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A New Type of Mappings in Nano Topological Spaces

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Abstract
The study of nano topology was initiated by M.Lellis Thivagar with regard to a subset X of a universe which is described in terms of lower, upper and boundary approximations of X. He also described nano interior and nano closure in nano topological spaces. In this paper, we define the concept of Nano bc-open mapping, Nano bc-closed mapping in Nano topological spaces. Also we poster some essential comparative notions with another open mappings and engage into a deeper analysis of their characterizations.

Key Words: *nano bc-closed set, nano bc-open set, nano bc–open mapping, nano bc–closed mapping.*

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I. INTRODUCTION

The study of ***nano topology*** was started by M. Lellis Thivagar et al [8] with regard to a subset X of a universe that is described in terms of lower, upper and boundary approximations of X. He additionally described ***nano interior*** and ***nano closure*** in ***nano topological spaces***.(or briefly №T Spaces)**.** Andrijevic [1] presented and studied a category of generalized open sets in a topological space referred to as b-open sets. Further C. Indirani et al [4] created and studied *nano* b-open sets **(**№ bo sets) in ***nano topological spaces*** **(**№TS)**.** B𝕔 open sets were first introduced in topological spaces by Hariwan Z. Ibrahim [6]. Here we proceed to present our findings on **nano b𝕔-open mappings in Nano Topological Spaces.**

II. PRELIMINARIES

Definition 2.1. [8] Let U denote a non-empty finite set of elements referred to as universe and R represents an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is called as the **approximation space.** Let $X\subseteq U$.

(i) ***The lower approximation*** of X with respect to R is the set of all elements, which can be for certain classified as X with respect to Rand it is denoted by $L_{\mathcal{R}}(X)$.That is, $L_{\mathcal{R}}(X)=\cup_{X\in U}\{R_x : R_x \subseteq X\}$ where R_x denotes the equivalence class determined by $x \in U$.

(ii) ***The upper approximation*** of X with respect to R is the set of all elements, which can be possibly classified as X with respect to R and it is denoted by$U_{\mathcal{R}}(X)$. That is$U_{\mathcal{R}}(X) = \cup_{X\in U}\{ R_x: R_x \cap X\neq \emptyset \}$.

(iii) ***The boundary region*** of X with respect to R is the set of all elements, that can be classified neither as X nor as not-X with respect to R and it is denoted by$B_{\mathcal{R}}(X)$. That is, $B_R(X)=U_{\mathcal{R}}(X)-L_{\mathcal{R}}(X)$.

Definition 2.2. [8] Let U represent the universe and R represent an equivalence relation on U. Then$\tau_R(X)$=$№^T$ = $\{ U, \emptyset, L_{\mathcal{R}}(X), U_{\mathcal{R}}(X), B_{\mathcal{R}}(X)\}$ where $X\subseteq U$.Then, $\tau_{\mathcal{R}}(X)$satisfies the axioms listed below.
(i) U and $\emptyset \in \tau_{\mathcal{R}}(X)$.
(ii) The union of the elements of any sub collection of $\tau_{\mathcal{R}}(X)$ is in $\tau_{\mathcal{R}}(X)$.

(iii) The intersection of the elements of any finite subcollection of $\tau_{\mathcal{R}}$(X) is in $\tau_{\mathcal{R}}$(X).That is, $\tau_{\mathcal{R}}$(X)is a topology on U referred to as the ***nano topology*** (№^{T}) on U with respect to X. We call (U,$\tau_{\mathcal{R}}$(X)) (or) (U,№^{T})as the ***nano topological space***(№TS-in short).The elements of№^{T} are known as № ***open sets***(briefly,№-OS). The complement of №-open sets are №-***closed***sets(briefly, №-CS).

Example 2.3. [8] Let U={p,q,r,s} with U/R = {{p},{q},{r,s}}and X={p,r} ⊂U. Then the nano topology is $\tau_{\mathcal{R}}$(X) =№^{T}={ U, ∅,{p},{r,s},{p,r,s}}.

Remark 2.4. [8] If $\tau_{\mathcal{R}}$(X) =№^{T}is the nano topology on Uwith respect to X and B_N isanano subset of №TS,then B_N ={ U, $L_{\mathcal{R}}$(X), $B_{\mathcal{R}}$(X)} is referred to as the basis for $\tau_{\mathcal{R}}$(X) .

