

New Forms of Continuous Maps in Topological Spaces

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ABSTRACT: In this Article, we introduced a new forms of functions called pre- β wg-continuous functions using β wg-closed sets in topological spaces and obtain some of their properties. Further also, we defined and studied the new notions of pre- β wg-open maps and pre β wg closed maps.

Mathematics Subject Classification (2010): 54C05, 54C08.

KEYWORDS: β wg-closed sets, β wg-continuous maps, pre- β wg-open maps, β wg-closed maps and contra β wg-irresolute maps.

I. INTRODUCTION

In 1970, Levine [9], introduced the concept of g-closed sets and a new class of spaces called $T_{1/2}$ – spaces in topology. Thereafter, many topologists have obtained several interesting results on these g-closed sets. In 1994, Maki et al.,[10] have defined and studied the α -generalised closed sets and α -generalised open sets making use of α -interior and α -closure due to A.S. Mashhour et al.,[12]. Caldas [6] and Balachandran et al.,[3] defined and studied the notion of g-continuous maps by using g-closed sets and discussed some of their properties. Further they have investigated and studied the new concept of gc-irresolute, perfectly g-continuous, strongly g-continuous maps. Recently, Govindappa. Navalagi and Kantappa. M. Bhavikatti [14] introduced and studied new concept of closed sets called β wg-closed sets and in [4, 15 & 16], contra β wg-continuous maps, β wg-continuous maps & β wg-irresolute and strongly β wg-continuous maps were studied. In this paper, we define and study the new concept of pre- β wg-continuous maps and their properties. Further, we also introduce pre β wg-open maps in topological spaces.

II. PRELIMINARIES

Throughout this paper, S , R , and P (or (S, τ) , (R, σ) , (P, η)) always denote topological spaces on which no separation axioms are assumed unless otherwise explicitly stated. For a subset $A \subseteq S$, closure and interior of A is denoted as $Cl(A)$ and $Int(A)$ respectively.

We have the following known definitions and results which are useful in the sequel.

Definition 2.1: A subset A of a space S is known as

- (i) Preopen[12] if $A \subseteq Int(Cl(A))$
- (ii) α -open[18] if $A \subseteq Int(Cl(Int(A)))$
- (iii) semipreopen[2] ($=\beta$ -open[1]) if $A \subseteq Cl(Int(A))$.

The compliments of above open sets are their closed sets.

Definition 2.2: A subset A of S said to be

- (i) generalized closed[9] (in brief, g-closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in S .
- (ii) α g-closed[10] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ & U is open in S .

- (iii) gsp-closed [7] if $\text{spcl}(A) \subseteq U$ whenever $A \subseteq U$ moreover U is open in S .
- (iv) β wg-closed set [14] if $\beta\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ & U is α g-open in S .

Definition 2.3: A function $h: S \rightarrow R$ is called a

- (i) β -continuous [1] if for each closed V of R , $f^{-1}(V)$ is β -closed in S .
- (ii) gc-irresolute [19] if for each g -closed set V of R , $f^{-1}(V)$ is g -closed in S .
- (iii) β wg-continuous [15] if for each closed set V of R , $f^{-1}(A)$ is β wg-closed in S .
- (iv) Strongly β wg-continuous [4] if for each β wg-closed V of R , $f^{-1}(V)$ is closed in S .
- (v) contra continuous [8] if $f^{-1}(V)$ is closed set in S for each open set V of R .
- (vi) contra β wg-continuous [16] if $f^{-1}(V)$ is β wg-closed set in S for every open set V of R .
- (vii) Pre gsp-continuous [17] if for each sp-closed set V of R , $f^{-1}(V)$ is gsp-closed in S .
- (viii) β -irresolute [11] if for each β -closed set V of R , $f^{-1}(V)$ is β -closed in S .
- (ix) β wg-irresolute [15] if for each β wg-closed set V of R , $f^{-1}(V)$ is β wg-closed in S .
- (x) quasi sgp-closed [5] if the image of each sgp-closed set of S being closed in R .
- (xi) sgp*-closed [5] if the image of sgp-closed subset of S is sgp-closed in R .

