 0 |
| $S_3$ | 0 | $S_{45}$ | 0.062906 |
| $S_4$ | 0.058201 | $S_{46}$ | 0.007997 |
| $S_5$ | 0.061433 | $S_{47}$ | 0 |
| $S_6$ | 0 | $S_{48}$ | 0.012807 |
| $S_7$ | 0 | $S_{49}$ | 0 |
| $S_8$ | 0 | $S_{50}$ | 0 |
| $S_9$ | 0 | $S_{51}$ | 0 |
| $S_{10}$ | 0 | $S_{52}$ | 0 |
| $S_{11}$ | 0.059079 | $S_{53}$ | 0 |
| $S_{12}$ | 0.167203 | $S_{54}$ | 0 |
| $S_{13}$ | 0 | $S_{55}$ | 0 |
| $S_{14}$ | 0 | $S_{56}$ | 0.081544 |
| $S_{15}$ | 0.016798 | $S_{57}$ | 0.06617 |
| $S_{16}$ | 0.030037 | $S_{58}$ | 0 |
| $S_{17}$ | 0.030037 | $S_{59}$ | 0 |
| $S_{18}$ | 0 | $S_{60}$ | 0.120544 |
| $S_{19}$ | 0 | $S_{61}$ | 0.006548 |
| $S_{20}$ | 0 | $S_{62}$ | 0 |
| $S_{21}$ | 0.01884 | $S_{63}$ | 0.014909 |
| $S_{22}$ | 0 | $S_{64}$ | 0 |
| $S_{23}$ | 0 | $S_{65}$ | 0 |
| $S_{24}$ | 0 | $S_{66}$ | 0 |
| $S_{25}$ | 0 | $S_{67}$ | 0 |
| $S_{26}$ | 0 | $S_{68}$ | 0 |
| $S_{27}$ | 0 | $S_{69}$ | 0 |
| $S_{28}$ | 0 | $S_{70}$ | 0.122543 |
| $S_{29}$ | 0 | $S_{71}$ | 0.011657 |
| $S_{30}$ | 0 | $S_{72}$ | 0 |
| $S_{31}$ | 0 | $S_{73}$ | 0.013562 |
| $S_{32}$ | 0 | $S_{74}$ | 0 |
| $S_{33}$ | 0.0734 | $S_{75}$ | 0 |
| $S_{34}$ | 0 | $S_{76}$ | 0 |
| $S_{35}$ | 0 | $S_{77}$ | 0 |
| $S_{36}$ | 0 | $S_{78}$ | 0 |
| $S_{37}$ | 0 | $S_{79}$ | 0.01352 |
| $S_{38}$ | 0.015367 | $S_{80}$ | 0 |
| $S_{39}$ | 0 | $S_{81}$ | 0 |
| $S_{40}$ | 0 | $S_{82}$ | 0 |
| $S_{41}$ | 0.020541 | $S_{83}$ | 0 |
| $S_{42}$ | 0.018385 | $S_{84}$ | 0 |

It is observed that the inclusion probabilities are proportional to initial probability of selection of the units and $\pi_{ij} = 0$ for $\delta(i,j) = 1, i \neq j = 1, 2, \ldots, 9$. Therefore, the above sampling plan is IPPSEA plan.

*Example 3*. Consider a population with *N=10, n=3* and *m=1* with initial probability of selection units as given below:

| **Units (i)** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Initial probability of selection** | 0.18 | 0.14 | 0.13 | 0.11 | 0.10 | 0.10 | 0.08 | 0.07 | 0.05 | 0.04 |

The sample space S consists of all possible $\binom{10}{3}$ = 120 samples of size 3 and $S_1$ consists of the samples which contains the pair $(i, j)$ for $\delta(i, j) = 1, i \neq j = 1,2, . . ,10$.