Definition 2. [8] If (U,№^{T}) is a №TS with respect to X where
X ⊆U and if A_N is a nano subset in №TS and if A_N ⊆ U, then
(1) The **Nano interior** of A_N is defined as the union of all nano-open subsets of A and it is denoted by №-int(A_N). That is, №-int(A_N) is the largest nano-open subset of A_N.
(2) The **Nano closure** of A_N is defined as the intersection of all nano closed sets containing A_N and it is denoted by №-cl(A_N). That is, №-cl(A_N) is the smallest nano closed set containing A_N.

Definition 2.6 . Let (U, $\tau_{\mathcal{R}}$(X)) be a №TS and A_N ⊆ U. Then A_N is said to be
(1) **Nano-semi open** set (№-SO set) [8] if A_N⊆№-cl[№-int(A_N)] and **Nano semi-closed** (№-SC set) [7] if №-int [№-cl(A_N)] ⊆ A_N.
(2) **Nano-θopen** set (№-θO set) [3] if for each x∈A_N, there exists a nano open set (№-OS) G such that x ∈ G ⊂№cl(G)⊂A_N.
(3) **Nano-θsemiopen** (№-θSO) [3] if for each x ∈A_N ,there exists a nano semi open set (№-SO set) G such that x∈G⊂№cl(G)⊂A_N.

№-SO(U, X), №-θO(U, X) and №-θSO(U, X) respectively denote the families of all nano semi-open(№-SO), nano θ-open(№-θO) and nano θ semi-open(№-θSO)subsets of U.

Definition 2.7. [3] Let (U,$\tau_{\mathcal{R}}$(X)) is a №TS and A_N ⊆ U. Then A_N is said to be nano- bopen set (№- bo- set) if A_N ⊆ №-cl(№-int(A_N)) ∪ №-int(№-cl(A_N)). The complement of nano- b-open set is called nano- b-closed set (№ b𝕔-set).

Example 2.8. [3] Let U = {p,q,r,s} with U/R ={p}, {r}, {q,s} and X = {p,q}.
Then the nano topology τ_R(X)= №^{T}= { U, ∅, {p}, {p,q,s}, {q,s}} and nano b-open sets are U, ∅, {p}, {q}, {s},{p, q},{p,r},{p,s},{q, s},{p,q, r},{p,q, s},{q,r,s}.

Definition 2.9. [10] A №TS (U, №^{T}) is referred to as nano locally Indiscrete space if every nano open set (№-OS) is nano closed set.(№-CS).

Definition 2.10. Let ($\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}}$) and ($\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}}$) be $\mathcal{NTS}$. A mapping : ($\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}}$) → ($\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}}$) is said to be
1.№-continuous (№-cts for short) [9] $\zeta^{-1}(\mathcal{Z}_{\mathcal{N}})$ is №-$\mathcal{OS}$ in $\tilde{U}_{\mathcal{N}}$ for every №-$\mathcal{OS}$ $\mathcal{Z}_{\mathcal{N}}$ in $\tilde{V}_{\mathcal{N}}$.
2.№-α-continuous (№-α-cts for short) [14] $\zeta^{-1}(\mathcal{Z}_{\mathcal{N}})$ is №-α-$\mathcal{OS}$ in $\tilde{U}_{\mathcal{N}}$ for every №-$\mathcal{OS}$ $\mathcal{Z}_{\mathcal{N}}$ in $\tilde{V}_{\mathcal{N}}$.
3.№-semi-continuous (№-$\mathcal{S}$-cts for short) [9] $\zeta^{-1}(\mathcal{Z}_{\mathcal{N}})$ is №-$\mathcal{S}$-$\mathcal{OS}$ in $\tilde{U}_{\mathcal{N}}$ for every №-$\mathcal{OS}$ $\mathcal{Z}_{\mathcal{N}}$ in $\tilde{V}_{\mathcal{N}}$.
4.№-pre-continuous (№-$\mathcal{P}$-cts for short) [9] $\zeta^{-1}(\mathcal{Z}_{\mathcal{N}})$ is № $\mathcal{P}$-$\mathcal{OS}$ in $\tilde{U}_{\mathcal{N}}$ for every №-$\mathcal{OS}$ $\mathcal{Z}_{\mathcal{N}}$ in $\tilde{V}_{\mathcal{N}}$.
5.№-$\mathscr{b}$-continuous (№-$\mathscr{b}$-cts for short) [4] $\zeta^{-1}(\mathcal{Z}_{\mathcal{N}})$ is №-$\mathscr{b}$-$\mathcal{OS}$ in $\tilde{U}_{\mathcal{N}}$ for every №-$\mathcal{OS}$ $\mathcal{Z}_{\mathcal{N}}$ in $\tilde{V}_{\mathcal{N}}$.
6.№- θ-continuous (№-θ-cts for short) [3] $\zeta^{-1}(\mathcal{Z}_{\mathcal{N}})$ is №-θ-$\mathcal{OS}$ in $\tilde{U}_{\mathcal{N}}$ for every №-$\mathcal{OS}$ $\mathcal{Z}_{\mathcal{N}}$ in $\tilde{V}_{\mathcal{N}}$.