Definition 2.4: A space B of S called

- (i) a $T_{1/2}$ -space [8] if every g -closed set in it is closed.
- (ii) a $\beta_{wb}T_b$ -space [14] if every β wg-closed subset in S is closed.

III. Some More Properties of Pre- β wg – Continuous Maps

In this section, we define and obtain the followings

DEFINITION 3.1.

A function $h: (S, \tau) \rightarrow (R, \sigma)$ is called a pre- β wg-continuous (Shortly written, pre- β wg-continuous) functions if for each β -closed in set C of R , $h^{-1}(C)$ is β wg-closed set in S .

THEOREM 3.2.

If $h: S \rightarrow R$ be pre- β wg-continuous thereupon, it is β wg-continuous.

PROOF:

Allow D is any closed subset of R . So D be β -closed in R . Since h be pre- β wg-continuous map, $h^{-1}(D)$ is β wg-closed set in S . Therefore, h be β wg-continuous.

THEOREM 3.3.

Allow $h: S \rightarrow R$ be a map, it is Strongly β wg-continuity iff the inverse image of each β wg-closed set of R being β -closed in S .

PROOF: Obvious.

PROPOSITION 3.4.

Allow $h: S \rightarrow R$ be pre- β wg-continuity iff the image of each β wg – closed set in S being β – closed set in R .

PROOF:

Suppose h be pre- β wg-continuity. Allow set Q be β -open in R . Then Q^c is β -closed in R . As h be pre- β wg-continuous, $h^{-1}(Q^c)$ is β wg- closed set in S . Hence $h^{-1}(Q)$ is β wg-open in S .

Conversely, Let D be β – open set in R . Thereupon D^c is β -closed in R . But $h^{-1}(D^c) = S - h^{-1}(D)$ by assumption. Therefore $h^{-1}(D)$ is β wg-closed in S . h is pre- β wg-continuous.

PROPOSITION 3.5.

Allow a map $h: S \rightarrow R$ be pre- β wg- continuity then h be β -irresolute map.

PROOF: Obvious

THEOREM 3.6.

If a function $h: S \rightarrow R$ be pre- β wg - continuity then h be β wg- irresolute map.

PROOF: Follows by Theorem 3.4.

THEOREM 3.7.

Every is pre- β wg-continuous map being pre – gsp continuity.

PROOF:

Allow set D be β -closed in R . As h be pre- β wg-continuous, $h^{-1}(D)$ is β wg –closed set in S . Then $h^{-1}(D)$ be gsp – closed set of S . As every β wg-closed is gsp-closed. Hence h is pre-gsp-continuity map.

EXAMPLE 3.8.

Take $R = P = \{c_1, c_2, c_3\}$, $\sigma = \{R, \{c_1\}, \varnothing\}$, $\eta = \{P, \{c_1, c_3\}, \varnothing\}$. Define a function $k: R \rightarrow P$ is identity function. Now k be pre-gsp – continuity yet never pre- β wg- continuity. As β -closed subset $\{c_1, c_2\}$ of P , $k^{-1}(\{c_1, c_2\}) = \{c_1, c_2\}$ is gsp-closed but β wg-closed in R .

REMARK 3.9.

The composition of two pre- β wg-continuous functions is not pre- β wg- continuous.

EXAMPLE 3.10.

Allowing $S = R = \{e, j, t\} = P$, $\tau = \{\varnothing, \{j\}, \{t\}, \{j, t\}\}$, $\sigma = \{\varnothing, \{e\}, R\}$ and $\eta = \{P, \{t\}, \{e, t\}, \varnothing\}$.

