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# The Empirical Investigation of the effect on Mixture Distributions in Black-Scholes Option Pricing Model

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***Abstract***
*In this paper we empirically evaluates the effect of mixed lognormal-Weibull Distributions (MLWD) in Black-Scholes Call Option Pricing Model.The data for this study were obtained from Australian Clearing House of Australian Securities Exchange (ASX) which consists of 50 enlisted stocks in the clearing house as products of monthly market summary for long term options from 3rd January, 2017 to 31st December, 2019. The data were properly arranged according to 25, 27, 28, 29 and 30 maturity days. With the help R-package, the maximum Likelihood Estimate (MLE) was used to obtain the parameters of MLWD and the goodness of fit test was conducted to find how best fit the MLWD is in Black-Scholes Option Pricing Model and the result revealed that under the null hypothesis of a good fit is accepted (P>0.05) only at the maturity days of 25 and 27 and rejected (P<0.05) at the maturity days of 28, 29 and 30. Hence, we affirmed that MLWD is a good fit in Black- Scholes Option Pricing Model at shorter maturity days and at small sample size but not useful when options have longer days of expiration and when the options are so large.*

***Key Words: Black-Scholes, Option Pricing, Mixture Distribution, Goodness – Of – Fit Test***

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## I. Introduction

The original Black-Scholes model of 1973 has undergone several theoretical developments in recent times. One of such developments for the valuation of options is introduced by Black (1976) where he proposed a formula for options under the assumption that investors generalized risk less hedges between options.

A number of option pricing models are now available in recent times after the foundation of models laid by Fisher Black and Myron Scholes and Black of 1973 and 1976 respectively. See for example; Black-Scholes-Merton (1979), GARCH option of Heston (1993), Stochastic Volatility model of Heston and nandi (2000), German-Kohagen-Black-Scholes-Merton model (1983), Jump and Jump Diffusion model of Bates (1996), Variance-Gamma model by Madan and Seneta (1991), Trautman and Beinert (1994), Carr and Madan (1999), Savickas (2002), Nwobi, Ugomma and Ohaegbulem (2021) and Ugomma and Nwobi (2022).

The mixture of two or more component distributions is the newest area of concern in option pricing and reliability studies. Several researchers have proposed the mixture of two or more distributions in pricing options as alternative to the original Black-Scholes option pricing model of 1973. See for example, KaceciloguandWang (1998), Neumann (1998), Razali, et al (2008), Kollu, (2012), Sultan and Al-Moisher (2015) and Elmahdy (2017).

Therefore, in this study, we combine the lognormal and Weibull distributions using Maximum Likelihood Estimate method to obtain their parameters, their properties and then apply empirically the mixed models to Black-Scholes option pricing model to ascertain the fit of the mixed Model to Black-Scholes.

## II. Method

### *2.1 Mixed Lognormal-Weibull Distribution (MLWD)*

Assume that the population consists of a mixture of two independent sub-population with zero correlation and each population has its unique properties.

The distribution for the mixed population can be expressed as:

$$f\left(x_i\right)=\sum_{i=1}^{n} w_i g_i\left(x_i;\theta_i\right) \quad (1)$$

where $0\le w_i\le 1, \sum w_i=1, i=1,2,...,n$, $\theta_i$ are the parameters representing the mixed distribution and $w_i$ are mixing parameters which represents the proportion of combining a number of distribution The probability density function (pdf) of the mixture distribution in (1) is expressed as:

$$f\left(x_i;w,\theta_i\right)=wf_1\left(x_i;\theta_1\right)+\left(1-w\right)f_2\left(x_i;\theta_2\right) \quad (2)$$

Where w and (1 – w) are the mixing parameters whose sum is equal to 1.
The pdf of lognormal and Weibull distributions are given respectively as:

$$f\left(x_1\right)=\frac{1}{\sqrt{2\pi\sigma^2 x}}\exp\left\{-\frac{1}{2}\frac{\left(\ln x-\mu\right)^2}{\sigma^2}\right\},x\geq 0,\sigma>0 \quad (3)$$

$$f\left(x_2\right)=\frac{\beta}{\sigma}\left(\frac{x}{\alpha}\right)^{\beta-1}\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\},0\leq x\leq\infty,\beta,\sigma>0 \quad (4)$$

and the respective cumulative distribution function of (3) and (4) are given as

$$F\left(x_1\right)=\frac{\left(\ln x-\mu\right)}{\sigma},x\geq 0,-\infty<\mu<\infty,\sigma>0 \quad (5)$$

and

$$F\left(x_1\right)=\exp\left\{-\frac{x}{\alpha}\right\}^{\beta},x\geq 0,\beta,\alpha>0 \quad (6)$$