Definition 2.11. [13] A nano subset $A_{\mathcal{N}}$ of a nano topological space ($\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}}$) is called nano b𝕔 - open set (№-b𝕔- $\mathcal{OS}$) if for every $\varkappa \in A_{\mathcal{N}}$ ∈ №-BO($\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}}$), there exists a nano closed set (№-$\mathcal{CS}$) $\mathcal{H}_{\mathcal{N}}$ such that $\varkappa \in \mathcal{H}_{\mathcal{N}} \subset A_{\mathcal{N}}$.
The family of all nano $\mathscr{b}$𝕔-open sets of a Nano topological space ($\mathcal{NTS}$ in short) ($\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}}$) is denoted by №-$\mathcal{B}$𝕔O($\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}}$).
Example 2.12 [13] Let $\tilde{U}_{\mathcal{N}}$ ={ϖ1, ϖ2, ϖ3, ϖ4},$\tilde{U}_{\mathcal{N}}$ / $\mathcal{R}$ ={{ϖ1},{ϖ3},{ϖ2, ϖ4}} and $\mathcal{X}_{\mathcal{N}}$ ={ϖ1, ϖ2} ⊂ $\tilde{U}_{\mathcal{N}}$. Then the Nano topology №^{T} = $\tau_{\mathcal{R}}(\mathcal{X}_{\mathcal{N}})$= {$\tilde{U}_{\mathcal{N}}$, $\emptyset_{\mathcal{N}}$, {ϖ1},{ϖ1, ϖ2, ϖ4},{ϖ2, ϖ4}}. Then the nano Closed sets are $\tilde{U}_{\mathcal{N}}$, $\emptyset_{\mathcal{N}}$, {ϖ2, ϖ3, ϖ4},{ϖ3} and {ϖ1, ϖ3}. Then the collection of all №-$\mathscr{b}$-open sets are №-$\mathscr{b}$O($\tilde{U}_{\mathcal{N}}$, $\mathcal{X}_{\mathcal{N}}$) = {$\tilde{U}_{\mathcal{N}}$, $\emptyset_{\mathcal{N}}$,

{ɷ$_1$},{ɷ$_2$},{ɷ$_4$},{ɷ$_1$, ɷ$_2$},{ɷ$_1$, ɷ$_3$}, {ɷ$_1$, ɷ$_4$},{ɷ$_2$, ɷ$_4$},{ɷ$_1$, ɷ$_2$, ɷ$_3$},{ɷ$_1$, ɷ$_2$, ɷ$_4$},{ɷ$_2$, ɷ$_3$, ɷ$_4$}} and №-$\mathcal{b}$ꞇO(Ũ$_{\mathcal{N}}$, $\mathcal{X}_{\mathcal{N}}$) = {Ũ$_{\mathcal{N}}$, ∅$_{\mathcal{N}}$, {ɷ$_2$, ɷ$_3$, ɷ$_4$},{ɷ$_1$, ɷ$_3$}}.

III. NANO-Bꞇ OPEN MAPPINGS IN NANO TOPOLOGICAL SPACES (№-bꞇ-$\mathcal{OM}$)

Definition 3.1 Let (Ũ, $N_R(x)$) and (Ṽ, N_R (y)) be the two $\mathcal{NTS}$. A function η: (Ũ, $N_R(x)$) → (Ṽ, N_R (y)) with respect to X and y respectively is called **№-bc-open mapping** if the image of every №-OS in (Ũ, $N_R(x)$) is №-bc-open in (Ṽ, N_R (y)).