Now define $h: S \rightarrow R$ as $h(e) = t, h(j) = e, h(t) = j$ & $k: R \rightarrow P$ as $k(e) = e, k(j) = t, k(t) = j$. Thereupon, both h and k are pre- β wg - continuous maps. Yet their composition $koh: S \rightarrow P$ be never pre- β wg - continuous function. As β - closed subset $\{e, j\}$ in P , but $(koh)^{-1}(\{e, j\}) = h^{-1}(k^{-1}\{j, t\}) = \{j, t\}$ be never β wg closed in S .

PROPOSITION 3.11.

If a function $h: S \rightarrow R$ be pre- β wg - continuous & $k: R \rightarrow P$ be strongly β wg - continuous map, thereupon the composition $koh: S \rightarrow P$ be β wg-irresolute.

PROOF:

Let F be β wg-closed set in P . Since k is strongly β wg-continuous, then $k^{-1}(F)$ is closed set and so $k^{-1}(F)$ is β -closed in R . Again since h is pre- β wg-continuous, $k^{-1}(h^{-1}(F)) = (koh)^{-1}(F)$ is β wg-closed in S . Hence $koh: S \rightarrow R$ is β wg-irresolute.

THEOREM 3.12:

If a function $h: S \rightarrow R$ is β wg-continuous and $k: R \rightarrow P$ is strongly β wg-continuous. Thereupon the composition $hok: S \rightarrow P$ is β wg-irresolute.

PROOF:

Let F be β wg-closed set in P . Since k is strongly β wg-continuous, then $k^{-1}(F)$ is closed in R . Again since h is pre- β wg-continuous, $h^{-1}(k^{-1}(F)) = (koh)^{-1}(F)$ is β wg-closed in S . Hence $koh: S \rightarrow P$ is β wg-irresolute.

PROPOSITION 3.13.

Allow a function $h: S \rightarrow R$ being β wg-irresolute & $k: R \rightarrow P$ be pre- β wg-continuous map, their composition $koh: S \rightarrow P$ be pre - β wg - conti. map.

Easy Proofs & follows by Theorem 3.11.

THEOREM 3.14.

If $h: S \rightarrow R$ and $k: R \rightarrow P$ are pre- β wg-continuous functions and R is β wg T_b -space. Then the composition $koh: S \rightarrow P$ is pre- β wg-continuous.

PROOF:

Let F be β -closed set in P . Then $k^{-1}(F)$ is β wg-closed in R as k is pre- β wg-continuous. Since, R is β wg T_b -space, $h^{-1}(F)$ is closed set and so β -closed in R . Again since h is pre- β wg-continuous, $h^{-1}(k^{-1}(F)) = (koh)^{-1}(F)$ is β wg-closed in S .

THEOREM 3.15.

Allowing both functions $h: S \rightarrow R$ be β wg – continuous, $k: R \rightarrow P$ be pre- β wg continuous functions & R be β wg T_b –space. Thereupon their composition $koh: S \rightarrow P$ being is pre- β wg-continuous.

PROOF:

Let F be β -closed set in P . Then $k^{-1}(F)$ is β wg-closed in R as k is pre- β wg-continuous. Since, R is β wg T_b -space, $h^{-1}(F)$ is closed set in R . Again since h is β wg-continuous, $h^{-1}(k^{-1}(F)) = (koh)^{-1}(F)$ is pre- β wg-closed in S .

DEFINITION 3.16.

A function $h: S \rightarrow R$ is said to be contra strongly β wg-continuous if the inverse image of each β wg-open set of R is closed in S .

Clearly it is easy to see that a map $h: S \rightarrow R$ is contra strongly β wg-continuous if and only if inverse image of each wg-closed set of R is open in S .

PROPOSITION 3.17.

If a function $h: S \rightarrow R$ is contra strongly β wg-continuous & $k: R \rightarrow P$ is β wg-continuous function then $koh: S \rightarrow P$ is contra continuous.