So, the joint Pdf of (3) and (4) can be expressed as

$$\begin{aligned} f\left(x_i;\theta_i\right)&=g_1\left(x_i;\theta_1\right)+g_2\left(x_i;\theta_2\right)\\ &=wf_1\left(x_i;\theta_1\right)+\left(1-w\right)f_2\left(x_i;\theta_2\right) \end{aligned} \quad (7)$$

Substituting (3) and (4) into (2), we obtain the joint pdf of the mixing distributions as

$$f\left(x_i,w,\mu,\sigma^2,\beta,\alpha\right)=w\left(\begin{array}{l}\frac{1}{\sqrt{2\pi\sigma^2 x}}\exp\left\{-\frac{1}{2}\left(\frac{\ln x-\mu}{\sigma^2}\right)^2\right\}\\ +\left(1-w\right)\frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1}\exp\left\{-\frac{x}{\alpha}\right\}^{\beta}\end{array}\right) \quad (8)$$

and the joint cumulative distribution function in (6) can be given as

$$F\left(x_i,\mu,\sigma^2,\beta,\alpha\right)=\frac{w\left(\ln x-\mu\right)}{\sigma}+\left(1-w\right)\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\} \quad (9)$$

### *2.1.1 **The Maximum Likelihood Estimation of the parameters of MLWD***

Menden and Harter (1958) obtained the MLE for the scale and mixing parameters where the shape parameter was assumed to be known. Several authors have found Maximum Likelihood Estimation method very helpful in obtaining the parameters of mixture distributions (see, for example, Ashour, (1987); Ahmed and Abdurahman, (1994))

The maximum likelihood approach considered for this study for the estimation of the parameters of the mixed distribution density function in (8) is based on a random sample of size n. The MLE $\hat{\theta}$ is obtained as the solution of the likelihood equation as:

$$\frac{\partial\theta}{\partial\theta_i}=0,i=1,2,...,n \quad (10)$$

Or equivalently, the partial derivative of the log likelihood function given as

$$\frac{\partial\log L\left(\theta\right)}{\partial\theta_i}=0$$

$$=\sum_{i=1}^{n}\log\left(f\left(x\right)\right)$$

where

$$L\left(\theta\right)=\prod_{i=1}^{n}f\left(x_i;\theta_i\right) \quad (11)$$

Therefore, the likelihood function corresponding to the mixture density in (8) is then expressed as

$$L(\theta)=\prod_{i=1}^{n}\left[w\left(f_1(x_i;\theta_1)\right)+(1-w)\left(f_2(x_i;\theta_2)\right)\right] \qquad (12)$$

where $\theta_1=(\mu,\sigma)$ and $\theta_2=(\alpha,\beta)$.

This implies that

$$f\left(x_i,w,\mu,\sigma^2,\beta,\alpha\right)=\prod_{i=1}^{n}\begin{bmatrix} w\left(2\pi\sigma^2\right)^{-\frac{1}{2}}\frac{1}{x}\exp\left\{-\frac{1}{2}\left(\frac{\ln x-\mu}{\sigma^2}\right)^2\right\} \\ +(1-w)\frac{\beta}{\alpha}\left(\frac{x}{\sigma}\right)^{\beta-1}\exp\left\{-\frac{x}{\alpha}\right\}^{\beta}\end{bmatrix} \qquad (13)$$

$$=w\left[\left(2\pi\sigma^2\right)^{-\frac{n}{2}}\prod_{i=1}^{n}\frac{1}{x}\exp\left\{\sum_{i=1}^{n}-\frac{1}{2}\left(\frac{\ln x-\mu}{\sigma^2}\right)^2\right\}\right]+$$

$$\left[(1-w)\frac{\beta}{\alpha}\left(\frac{x}{\sigma}\right)^{n\beta-n}\sum_{i=1}^{n}x_i^{\beta-1}-\ln\left(\alpha^{\beta-1}\right)\exp\left\{-\sum_{i=1}^{n}\frac{x}{\alpha}\right\}^{\beta}\right] \qquad (14)$$