| Sample No. | Samples | Sample No. | Samples | Sample No. | Samples |
|---|---|---|---|---|---|
| 1 | 1,2,3 | 41 | 2,3,8 | 81 | 3,7,9 |
| 2 | 1,2,4 | 42 | 2,3,9 | 82 | 3,7,10 |
| 3 | 1,2,5 | 43 | 2,3,10 | 83 | 3,8,9 |
| 4 | 1,2,6 | 44 | 2,4,5 | 84 | 3,8,10 |
| 5 | 1,2,7 | 45 | 2,4,6 | 85 | 3,9,10 |
| 6 | 1,2,8 | 46 | 2,4,7 | 86 | 4,5,6 |
| 7 | 1,2,9 | 47 | 2,4,8 | 87 | 4,5,7 |
| 8 | 1,2,10 | 48 | 2,4,9 | 88 | 4,5,8 |
| 9 | 1,3,4 | 49 | 2,4,10 | 89 | 4,5,9 |
| 10 | 1,3,5 | 50 | 2,5,6 | 90 | 4,5,10 |
| 11 | 1,3,6 | 51 | 2,5,7 | 91 | 4,6,7 |
| 12 | 1,3,7 | 52 | 2,5,8 | 92 | 4,6,8 |
| 13 | 1,3,8 | 53 | 2,5,9 | 93 | 4,6,9 |
| 14 | 1,3,9 | 54 | 2,5,10 | 94 | 4,6,10 |
| 15 | 1,3,10 | 55 | 2,6,7 | 95 | 4,7,8 |
| 16 | 1,4,5 | 56 | 2,6,8 | 96 | 4,7,9 |
| 17 | 1,4,6 | 57 | 2,6,9 | 97 | 4,7,10 |
| 18 | 1,4,7 | 58 | 2,6,10 | 98 | 4,8,9 |
| 19 | 1,4,8 | 59 | 2,7,8 | 99 | 4,8,10 |
| 20 | 1,4,9 | 60 | 2,7,9 | 100 | 4,9,10 |
| 21 | 1,4,10 | 61 | 2,7,10 | 101 | 5,6,7 |
| 22 | 1,5,6 | 62 | 2,8,9 | 102 | 5,6,8 |
| 23 | 1,5,7 | 63 | 2,8,10 | 103 | 5,6,9 |
| 24 | 1,5,8 | 64 | 2,9,10 | 104 | 5,6,10 |
| 25 | 1,5,9 | 65 | 3,4,5 | 105 | 5,7,8 |
| 26 | 1,5,10 | 66 | 3,4,6 | 106 | 5,7,9 |
| 27 | 1,6,7 | 67 | 3,4,7 | 107 | 5,7,10 |
| 28 | 1,6,8 | 68 | 3,4,8 | 108 | 5,8,9 |
| 29 | 1,6,9 | 69 | 3,4,9 | 109 | 5,8,10 |
| 30 | 1,6,10 | 70 | 3,4,10 | 110 | 5,9,10 |
| 31 | 1,7,8 | 71 | 3,5,6 | 111 | 6,7,8 |
| 32 | 1,7,9 | 72 | 3,5,7 | 112 | 6,7,9 |
| 33 | 1,7,10 | 73 | 3,5,8 | 113 | 6,7,10 |
| 34 | 1,8,9 | 74 | 3,5,9 | 114 | 6,8,9 |
| 35 | 1,8,10 | 75 | 3,5,10 | 115 | 6,8,10 |
| 36 | 1,9,10 | 76 | 3,6,7 | 116 | 6,9,10 |
| 37 | 2,3,4 | 77 | 3,6,8 | 117 | 7,8,9 |
| 38 | 2,3,5 | 78 | 3,6,9 | 118 | 7,8,10 |
| 39 | 2,3,6 | 79 | 3,6,10 | 119 | 7,9,10 |
| 40 | 2,3,7 | 80 | 3,7,8 | 120 | 8,9,10 |

These 120 possible samples are denoted by $S_1, S_2, ..., S_{120}$ and their probabilities are $p(S_1), p(S_2), \ldots, p(S_{120})$. Let us denote these probabilities by $p_1, p_2, \ldots, p_{120}$ respectively.

The preferred samples are the samples without contiguous units which as follows:

| **Sample No** | **Probability** | **Samples** |
|---|---|---|
| $S_{10}$ | $p_{10}$ | 1,3,5 |
| $S_{11}$ | $p_{11}$ | 1,3,6 |
| $S_{12}$ | $p_{12}$ | 1,3,7 |
| $S_{13}$ | $p_{13}$ | 1,3,8 |
| $S_{14}$ | $p_{14}$ | 1,3,9 |
| $S_{17}$ | $p_{17}$ | 1,4,6 |
| $S_{18}$ | $p_{18}$ | 1,4,7 |
| $S_{19}$ | $p_{19}$ | 1,4,8 |
| $S_{20}$ | $p_{20}$ | 1,4,9 |
| $S_{23}$ | $p_{23}$ | 1,5,7 |
| $S_{24}$ | $p_{24}$ | 1,5,8 |
| $S_{25}$ | $p_{25}$ | 1,5,9 |
| $S_{28}$ | $p_{28}$ | 1,6,8 |
| $S_{29}$ | $p_{29}$ | 1,6,9 |
| $S_{32}$ | $p_{32}$ | 1,7,9 |
| $S_{45}$ | $p_{45}$ | 2,4,6 |
| $S_{46}$ | $p_{46}$ | 2,4,7 |
| $S_{47}$ | $p_{47}$ | 2,4,8 |
| $S_{48}$ | $p_{48}$ | 2,4,9 |
| $S_{49}$ | $p_{49}$ | 2,4,10 |
| $S_{51}$ | $p_{51}$ | 2,5,7 |
| $S_{52}$ | $p_{52}$ | 2,5,8 |
| $S_{53}$ | $p_{53}$ | 2,5,9 |
| $S_{54}$ | $p_{54}$ | 2,5,10 |
| $S_{56}$ | $p_{56}$ | 2,6,8 |
| $S_{57}$ | $p_{57}$ | 2,6,9 |
| $S_{58}$ | $p_{58}$ | 2,6,10 |
| $S_{60}$ | $p_{60}$ | 2,7,9 |
| $S_{61}$ | $p_{61}$ | 2,7,10 |
| $S_{63}$ | $p_{63}$ | 2,8,10 |
| $S_{72}$ | $p_{72}$ | 3,5,7 |
| $S_{73}$ | $p_{73}$ | 3,5,8 |
| $S_{74}$ | $p_{74}$ | 3,5,9 |
| $S_{75}$ | $p_{75}$ | 3,5,10 |
| $S_{77}$ | $p_{77}$ | 3,6,8 |
| $S_{78}$ | $p_{78}$ | 3,6,9 |
| $S_{79}$ | $p_{79}$ | 3,6,10 |
| $S_{81}$ | $p_{81}$ | 3,7,9 |
| $S_{82}$ | $p_{82}$ | 3,7,10 |
| $S_{84}$ | $p_{84}$ | 3,8,10 |
| $S_{92}$ | $p_{92}$ | 4,6,8 |
| $S_{93}$ | $p_{93}$ | 4,6,9 |
| $S_{94}$ | $p_{94}$ | 4,6,10 |
| $S_{96}$ | $p_{96}$ | 4,7,9 |
| $S_{97}$ | $p_{97}$ | 4,7,10 |
| $S_{99}$ | $p_{99}$ | 4,8,10 |
| $S_{106}$ | $p_{106}$ | 5,7,9 |
| $S_{107}$ | $p_{107}$ | 5,7,10 |
| $S_{109}$ | $p_{109}$ | 5,8,10 |
| $S_{115}$ | $p_{115}$ | 6,8,10 |

When samples are arranged then it is observed that $S_1$ consists of sample numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 15, 16, 21, 22, 26, 27, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 50, 55, 59, 62, 64, 65, 66, 67, 68, 69, 70, 71, 76, 80, 83, 85, 86, 87, 88, 89, 90, 91, 95, 98, 100, 101, 102, 103, 104, 105, 108, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120. Hence, objective function is

$$\phi = p(s_1) + p(s_2)+\ldots+p(s_8) + p(s_{15}) + p(s_{16}) + p(s_{21})+\ldots+p(s_{120})$$

$$= p(s_1) + p(s_2)+\ldots+p(s_8)+\ldots+0.p(s_{15}) + 0.p(s_{16})+\ldots+0.p(s_{21}) + 0.p(s_{22}) + p(s_{23}) + p(s_{24})+\ldots+p(s_{120})$$

Unit 1 appears in 15 samples. For $i$ = 1, the constraint (i) is

$$\pi_1 = p_{10} + p_{11} + p_{12} + p_{13} + p_{14} + p_{17} + p_{18} + p_{19} + p_{20} + p_{23} + p_{24} + p_{25} + p_{28} + p_{29} + p_{32}$$

Similarly, other constraints for $i$ = 2, 3, …., 120 are set.