Example 3.2 Let Ũ = {ɷ$_1$, ɷ$_2$, ɷ$_3$, ɷ$_4$} with Ũ/R ={{ɷ$_1$,ɷ$_2$},{ɷ$_3$}, ɷ$_4$}} and X ={ɷ$_1$, ɷ$_3$}. Then N.T $N_R(x)$ = {Ũ,∅, {ɷ$_3$},{ɷ$_1$, ɷ$_2$, ɷ$_3$},{ɷ$_1$, ɷ$_2$}}. Then №-bcO(x) = {Ũ, ∅, {ɷ$_2$, ɷ$_3$, ɷ$_4$ },{ɷ$_2$, ɷ$_3$}}. Let Ṽ = { ʐ$_1$, ʐ$_2$, ʐ$_3$, ʐ$_4$} with Ṽ /∅ ={{ʐ$_1$},{ʐ$_3$},{ʐ$_2$, ʐ$_4$}} and y ={ ʐ$_1$, ʐ$_2$}. Then N_R(y) = {Ṽ,∅, {ʐ$_1$},{ʐ$_1$, ʐ$_2$, ʐ$_4$},{ʐ$_2$, ʐ$_4$}}. №- bcO(y) ={Ṽ,∅, {ʐ$_1$},{ʐ$_1$, ʐ$_3$},{ʐ$_2$, ʐ$_4$}, { ʐ$_1$, ʐ$_2$, ʐ$_4$}, { ʐ$_2$, ʐ$_3$, ʐ$_4$ }}. Define η: (Ũ, $N_R(x)$) → (Ṽ, N_R (y)) by η(ɷ$_1$) = ʐ$_2$, η(ɷ$_2$) = ʐ$_4$, η(ɷ$_3$) = ʐ$_1$, η(ɷ$_4$) = ʐ$_3$. Then η is №-bc-$\mathcal{OM}$.

Theorem 3.3 Every №-θ-$\mathcal{OM}$ is №-bc-$\mathcal{OM}$.

Proof : Consider f : (Ũ, $N_R(x)$) → (Ṽ, N_R (y)) is №-θ-$\mathcal{OM}$ and H be a №- θ open set in Ũ ,then f(H) is №- θ - open in Ṽ. Since every №-θ-open set is №- bc-open, f(H) is №-bc-open in Ṽ. Hence f is №-bc-$\mathcal{OM}$.

Remark 3.4 The Reverse implication of the above theorem need not be true as shownin the following example.

Example 3.5 Let Ũ = {ɷ$_1$, ɷ$_2$, ɷ$_3$, ɷ$_4$} with Ũ/R ={{ɷ$_1$,ɷ$_3$},{ɷ$_2$}, ɷ$_4$}} and X ={ɷ$_1$, ɷ$_3$}. Then N.T $N_R(x)$ = {Ũ,∅, {ɷ$_2$},{ɷ$_1$, ɷ$_2$, ɷ$_4$},{ɷ$_1$, ɷ$_3$}}.№- θ-OS ={ Ũ, ∅, {ɷ$_1$, ɷ$_3$},{ɷ$_2$}}. Then №-bcO(x) = {Ũ, ∅, {ɷ$_1$, ɷ$_3$, ɷ$_4$ },{ɷ$_2$, ɷ$_4$}}. Let Ṽ = { ʐ$_1$, ʐ$_2$, ʐ$_3$, ʐ$_4$} with Ṽ /∅ ={{ʐ$_1$},{ʐ$_3$},{ʐ$_2$, ʐ$_4$}} and y ={ ʐ$_1$, ʐ$_2$}. Then N_R(y) = {Ṽ,∅, {ʐ$_1$},{ʐ$_1$, ʐ$_2$, ʐ$_4$},{ʐ$_2$, ʐ$_4$}}. №- θ-OS ={ Ũ, ∅, {ʐ$_1$,ʐ$_2$},{ ʐ$_4$}}. №- bcO(y) ={Ṽ,∅, {ʐ$_1$},{ʐ$_1$, ʐ$_3$},{ʐ$_2$, ʐ$_4$}, { ʐ$_1$, ʐ$_2$, ʐ$_4$}, { ʐ$_2$, ʐ$_3$, ʐ$_4$ }}. Define η: (Ũ, $N_R(x)$) → (Ṽ, N_R (y)) by η(ɷ$_1$) = ʐ$_3$, η(ɷ$_2$) = ʐ$_4$, η(ɷ$_3$) = ʐ$_1$, η(ɷ$_4$) = ʐ$_2$. Then η is №-bc-$\mathcal{OM}$ but not №-θ- $\mathcal{OM}$.

Theorem 3.6 Every №-θS-$\mathcal{OM}$ is №-bc-$\mathcal{OM}$.