Taking the log likelihood function for the mixture distribution in (14), we obtain

$$\ell(x_i;\theta_1,\theta_2)=\left(\log\left(w\left(-\frac{n}{2}\ln\left(2\pi\sigma^2\right)-\sum_{i=1}^{n}\ln x-\sum_{i=1}^{n}\frac{\ln x^2}{2\sigma^2}+\sum_{i=1}^{n}\left(\frac{2\ln x-\mu}{2\sigma^2}\right)-\frac{n\mu}{2\sigma^2}\right)\right)\right)$$

$$+\left(\log\left((1-w)n\ln\left(\frac{\beta}{\alpha}\right)+(\beta-1)\sum_{i=1}^{n}x_i-\ln\left(\alpha^{\beta-1}\right)-\sum_{i=1}^{n}\left(\frac{x}{\alpha}\right)^{\beta}\right)\right) \qquad (15)$$

Let $Q$ be the function of the log likelihood such that

$$Q=\left(w,\mu,\sigma^2,\alpha,\beta\right)=\sum_{i=1}^{n}\left[\log\begin{pmatrix} w\left(-\frac{n}{2}\ln\left(2\pi\sigma^2\right)-\ln x-\frac{\ln x^2}{2\sigma^2}+\frac{\ln x-\mu}{2\sigma^2}-\frac{n\mu}{2\sigma^2}\right) \\ +\left((1-w)n\ln\left(\frac{\beta}{\alpha}\right)+(\beta-1)x_i-\ln\left(\alpha^{\beta-1}\right)-\left(\frac{x}{\alpha}\right)^{\beta}\right)\end{pmatrix}\right] \qquad (16)$$

Taking the partial derivative of the log likelihood function of (16) w.r.t the parameters and in turn equating to zero yields the following equations

$$\frac{\partial Q}{\partial\mu}=\sum_{i=1}^{n}\frac{\ln x}{\sigma^2}-\frac{n\mu}{\sigma^2}=0 \qquad (17)$$

$$=\frac{n\mu}{\sigma^2}=\sum_{i=1}^{n}\frac{\ln x}{\sigma^2}\Rightarrow\hat{\mu}_{mix}=\sum_{i=1}^{n}\frac{\ln x}{n}$$

$$\frac{\partial Q}{\partial\sigma^2}=-\frac{n}{2\sigma^2}-\sum_{i=1}^{n}\left(\frac{\ln x-\mu}{2}\right)^2\left(-\sigma^2\right)^{-2}$$

$$=-\frac{n}{2\sigma^2}-\sum_{i=1}^{n}\left(\frac{\ln x-\mu}{2\sigma^4}\right)^2=0$$

$$=-\frac{n}{2\sigma^2}=\sum_{i=1}^{n}\left(\frac{\ln x-\mu}{2\sigma^4}\right)^2\Rightarrow n=\sum_{i=1}^{n}\left(\frac{\ln x-\mu}{\sigma^2}\right)^2$$

$$=\hat{\sigma}^2_{mix}=\sum_{i=1}^{n}\left(\frac{\ln x-\hat{\mu}}{n}\right)^2 \qquad (18)$$

$$\frac{\partial Q}{\partial \beta}=\frac{n}{\beta}+\sum_{i=1}^{n}x_i-\frac{1}{\alpha}\sum_{i=1}^{n}x_i^{\beta}-\frac{1}{\alpha}=0$$

$$=\frac{1}{\beta}+\frac{1}{\alpha}\sum_{i=1}^{n}\ln x_i-\frac{\sum_{i=1}^{n}x_i^{\beta}\ln x_i}{\sum_{i=1}^{n}x_i^{\beta}}=0$$