As per constraint of equation $p_1 + p_2+,\ldots,+p_{120}$ = 1

Then the first order inclusion probabilities are:

$$\pi_1 = p_{10} + p_{11} + p_{12} + p_{13} + p_{14} + p_{17} + p_{18} + p_{19} + p_{20} + p_{23} + p_{24} + p_{25} + p_{28} + p_{29} + p_{32}$$
$$\pi_2 = p_{45} + p_{46} + p_{47} + p_{48} + p_{49} + p_{51} + p_{52} + p_{53} + p_{54} + p_{56} + p_{57} + p_{58} + p_{60} + p_{61}$$
$$\pi_3 = p_{10} + p_{11} + p_{12} + p_{13} + p_{14} + p_{72} + p_{73} + p_{74} + p_{75} + p_{77} + p_{78} + p_{79} + p_{81} + p_{82} + p_{84}$$
$$\pi_4 = p_{17} + p_{18} + p_{19} + p_{20} + p_{45} + p_{46} + p_{47} + p_{48} + p_{49} + p_{92} + p_{93} + p_{94} + p_{96} + p_{97}$$
$$\pi_5 = p_{10} + p_{23} + p_{24} + p_{25} + p_{51} + p_{52} + p_{53} + p_{54} + p_{72} + p_{73} + p_{74} + p_{75} + p_{106} + p_{107} + p_{109}$$
$$\pi_6 = p_{11} + p_{17} + p_{28} + p_{29} + p_{45} + p_{56} + p_{57} + p_{58} + p_{77} + p_{78} + p_{79} + p_{92} + p_{93} + p_{94} + p_{115}$$
$$\pi_7 = p_{12} + p_{18} + p_{23} + p_{32} + p_{46} + p_{51} + p_{60} + p_{61} + p_{72} + p_{81} + p_{82} + p_{96} + p_{97} + p_{106} + p_{107}$$
$$\pi_8 = p_{13} + p_{19} + p_{24} + p_{28} + p_{47} + p_{52} + p_{56} + p_{73} + p_{77} + p_{92} + p_{99} + p_{109} + p_{115}$$
$$\pi_9 = p_{14} + p_{20} + p_{25} + p_{29} + p_{32} + p_{48} + p_{53} + p_{57} + p_{60} + p_{74} + p_{78} + p_{81} + p_{93} + p_{96} + p_{106}$$
$$\pi_{10} = p_{49} + p_{54} + p_{58} + p_{61} + p_{75} + p_{79} + p_{82} + p_{84} + p_{94} + p_{97} + p_{99} + p_{107} + p_{109} + p_{115}$$

From equation for *N=10, n=3* and *m=1*, the first order inclusion probabilities are:

$$\pi_i = nP_i \qquad \forall \quad i = 1,2,..,10$$

| | |
|---|---|
| $\pi_1$ | 0.54 |
| $\pi_2$ | 0.42 |
| $\pi_3$ | 0.39 |
| $\pi_4$ | 0.33 |
| $\pi_5$ | 0.30 |
| $\pi_6$ | 0.30 |
| $\pi_7$ | 0.24 |
| $\pi_8$ | 0.21 |
| $\pi_9$ | 0.15 |
| $\pi_{10}$ | 0.12 |

And second order inclusion probabilities are:

$$\pi_{ij} = 0 \quad ; \quad \delta(i,j) \geq m \;, \quad i \neq j = 1,2,\ldots,N$$

The constraints arranged according to example 3 and minimizing the objective function gives the following optimal solution with $\phi = 0$.

Now, giving the first order inclusion probabilities and solving by Microsoft 2019 Package and the above formulation of linear programming problem gives the following sampling plan given in Table 3.

Table 3. Circular IPPSEA plan for *N=10, n=3 and m=1*.

| $S_i$ | p(s) | $S_i$ | p(s) |
|---|---|---|---|
| $S_1$ | 0 | $S_{61}$ | 0.006859 |
| $S_2$ | 0 | $S_{62}$ | 0 |
| $S_3$ | 0 | $S_{63}$ | 0 |
| $S_4$ | 0 | $S_{64}$ | 0 |
| $S_5$ | 0 | $S_{65}$ | 0 |
| $S_6$ | 0 | $S_{66}$ | 0 |
| $S_7$ | 0 | $S_{67}$ | 0 |
| $S_8$ | 0 | $S_{68}$ | 0 |
| $S_9$ | 0 | $S_{69}$ | 0 |
| $S_{10}$ | 0.145085 | $S_{70}$ | 0 |
| $S_{11}$ | 0.118605 | $S_{71}$ | 0 |
| $S_{12}$ | 0.041248 | $S_{72}$ | 0.000161 |