PROOF:

Let Q be an open set in P . Since k is β wg-continuous, $k^{-1}(Q)$ is β wg-open in R . Therefore, $h^{-1}(k^{-1}(Q))$ is closed in S . Since, as h is contra strongly β wg-continuous. So, $(koh)^{-1}(Q) = h^{-1}(k^{-1}(Q))$ is closed in S . Hence koh is contra continuous.

DEFINITION 3.18.

A function $h: S \rightarrow R$ is called contra β wg-irresolute if the inverse image of each β wg-open set of R is β wg-closed in S .

THEOREM 3.19.

If a function $h: S \rightarrow R$ is contra β wg-continuous & $k: R \rightarrow P$ is strongly β wg-continuous then $koh: S \rightarrow P$ is contra β wg-irresolute.

PROOF:

Let D be β wg-open set in P . Since k is strongly β wg-continuous, $k^{-1}(D)$ is open in R . Therefore $h^{-1}(k^{-1}(D))$ is β wg-closed in S since h is contra β wg-continuous. But $(koh)^{-1}(D) = h^{-1}(k^{-1}(D))$ is β wg-closed in S . Hence, koh is contra β wg-irresolute.

IV. Pre- β wg-closed Functions and Pre- β wg-open Functions

In this section next, we define the followings

DEFINITION 4.1.

A map $h: (S, \tau) \rightarrow (R, \sigma)$ is termed as **Pre- β wg-closed** (resp. Pre- β wg-open) if for each β -closed (resp. β -open) subset N of S , $h(N)$ is β wg-closed set (resp. β wg-open set) of R .

DEFINITION 4.2.

A map $h: S \rightarrow R$ being said to be strongly β wg-open if for each β wg-open set D of S , then $h(D)$ is open in R .

THEOREM 4.3.

If function $h: (S, \tau) \rightarrow (R, \sigma)$ being said to be Pre- β wg-open then it is β wg-open.

PROOF:

Let V is open set of S . Thereupon, V is β -open set in S . As h is pre- β wg-open, $h(V)$ is β wg-open set of R . So it shows that f is β wg-open map.

THEOREM 4.4.

If a function $h: (S, \tau) \rightarrow (R, \sigma)$ being Pre β wg-open iff for each β -closed subset of S is β wg-closed in R .

PROOF:

Suppose h be pre β wg-closed. As Q is β -open set of S . Thereupon, Q^c is β -closed subset of S . Again h be pre β wg-open, $h(Q^c) = S - h(Q)$ being β wg-closed set of R . Hence $h(Q)$ is β wg-open in R .

Conversely, allow D is β -closed in S . Thereupon, D^c be β -open subset of X . But $h(R - D) = R - h(D)$ being β wg-open set of R by assumption. Therefore $h(D)$ be pre- β wg-closed subset of R . So h being Pre- β wg-closed.

DEFINITION 4.5.

If a function $k: (R, \sigma) \rightarrow (P, \eta)$ is known as always β wg-closed map if for each β wg-closed set D of R , $k(D)$ being β wg-closed in P .

DEFINITION 4.6.

Allowing $k: (R, \sigma) \rightarrow (P, \eta)$ is known as completely β wg-closed map if for each β wg-closed set M of R , $k(M)$ be regular closed set of P .

Now We prove the followings

PROPOSITION 4.7.

Allow a function $h: (S, \tau) \rightarrow (R, \sigma)$ being completely β wg-open and $k: (R, \sigma) \rightarrow (P, \eta)$ being pre- β wg-open. Then their composition $koh: S \rightarrow P$ is always β wg – open function.

PROOF:

Take set V is any β wg – open subset of S . Since h be completely β wg – open set, $h(V)$ be β wg –regular open set of R . Hence $k(h(V)) = koh(V)$ being β wg – open in P , as h be pre β wg-open function. Therefore $koh: S \rightarrow P$ is always β wg-open function.

REMARK 4.8.

Clearly, note that composition of two pre- β wg - closed functions, again not being pre- β wg - closed functions true as seen from below example.