(19)

$$=\frac{1}{\beta}+\frac{1}{n}\sum_{i=1}^{n}\ln x_i-\ln x_i=0\Rightarrow\hat{\beta}_{mix}=\frac{1}{\ln x_i-\frac{1}{n}\sum_{i=1}^{n}x_i}$$

$$\frac{\partial Q}{\partial \alpha}=-\frac{n}{2}+\frac{1}{\alpha^2}\sum_{i=1}^{n}x_i^{\beta}=0\Rightarrow\hat{\alpha}_{mix}=\frac{1}{n}\sum_{i=1}^{n}x_i^{\hat{\beta}_{mix}} \tag{20}$$

$$\frac{\partial Q}{\partial w}=\frac{f_1(x_i;\theta_1)-f_2(x_i;\theta_2)}{wf_1(x_i;\theta_1)+(1-w)f_2(x_i;\theta_2)}=\frac{f_1(x_i;\mu,\sigma^2)-f_2(x_i;\alpha,\beta)}{wf_1(x_i;\mu,\sigma^2)+(1-w)f_2(x_i;\alpha,\beta)} \tag{21}$$

2.1.2 **Some Properties of MLWD**

(i) The Mean:

$$E(X)=w\left(\exp\left\{\mu+\frac{1}{2}\sigma^2\right)+(1-w)\alpha\Gamma\left(1+\frac{1}{\beta}\right) \tag{22}$$

(ii) The variance

$$Var(X)=E(X)-[EX]^2$$

$$=w\left(\exp\left\{-2\mu+\sigma^2\right)\left(\exp\left\{-\left(\sigma^2-w\right)\right)+(1-w)\alpha^2\Gamma\left(1+\frac{2}{\beta}\right)$$

(23)

$$-(1-w)\Gamma^2\left(1+\frac{1}{\beta}\right)-2w(1-w)\left(\exp\left\{-\left(\mu+\frac{1}{2}\sigma^2\right)\Gamma\left(1+\frac{2}{\beta}\right)\right)$$

(iii) The Skewness

$$skew(X)=w\left(\exp\left\{\sigma^2+2\right)\sqrt{\left(\exp\left\{\sigma^2-1\right)}$$

(24)

$$+(1-w)\frac{1}{\alpha^{\frac{1}{\beta}}}\left[(2\gamma_1)^3-3\gamma_1\Gamma\left(1+\frac{2}{\beta}\right)+\Gamma\left(1+\frac{3}{\beta}\right)\right]$$

(iv) The kurtosis

$$kurt(X)=w\left(\exp\left\{4\sigma^2\right)+2\left(\exp\left\{\sigma^4\right)+3\exp\left(\sigma^4\right)-3\right)$$

(25)

$$+(1-w)\Gamma\left(1+\frac{4}{\beta}\right)-4\gamma_1\Gamma\left(1+\frac{4}{\beta}\right)+6\gamma_1^2\Gamma\left(1+\frac{2}{\beta}\right)-\frac{3\gamma_1^4}{\gamma_2}$$

Where $\gamma_1=\Gamma\left(1+\frac{1}{\beta}\right),\ \gamma_2=\left[\Gamma\left(1+\frac{2}{\beta}\right)-\Gamma^2\left(1+\frac{1}{\beta}\right)^2\right]$

(i) Reliability (survival) function

$$R\left(X_t/\theta_i\right)=\sum_{i=1}^{n}\left[w\left(1-\Phi\left(\frac{\ln(x)-\mu}{\sigma}\right)^2\right)+(1-w)\left(1-\exp\left\{\frac{X_t}{\alpha}\right)^2\right] \tag{26}$$

(ii) Hazard function

$$h\left(X_t/\theta_i\right)=\sum_{i=1}^{n}\left(w\left[\frac{\frac{1}{\sqrt{2\pi\sigma x}}\exp\left\{-\frac{1}{2}\left(\frac{\ln(x)-\mu}{\sigma}\right)^2\right]}{1-\Phi\left(\frac{\ln(x)-\mu}{\sigma}\right)^2}\right]+(1-w)\left(\frac{\beta}{\sigma}\right)\left(\frac{X_t}{\alpha}\right)^{\beta-1}\right. \tag{27}$$

## 2.2 The Black-Scholes Option Pricing Model under MLWD.

The price of the call option is given as

$$C_t\left(X_t,K\right)=e^{-rT}\int_0^\infty\left(X_t-K\right)f\left(X_t\right)dX_t \tag{28}$$

where $f\left(X_t\right)$ denotes the probability density function of $X_t$ evaluated as the result of $X_t$.