| | | | |
|---|---|---|---|
| $S_{13}$ | 0.066686 | $S_{73}$ | 0 |
| $S_{14}$ | 0 | $S_{74}$ | 0.000602 |
| $S_{15}$ | 0 | $S_{75}$ | 0 |
| $S_{16}$ | 0 | $S_{76}$ | 0 |
| $S_{17}$ | 0.0214 | $S_{77}$ | 0 |
| $S_{18}$ | 0.02496 | $S_{78}$ | 0.008889 |
| $S_{19}$ | 0.019439 | $S_{79}$ | 0.070031 |
| $S_{20}$ | 0 | $S_{80}$ | 0 |
| $S_{21}$ | 0 | $S_{81}$ | 0.008889 |
| $S_{22}$ | 0 | $S_{82}$ | 0 |
| $S_{23}$ | 0.044739 | $S_{83}$ | 0 |
| $S_{24}$ | 0.017727 | $S_{84}$ | 0 |
| $S_{25}$ | 0 | $S_{85}$ | 0 |
| $S_{26}$ | 0 | $S_{86}$ | 0 |
| $S_{27}$ | 0 | $S_{87}$ | 0.006859 |
| $S_{28}$ | 0.017727 | $S_{88}$ | 0 |
| $S_{29}$ | 0.012064 | $S_{89}$ | 0 |
| $S_{30}$ | 0 | $S_{90}$ | 0 |
| $S_{31}$ | 0 | $S_{91}$ | 0 |
| $S_{32}$ | 0.010325 | $S_{92}$ | 0 |
| $S_{33}$ | 0 | $S_{93}$ | 0.010757 |
| $S_{34}$ | 0 | $S_{94}$ | 0 |
| $S_{35}$ | 0 | $S_{95}$ | 0 |
| $S_{36}$ | 0 | $S_{96}$ | 0.010757 |
| $S_{37}$ | 0 | $S_{97}$ | 0 |
| $S_{38}$ | 0 | $S_{98}$ | 0 |
| $S_{39}$ | 0 | $S_{99}$ | 0 |
| $S_{40}$ | 0 | $S_{100}$ | 0 |
| $S_{41}$ | 0 | $S_{101}$ | 0 |
| $S_{42}$ | 0 | $S_{102}$ | 0 |
| $S_{43}$ | 0 | $S_{103}$ | 0 |
| $S_{44}$ | 0 | $S_{104}$ | 0 |
| $S_{45}$ | 0.053296 | $S_{105}$ | 0 |
| $S_{46}$ | 0.036137 | $S_{106}$ | 0 |
| $S_{47}$ | 0.030616 | $S_{107}$ | 0 |
| $S_{48}$ | 0.023214 | $S_{108}$ | 0 |
| $S_{49}$ | 0.099424 | $S_{109}$ | 0 |
| $S_{50}$ | 0 | $S_{110}$ | 0 |
| $S_{51}$ | 0.034424 | $S_{111}$ | 0 |
| $S_{52}$ | 0.028904 | $S_{112}$ | 0 |
| $S_{53}$ | 0.021502 | $S_{113}$ | 0 |
| $S_{54}$ | 0.006859 | $S_{114}$ | 0 |
| $S_{55}$ | 0 | $S_{115}$ | 0 |
| $S_{56}$ | 0.028904 | $S_{116}$ | 0 |
| $S_{57}$ | 0.021502 | $S_{117}$ | 0 |
| $S_{58}$ | 0.006859 | $S_{118}$ | 0 |
| $S_{59}$ | 0 | $S_{119}$ | 0 |
| $S_{60}$ | 0.021502 | $S_{120}$ | 0 |

It is observed that the inclusion probabilities are proportional to initial probability of selection of the units and $\pi_{ij} = 0$ for $\delta(i,j) = 1, i \neq j = 1, 2, \dots, 10.$ Therefore, the above sampling plan is IPPSEA plan.

## IV. Conclusion

Some previous researchers have given IPPSEA plan based on SAS coding, comparing my methodology with them. I observed that in proposed methodology, no. of preferred samples selected are more. Again, probability of these preferred samples is also high in most of the cases. This methodology of IPPSEA is more efficient on comparing with the SRSWOR. SAS software is an expensive software and one needs to a license to operate it. It is not affordable by everyone as well as its courses are also costly. One must have an interest in complex programming to understand it easily, whereas Microsoft is available with installed windows in computers and it is accessible to all. It is simple to use and ease the calculation to understand. I can say that this methodology is less cumbersome and equally or more efficient than previous work.

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