EXAMPLE 4.9.

Allowing $S = \{1, 3, 5\} = R$, $\tau = \{\emptyset, \{1\}, \{1, 5\}, S\}$, $\sigma = \{\emptyset, \{1\}, R\}$, $P = \{\emptyset, \{1\}, \{1, 3\}, P\}$. Now a function $k: R \rightarrow P$ be defined as by $k(1) = 5$, $k(3) = 1$, $k(5) = 3$ & let $h: S \rightarrow R$ be identity map, both h , k are pre- β wg-closed maps. Yet koh being never pre- β wg – closed map. As β - closed set $\{3, 5\}$ of S .

Now $koh(\{3, 5\}) = k(h(\{3, 5\})) = k(\{3, 5\}) = \{1, 3\}$ be never β wg – closed set of P .

PROPOSITON 4.10.

If function $h: S \rightarrow R$ being always β wg open, $k: R \rightarrow P$ being completely β wg-open. Thereupon, their composition $hof: S \rightarrow P$ is completely β wg-open.

PROOF:

Follows by Theorem 4.7.

Easy proofs of the following results omitted

PROPOSITION 4.11.

Authorize the functions $h: S \rightarrow R$ be β wg-open and $k: R \rightarrow P$ being pre- β wg-open, $koh: S \rightarrow P$ be β wg-open.

PROPOSITION 4.12.

Allow a function $h: S \rightarrow R$ be always β -open and $k: R \rightarrow P$ be pre- β wg-open, thereupon koh being pre- β wg-open.

THEOREM 4.13.

Allow a function $h: S \rightarrow R$ be sgp*-closed & $k: R \rightarrow P$ be strongly β wg-closed, thereupon $koh: S \rightarrow P$ being quasi sgp-closed.

PROPOSITION 4.14.

Allowing a function $h: S \rightarrow R$ be completely β wg-open and $k: R \rightarrow P$ being pre- β wg-open, $koh: S \rightarrow P$ be always β wg-open.

THEOREM 4.15.

If a function $h: S \rightarrow R$ is β wg-closed and R is β wg T_b -space. Then f is closed map.

We define the following

DEFINITION 4.16.

A map $h: S \rightarrow R$ is called contra strongly β wg-open if the image of each β wg-open set of S is closed in R .

Clearly, it is easy to see that a map $h: S \rightarrow R$ is contra strongly β wg-open if and only if image of each β wg-closed set of S is open in R .

THEOREM 4.17.

If $h: S \rightarrow R$ is contra strongly β wg-open & $k: R \rightarrow P$ is contra-closed map then $koh: S \rightarrow P$ is contra strongly β wg-open map.

PROOF: Obvious.

DEFINITION 4.18.

A map $h: S \rightarrow R$ is said to be contra β wg-open if the image of every open set of S is β wg -closed in R .

THEOREM 4.19: If $h: S \rightarrow R$ & $k: R \rightarrow P$ be two maps, then the following statements holds:

- (i) If h is pre- β wg-open & k is strongly β wg-open, then koh is β -open.
- (ii) If koh is always β wg-open and h is β wg-irresolute surjection, then k is always β wg-open.
- (iii) If koh is pre- β wg-open and h is completely β wg-continuous, then k is always β wg-open.
- (iv) If koh is strongly β wg-open and h is β wg-continuous surjection, then k is an open.
- (v) If koh is always β wg-open and k is β wg-irresolute injection, then h is always β wg-open.
- (vi) If koh is contra β wg-open & k is β wg-irresolute injection, then h is β wg-open.
- (vii) If koh is β wg-open & k is strongly β wg-continuous, then h is open.

V. CONCLUSION

In this article, We introduced and studied a new class of maps termed as pre- β wg-continuous maps using β wg-closed sets in topological spaces and obtain some of their properties. Further also, we defined and studied the new notions of pre- β wg-open maps and pre β wg closed maps.

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