Consequently, the mixture distribution for the Black-Scholes Option Pricing Model is expressed as

$$f_{\theta_i}^{mix}\left(x_t,\theta_1,\theta_2\right)=wf_1\left(x_t,\theta_1\right)+(1-w)\,f_2\left(x_t,\theta_2\right)$$

$$=wf_1^{\log}\left(x_t,\theta_1\right)+(1-w)\,f_2^{web}\left(x_t,\theta_2\right)$$

$$=e^{-rT}\left[\int_0^\infty wf_1^{\log}\left(x_t,\theta_1\right)+(1-w)\,f_2^{web}\left(x_t,\theta_2\right)\right. \tag{29}$$

Substituting the Black-Scholes Models for both lognormal and Weibull respectively, we obtain

$$e^{-rT}\left[w\left(\frac{X_0\Phi\ln\left(\frac{X_0}{K}\right)-\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}-Ke^{-rT}\ln\left(\frac{X_0}{K}\right)-\left(r-\frac{\sigma^2}{2}\right)T\right)+(1-w)X_0\left(\alpha\Gamma\left(1+\frac{1}{\beta}\right)-\Gamma_y\left(\frac{1+\frac{1}{\beta}}{\Gamma\left(1+\frac{1}{\beta}\right)}\right)\right)-Ke^{-rT}\exp(-y)\right] \tag{30}$$

$$e^{-rT}\left[w\left(\frac{X_0\Phi\ln\left(\frac{X_0}{K}\right)-\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}-w\left(Ke^{-rT}\ln\left(\frac{X_0}{K}\right)-\left(r-\frac{\sigma^2}{2}\right)T\right)\right)+(1-w)X_0\left[\alpha\Gamma\left(1+\frac{1}{\beta}\right)-\Gamma_y\left(\frac{1+\frac{1}{\beta}}{\Gamma\left(1+\frac{1}{\beta}\right)}\right)-(1-w)\left(Ke^{-rT}\exp(-y)\right)\right]\right] \tag{31}$$

Collecting like terms together, we have

$$e^{-rT}\left[w(1-w)X_0\left(\frac{\Phi\ln\left(\frac{X_0}{K}\right)-\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}-\left(\alpha\Gamma\left(1+\frac{1}{\beta}\right)-\left(\frac{\Gamma_y\left(1+\frac{1}{\beta}\right)}{\Gamma\left(1+\frac{1}{\beta}\right)}\right)\right)\right)-w(1-w)Ke^{-rT}\left(\frac{\ln\left(\frac{X_0}{K}\right)-\left(r-\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}+\exp(-y)\right)\right] \tag{32}$$

$$=\left[w(1-w)X_0\Phi d_1-w(1-w)Ke^{-rT}\Phi d_2\right]$$

$$\therefore C_{tMix}=w(1-w)\left[X_0\Phi d_1-Ke^{-rT}\Phi d_2\right] \tag{33}$$

## *2.3 Testing Procedure*

The technique adopted for this study will estimate the absolute returns of the underlying price and the volatility from annualized standard deviation (implied volatility) using log – difference of option prices that equates to theoretical option pricing models.

The data in each of the maturity days (expiration time) were tested in accordance with 252 trading days. In order to get the implied volatility of the models, we first estimate the historic volatility (standard deviation) of option prices using opening and closed prices as underlying and strike prices respectively.

*Computation of the Annualize (Implied Volatility)*

Let $X_i = ABS\ln\left(\frac{X_t}{X_{t-1}}\right)$ , $X_t$ is the underlying option price at time t.