Proof : Consider f : (Ũ, $N_R(x)$) → (Ṽ, N_R (y)) is №-θS-$\mathcal{OM}$ and H be a №- open set in Ũ ,then f(H) is №- θS - open in Ṽ. Since every №-θS-open set is №- bc-open, f(H) is №-bc-open in Ṽ. Hence f is №-bc-$\mathcal{OM}$.

Example 3.7 Let Ũ = {ɷ$_1$, ɷ$_2$, ɷ$_3$, ɷ$_4$} with Ũ/R ={{ɷ$_2$},{ɷ$_1$, ɷ$_4$,}, ɷ$_3$}} and X ={ɷ$_2$, ɷ$_4$}. Then N.T $N_R(x)$ = {Ũ,∅, {ɷ$_1$,ɷ$_3$,},{ɷ$_2$, ɷ$_4$},{ɷ$_1$, ɷ$_2$, ɷ$_3$}}.№- θS-OS ={ Ũ, ∅, {ɷ$_2$, ɷ$_4$},{ɷ$_1$},{ ɷ$_3$, ɷ$_4$,}. Then №-bcO(x) = {Ũ, ∅, { ɷ$_2$, ɷ$_4$ },{ɷ$_1$, ɷ$_3$},{ ɷ$_4$,}. Let Ṽ = { ʐ$_1$, ʐ$_2$, ʐ$_3$, ʐ$_4$} with Ṽ /∅ ={{ʐ$_1$},{ʐ$_3$},{ʐ$_2$, ʐ$_4$}} and y ={ ʐ$_1$, ʐ$_2$}. Then N_R(y) = {Ṽ,∅, {ʐ$_1$},{ʐ$_1$, ʐ$_2$, ʐ$_4$},{ʐ$_2$, ʐ$_4$}}. №- θ-OS ={ Ũ, ∅, { ʐ$_1$,ʐ$_4$},{ ʐ$_1$}}. №- bcO(y) ={Ṽ,∅, {ʐ$_1$},{ʐ$_1$, ʐ$_3$},{ʐ$_2$, ʐ$_4$}, { ʐ$_1$, ʐ$_2$, ʐ$_4$}, { ʐ$_2$, ʐ$_3$, ʐ$_4$ }}. Define η: (Ũ, $N_R(x)$) → (Ṽ, N_R (y)) by η(ɷ$_1$) = ʐ$_2$, η(ɷ$_2$) = ʐ$_1$, η(ɷ$_3$) = ʐ$_4$, η(ɷ$_4$) = ʐ$_3$. Then η is №-bc-$\mathcal{OM}$ but not №-θS-$\mathcal{OM}$.

Theorem 3.8 Every №-bc-$\mathcal{OM}$ is №-S-$\mathcal{OM}$.

Proof : Consider f : (Ũ, $N_R(x)$) → (Ṽ, N_R (y)) is №-bc-$\mathcal{OM}$ and H be a №- open set in Ũ ,then f(H) is №- bc - open in Ṽ. Since every №-bc-open set is №- semi-open, f(H) is №-semi-open in Ṽ. Hence f is №-S-$\mathcal{OM}$.

Example 3.9 Let Ũ = {ɷ$_1$, ɷ$_2$, ɷ$_3$, ɷ$_4$} with Ũ/R ={{ɷ$_1$},{ɷ$_1$, ɷ$_3$,}, ɷ$_2$}} and X ={ɷ$_1$, ɷ$_4$}. Then N.T $N_R(x)$ = {Ũ,∅, {ɷ$_1$,ɷ$_2$,},{ɷ$_2$, ɷ$_3$},{ɷ$_1$, ɷ$_3$, ɷ$_4$}}.№- S-OS ={ Ũ, ∅, {ɷ$_2$, ɷ$_4$},{ɷ$_1$},{ ɷ$_3$, ɷ$_4$,}. Then №-bcO(x) = {Ũ, ∅, { ɷ$_3$, ɷ$_4$ },{ɷ$_1$, ɷ$_4$},{ ɷ$_2$,}. Let Ṽ = { ʐ$_1$, ʐ$_2$, ʐ$_3$, ʐ$_4$} with Ṽ /∅ ={{ʐ$_1$},{ʐ$_3$},{ʐ$_2$, ʐ$_4$}} and y ={ ʐ$_1$, ʐ$_2$}. Then N_R(y) = {Ṽ,∅, {ʐ$_1$},{ʐ$_1$, ʐ$_2$, ʐ$_4$},{ʐ$_2$, ʐ$_4$}}. №- S-OS ={ Ũ, ∅, { ʐ$_1$,ʐ$_4$},{ ʐ$_1$}}. №- bcO(y) ={Ṽ,∅, {ʐ$_1$},{ʐ$_1$, ʐ$_3$},{ʐ$_2$, ʐ$_4$}, { ʐ$_1$, ʐ$_2$, ʐ$_4$}, { ʐ$_2$, ʐ$_3$, ʐ$_4$ }}. Define η: (Ũ, $N_R(x)$) → (Ṽ, N_R (y)) by η(ɷ$_1$) = ʐ$_2$, η(ɷ$_2$) = ʐ$_1$, η(ɷ$_3$) = ʐ$_4$, η(ɷ$_4$) = ʐ$_3$. Then η is №-S-$\mathcal{OM}$ but not №- bc-$\mathcal{OM}$.