$\bar{x} = \frac{1}{n}\sum_{i-1}^{n} x_i$ and $\sigma_X^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$

So that the implied volatility is obtained by

$$\hat{\sigma}_{im} = \sqrt{\frac{\sigma_x^2}{\Delta t}}$$

where, $\Delta t = t_i, t_{i-1}, ..$

$$= \sqrt{\frac{T}{n}\frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2} = \sqrt{\frac{T}{n}\text{var}(x)} \qquad (34)$$

where T is 252 trading days per annum and n is number of stocks.
and the rate of return is estimated by

$$r = \frac{1}{T}\ln\left(\frac{K}{X_0}\right) \qquad (35)$$

## III. Results and Discussions

### 3.1 *Data Description*

The data for this study were obtained from Australian Clearing House of Australian Securities Exchange (ASX). The sample consists of fifty (50) enlisted stocks in the clearing house as products of monthly market summary for long term options which consists of the period of January, 3rd 2017 to December, 31st 2019 when there are no significant structural changes among the products. For each transaction, our sample contains the following information: the opening and closing dates of the options, option prices comprising opening and closing prices otherwise referred in our case as the underlying and strike prices respectively. The final sample consists of 50 stocks for the period of 36 months (720 trading days). The maturity period of the options was gotten from the difference between the opening date and closing date of the options over the trading days. The data for the analysis were arranged in accordance to the maturity days of 25, 27, 28, 29 and 30 days. The data were actually obtained at http//www.asx.com/au/product/equity_options/options_statistics.htm.

#### 3.1.1 **Descriptive Statistics of *ASX Original Data***

***Table 1: Summary Statistics of ASX Original Data***

| Maturity Days | Sample Size | Sample Mean $\left(\bar{X}\right)$ | Sample Variance $\left(S^2\right)$ | Sample Std. Dev $\left(S\right)$ | Skewness $Skew\left(X\right)$ | Kurtosis $Kurt\left(X\right)$ | Implied Volatility $\left(\hat{\sigma}_{im}\right)$ | Rate of Return (r) |
|---|---|---|---|---|---|---|---|---|
| **25** | 99 | 0.0034 | 2.5495 | 1.5967 | -0.5583 | 4.2359 | 2.55 | -0.01 |
| **27** | 199 | 0.0019 | 2.6503 | 1.6280 | -0.5529 | 4.2529 | 1.83 | -0.02 |
| **28** | 399 | 0.00092 | 2.7046 | 1.6446 | -0.5453 | 3.9624 | 1.31 | -0.01 |
| **29** | 449 | 0.00053 | 2.8253 | 1.6809 | -0.5616 | 4.1640 | 1.26 | -0.03 |
| **30** | 499 | 0.00057 | 2.7430 | 1.6562 | -0.5544 | 4.0257 | 1.18 | 0.01 |

Table 1 displays the descriptive statistics of the original data of Australian Stock Exchange for the period under study. From the results, we observed that the skewness from the various maturity days are all negative indicating that the left tail of the distribution is longer than the right and their kurtosis also suggested that the distribution is perfectly peaked. This result indicates that the options from ASX is not normally distributed.

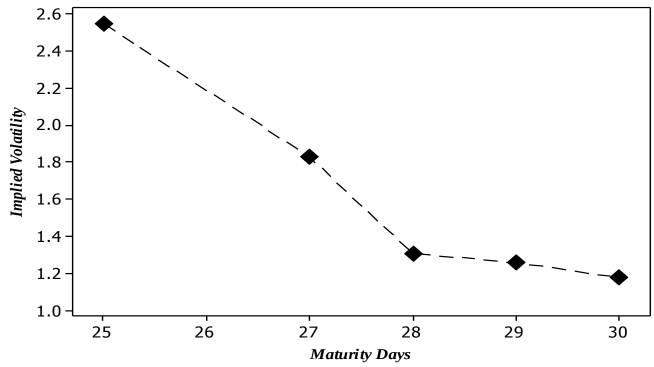


*Fig 1 The plot of Implied Volatility against Maturity Days*

Since volatility significantly affects the option prices, meaning the higher the volatility of the underlying asset; the higher the price of call options. Figure 1 displays high volatility of trading when options expired in 25 days, dropped significantly when the options were traded especially at maturity days of 27 to 30 days. This infers that there is more risk of trading options when the maturity or expiration days are less than when they are high.