Theorem 3.10 Every №-bc-$\mathcal{OM}$ is №-b-$\mathcal{OM}$.

Proof : Consider f : (Ũ, $N_R(x)$) → (Ṽ, N_R (y)) is №-bc-$\mathcal{OM}$ and H be a №- open set in Ũ ,then f(H) is №- bc - open in Ṽ. Since every №-bc-open set is №- b-open, f(H) is №-b-open in Ṽ. Hence f is №-b-$\mathcal{OM}$.

Example 3.11 Let Ũ = {ɷ$_1$, ɷ$_2$, ɷ$_3$, ɷ$_4$} with Ũ/R ={{ɷ$_1$,ɷ$_2$},{ɷ$_4$}, {ɷ$_3$,}} and X ={ɷ$_2$, ɷ$_3$}. Then N.T $N_R(x)$ = {Ũ,∅, {ɷ$_3$, ɷ$_1$,},{ɷ$_1$, ɷ$_4$},{ɷ$_1$, ɷ$_3$, ɷ$_4$}}.№- b-OS ={ Ũ, ∅, {ɷ$_2$, ɷ$_4$},{ɷ$_1$},{ ɷ$_3$, ɷ$_4$,}. Then №-bcO(x) = {Ũ, ∅, { ɷ$_3$, ɷ$_4$ },{ɷ$_1$, ɷ$_4$},{ ɷ$_2$,}. Let Ṽ = { ʐ$_1$, ʐ$_2$, ʐ$_3$, ʐ$_4$} with Ṽ /∅ ={{ʐ$_1$},{ʐ$_3$},{ʐ$_2$, ʐ$_4$}} and y ={ ʐ$_1$, ʐ$_2$}. Then N_R(y) = {Ṽ,∅, {ʐ$_1$},{ʐ$_1$, ʐ$_2$, ʐ$_4$},{ʐ$_2$, ʐ$_4$}}. №- b-OS ={ Ũ, ∅, { ʐ$_1$,ʐ$_4$},{ ʐ$_1$}}. №- bcO(y) ={Ṽ,∅, {ʐ$_2$},{ʐ$_1$, ʐ$_4$},{ʐ$_1$, ʐ$_3$}, { ʐ$_1$, ʐ$_2$, ʐ$_4$}, { ʐ$_2$, ʐ$_3$, ʐ$_4$ }}. Define η: (Ũ, $N_R(x)$) → (Ṽ, N_R (y)) by η(ɷ$_1$) = ʐ$_2$, η(ɷ$_2$) = ʐ$_1$, η(ɷ$_3$) = ʐ$_4$, η(ɷ$_4$) = ʐ$_3$. Then η is №-b-$\mathcal{OM}$ but not №- bc-$\mathcal{OM}$.

Theorem 3.12 Every №-bc-$\mathcal{OM}$ is №- β -$\mathcal{OM}$.

Proof : Consider f : (Ũ, $N_R(x)$) → (Ṽ, N_R (y)) is №-bc-$\mathcal{OM}$ and H be a №- open set in Ũ ,then f(H) is №- bc -open in Ṽ. Since every №-bc-open set is №- β -open, f(H) is №- β -open in Ṽ. Hence f is №- β -$\mathcal{OM}$.