***Table 2: Summary Statistics of Absolute Returns of ASX Original Data***

| Maturity Days | Sample Size | Sample Mean | Sample Standard Dev | Skewness | Kurtosis | Implied Volatility | Rate of Return (r) |
|---|---|---|---|---|---|---|---|
| **25** | 99 | 1.1983 | 1.0483 | 1.5414 | 6.0265 | 1.6726 | -0.01 |
| **27** | 199 | 1.2150 | 1.0801 | 1.5255 | 5.8071 | 1.2154 | -0.02 |
| **28** | 399 | 1.2408 | 1.0780 | 1.4158 | 5.2195 | 0.8564 | -0.01 |
| **29** | 449 | 1.2593 | 1.1117 | 1.4866 | 5.6435 | 0.8329 | -0.03 |
| **30** | 499 | 1.2466 | 1.0890 | 1.4330 | 5.3350 | 0.7740 | 0.01 |

The output in Table 2 shows an approximately equal sample means and standard deviations for all the maturity days. The result further indicates that all skewness are positive, thereby showing the right tail of the distribution is longer than left tail. Hence, this result proved that the absolute returns of ASX data for the period of study are normally distributed.

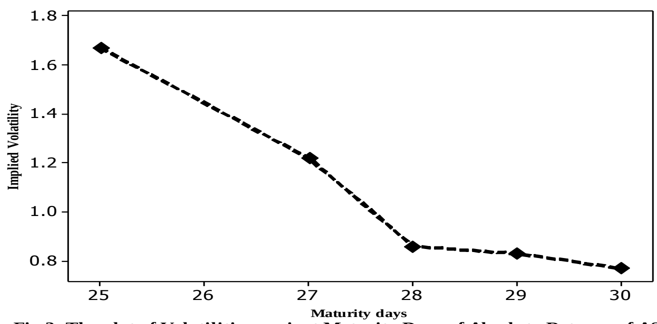


**Fig 2: The plot of Volatilities against Maturity Days of Absolute Returns of ASX Data**

Fig 2, shows that the implied volatility for 25 and 27 maturity days were higher than 28, 29 and 30 maturity days, hence, the lesser the maturity days, the higher the volatility, and vice-versa. This means that trading options reduces the risk involved if the options are allowed to mature at its expiration date mostly for the investors.

### *3.12 Evaluation of MLWD in Black-Scholes Call Option Pricing Model*

Here, we test the null hypothesis whether MLWD option pricing model is a good fit for pricing options against the alternative that it is not a good fit for option pricing model

$H_0$ : MLWD is a good fit for Black-Scholes Option Pricing Model

$H_1$ : MLWD is not a good fit for Black-Scholes Option Pricing Model

***Table 3: Summary Statistics of MLWD Parameters using the Absolute Returns of ASX Data***

| Maturity Days | Sample Size | Mean | Std. Dev. | Skewness | Kurtosis |
|---|---|---|---|---|---|
| **25** | 99 | 9.2541 | 11.5595 | 20.26 | 79.27 |
| **27** | 199 | 7.8939 | 7.6259 | 22.42 | 102.26 |
| **28** | 399 | 12.8939 | 18.3859 | 22.28 | 102.96 |
| **29** | 449 | 11.6112 | 15.7512 | 24.71 | 134.88 |
| **30** | 499 | 9.6934 | 11.5192 | 23.10 | 111.98 |

Table 3, shows that MLWD is positive skewed and has excess kurtosis. It indicates that the distribution is right tailed and also leptokurtic in nature.

***Table 4: Goodness – of – Test for MLWD Parameters using the Absolute Returns of ASX Data***

| Maturity Days | Sample Size | $\chi^2$ | $df$ | P-Value | Decision |
|---|---|---|---|---|---|
| **25** | 100 | 9408 | 9312 | 0.2401 | Accept |
| **27** | 200 | 38400 | 0.0822 | 0.2433 | Accept |
| **28** | 400 | 149600 | 148478 | 0.0199 | Reject |
| **29** | 450 | 187650 | 185565 | 0.0003 | Reject |
| **30** | 500 | 222055 | 220720 | 0.0224 | Reject |

From Table 4, we observed that the null hypothesis of a good fit is accepted (P>0.05) only at the maturity days of 25 and 27 and rejected (P<0.05) at the maturity days of 28, 29 and 30. Hence, we affirmed that MLWD is a good fit in Black- Scholes Option Pricing Model at shorter maturity days and at small sample size but not useful when options have longer days of expiration and when the options are so large.