Example 3.13 Let Ũ = {ɷ1, ɷ2, ɷ3, ɷ4} with Ũ/R ={{ɷ2,ɷ3},{ɷ1}, {ɷ4,}} and X ={ɷ1, ɷ3}. Then N.T $N_R(x)$ = {Ũ,∅, {ɷ2, ɷ3,},{ɷ4},{ɷ2, ɷ3, ɷ4}}. №- β -OS ={ Ũ, ∅, {ɷ2, ɷ4},{ɷ1},{ɷ3, ɷ4,}. Then №-bcO(x) = {Ũ, ∅, {ɷ3, ɷ4 },{ɷ1, ɷ4},{ ɷ2,}. Let Ṽ = { ʐ1, ʐ2, ʐ3, ʐ4} with Ṽ /∅ ={{ʐ1},{ʐ3},{ʐ2, ʐ4}} and y ={ ʐ1, ʐ2}. Then N_R(y) = {Ṽ,∅, {ʐ1},{ʐ1, ʐ2, ʐ4},{ʐ2, ʐ4}}. №- β-OS ={ Ũ, ∅, { ʐ1,ʐ4},{ ʐ1}}. №- bcO(y) ={Ṽ,∅, {ʐ2},{ʐ1, ʐ4},{ʐ1, ʐ3}, { ʐ1, ʐ2, ʐ4}, { ʐ2, ʐ3, ʐ4 }}. Define η: (Ũ, $N_R(x)$) → (Ṽ, N_R (y)) by η(ɷ1) = ʐ2, η(ɷ2) = ʐ1, η(ɷ3) = ʐ4, η(ɷ4) = ʐ3. Then η is №-b-$\mathcal{OM}$ but not №- bc-$\mathcal{OM}$.

Theorem : 3.14 A function f : (Ũ, $N_R(x)$) → (Ṽ, N_R (y)) is №- bc-open mapping if and only if f(№-int(H) ⊆ №- bc-int(f(H)) for each subset H of (Ũ, $N_R(x)$).

Proof : Suppose that f : (Ũ, $N_R(x)$) → (Ṽ, N_R (y)) is №- bc-open mapping. Let H ⊆ U. Then №-int (H) is №-open in (Ũ, $N_R(x)$). Since f is №- bc-open, f(№-int(H)) is №- bc-open in (Ṽ, N_R (y)). Also №-int (H) ⊆ H, it follows that f(№-int(H)) ⊆ f(H). Thus f(№-int(H)) is №- bc-open contained in f(H). Hence f(№-int(H)) is contained in №- bc-int(f(H)). Conversely, let f(№-int(H)) ⊆ №- bc-int(f(H)) for every subset H of (Ũ, $N_R(x)$). Let H be a №-open set in (Ũ, $N_R(x)$). Then (№-int(H)) = H. Now f(H) ⊆ f(№-int(H) ⊆ №- bc-int(f(H)) = f(H). Hence f(H) = №-bc-int(f(H)). Therfore f(H) is №- bc-open in (Ṽ, N_R (y)) for every №-openset H in (Ũ, $N_R(x)$).Hence f is №- bc-open mapping.

Theorem 3.15 If a function f : (Ũ, $N_R(x)$) → (Ṽ, N_R (y)) is №- bc-open mapping, then №-bc int(f^{-1}(H)) ⊆ f^{-1} (№-bc-int(H)) for every №-subset H of (Ṽ, N_R (y)).

Proof : Let H be №-subset of (Ṽ, N_R (y)). Then int(f^{-1}(H)) is №-OS in (Ũ, $N_R(x)$). Since f is №- bc-$\mathcal{OM}$, f(№-int (f^{-1}(H))) is №- bc-open set in (Ṽ, N_R (y)).Hence f (№-int(f^{-1}(H))) ⊆ №-bc-int(f (f^{-1}(H))) ⊆ №-bc-int(H). Hence №-bc-int(f^{-1}(H)) ⊆ f^{-1} (№-bc-int(H)) .

Theorem 3.16 A mapping f : (Ũ, $N_R(x)$) → (Ṽ, N_R(y)) is №-$\mathcal{OM}$ and g:(Ṽ,N_R(y)) → (Ẅ, N_R(z)) is №-bc-$\mathcal{OM}$ then gof : (Ũ, $N_R(x)$) → (Ẅ, N_R(z)) is №-bc-$\mathcal{OM}$.

Proof : Let H be №-OS in (Ũ, $N_R(x)$). Then by given condition, f(H) is№-OS in (Ṽ,N_R(y)). Also given that g:(Ṽ,N_R(y)) → (Ẅ, N_R(z)) is №-bc-$\mathcal{OM}$. Then g(f(H)) = (gof) (H) is №-bc-OS in (Ẅ, N_R(z)).