## IV. Conclusion

In this study, we empirically evaluate the effect of two mixture distributions namely the lognormally and Weibull distributions in Black-Scholes option pricing model using the goodness – of – fit test and we observed that the mixed model was a good fit only when the options have shorter maturity days and with small sample size than the longer days. Therefore, we conclude that this mixed model should only be applied when the options have shorter expiration (maturity) days with small sample sizes.


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## References


[1]. Ahmad, K.E &AbdulRaham, A.M (1994). Upgrading a Nonlinear discriminate Function Estimated from a Mixture of two Weibull Distributions. Mathematics and Computer Modeling, 18, 41 – 51.

[2]. Ashour, S.K (1987). Multicensored Sampling in Mixed Weibull Distribution. IAPQR Transaction, 12, 51 – 56.

[3]. Bates, D (1996): Jumps & Stochastic Volatility. Exchange rate process implicit in Deutschmark options. Review of financial studies. 9, 69 – 108.

[4]. Black, F (1976): Studies of stock Market Volatility Changes. Proceedings of American Statistical Association, Business and Economic Statistics section 1(1):177 – 181.

[5]. Black, F & Scholes, M (1973): The Pricing of Options and Corporate Liabilities. The Journal of political Economy. 8(3).

[6]. Carr, P and Madan, D.B (1999): Option Valuation Using Fast Fourier Transform. Journal Financial Economics.

[7]. Elmahdy. E.E (2017). Modeling Reliability Data with Finite Weibull or Lognormal Mixture Distributions. International Journal of Mathematics & Information Science.11 (4), 1081 – 1089.

[8]. German, M &Kohlhagen, S (1983): Foreign Currency Option Value. Journal of International Money and Finance, 2(3).

[9]. Heston, S & Nandi, S (2000). A closed – form GARCH option valuation model. Review of Financial studies 13, 585 – 625.

[10]. Heston, S.L (1993): A closed form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options. The Review of Financial Studies 6(2), 327 – 343.

[11]. Kacecilogu, D & Wang, W (1998). Parameter Estimation for Mixed-Weibull Distribution. IEEE Proceedings Annual Reliability and Maintainability Symposium, USA.

[12]. Kollu, R, Rayapudi, S.R, Narasimham, SVL and Pakkurth, K.M (2012). Mixture Probability Distribution Functions to Model Wind Speed distribution. International Journal of Energy and Environmental Engineering, 3(27)

[13]. Madan, D.B &Seneta, E (1991): The Variance – Gamma Model for share market Returns. Journal of Business. 63, 4, 511 – 524.

[14]. Mendell, H & Harter, R.J (1958). Estimation of Parameters of Mixed Exponentially Distributed Failure Time Distributions from censored life tested Data. Biometrika, 45, 504 – 520.

[15]. Neuman, M (1998). Options Pricing Under Mixture of Distributions Hypothesis. Institut fur Entscheidungstheorie und Unternehmensforschung, Universtat Karlsruhe, Germany

[16]. Nwobi, F.N, Ugomma, C.A &Ohaegbulem, E.U (2021). An Empirical Evaluation of Lognormality in Black-Scholes Option Pricing Model. International Journal of Scientific and Research Publications, 11(11), 374 – 383.

[17]. Razali, A.M, Salih, A.A, Mahdi, A.A, Zaharim, A, Ibrahim, K &Sopian, K (2008). On Simulation Study of Mixture of Two Weibull Distribution Proceedings of the 7th WSEAS International Conference, SYSTEM SCIENCE and SIMULATION in Engineering.
[18]. Savickas, R (2002). A simple Option Pricing Formula. Working Paper. Department of Finance George Washington University, Washington DC.
[19]. Sultan, K.S & Al-Moisher, A.S (2015). Mixture of Inverse Weibull and Lognormal Distributions: Properties, Estimation and illustration. Hindawaii Corporation, Mathematical Problems in Engineering. Article ID 526786.
[20]. Trautmann, S &Beinert, M (1994): Stock Price Jumps and their impact on Option Valuation. Johannes Gutenberg Universitat, Mainz, Germany Working Paper.
[21]. Ugomma, C.A &Nwobi, F.N (2022). Empirical Evaluation of Weibull Distribution in Black-Scholes Call Option Pricing Model. International Journal of Scientific Engineering and Applied Science (IJSEAS), 8(3), 58 – 65.