IV. NANO-Bc CLOSED MAPPINGS IN NANO TOPOLOGICAL SPACES (№-bc-$\mathcal{CM}$)

Definition 4.1 Let (Ũ, $N_R(x)$) and (Ṽ, N_R (y)) be the two $\mathcal{NTS}$. A function η: (Ũ, $N_R(x)$) → (Ṽ, N_R (y)) with respect to X and y respectively is called **№-bc-closed mapping** if the image of every №-CS in (Ũ, $N_R(x)$) is №-bc-closed in (Ṽ, N_R (y)).

Example 4.2 Let Ũ = {ɷ1, ɷ2, ɷ3, ɷ4} with Ũ/R ={{ɷ1,ɷ2},{ɷ3}, ɷ4}} and X ={ɷ1, ɷ3}. Then N.T $N_R(x)$ = {Ũ,∅, {ɷ3},{ɷ1, ɷ2, ɷ3},{ɷ1, ɷ2}}. Then №-bc-C(x) = {Ũ, ∅, {ɷ1, ɷ2, ɷ4 },{ɷ3, ɷ4}},{ɷ4}}.
Let Ṽ = { ʐ1, ʐ2, ʐ3, ʐ4} with Ṽ /∅ ={{ʐ1},{ʐ3},{ʐ2, ʐ4}} and y ={ ʐ1, ʐ2}. Then N_R(y) = {Ṽ,∅, {ʐ1},{ʐ1, ʐ2, ʐ4},{ʐ2, ʐ4}}. №- bc-C(y) ={Ṽ,∅, {ʐ1},{ʐ1, ʐ3},{ʐ2, ʐ4}, { ʐ1, ʐ2, ʐ4}, { ʐ2, ʐ3, ʐ4 }}. Define η: (Ũ, $N_R(x)$) → (Ṽ, N_R (y)) by η(ɷ1) = ʐ2, η(ɷ2) = ʐ4, η(ɷ3) = ʐ1, η(ɷ4) = ʐ3. Then η is №-bc-$\mathcal{CM}$.

Theorem 4.3 Every №-θ-$\mathcal{CM}$ is №-bc-$\mathcal{CM}$.

Proof : Consider f : (Ũ, $N_R(x)$) → (Ṽ, N_R (y)) is №-θ-$\mathcal{CM}$ and H be a №- θclosed set in Ũ ,then f(H) is №- θ -closed in Ṽ. Since every №-θ-C set is №- bc-closed, f(H) is №-bc-closed in Ṽ. Hence f is №-bc-$\mathcal{CM}$.

Remark 4.4 The Reverse implication of the above theorem need not be true as shown in the following example.

Example 4.5 Let Ũ = {ɷ1, ɷ2, ɷ3, ɷ4} with Ũ/R ={{ɷ1,ɷ3},{ɷ2}, ɷ4}} and X ={ɷ1, ɷ3}. Then N.T $N_R(x)$ = {Ũ,∅, {ɷ2},{ɷ1, ɷ2, ɷ4},{ɷ1, ɷ3}}.№- θ-CS ={ Ũ, ∅, {ɷ2, ɷ1},{ɷ2},{{ɷ3,ɷ4}}. Then №-bcC(x) = {Ũ, ∅, {ɷ1, ɷ3, ɷ4 },{ɷ2, ɷ4}}. Let Ṽ = { ʐ1, ʐ2, ʐ3, ʐ4} with Ṽ /∅ ={{ʐ1},{ʐ3},{ʐ2, ʐ4}} and y ={ ʐ1, ʐ2}. Then N_R(y) = {Ṽ,∅, {ʐ1},{ʐ1, ʐ2, ʐ4},{ʐ2, ʐ4}}. №- θ-CS ={ Ũ, ∅, {ʐ1,ʐ2},{ ʐ4}}. №- bc-C(y) ={Ṽ,∅, {ʐ1},{ʐ1, ʐ3},{ʐ2, ʐ4}, { ʐ1, ʐ2, ʐ4}, { ʐ2, ʐ3, ʐ4 }}. Define η: (Ũ, $N_R(x)$) → (Ṽ, N_R (y)) by η(ɷ1) = ʐ3, η(ɷ2) = ʐ4, η(ɷ3) = ʐ1, η(ɷ4) = ʐ2. Then η is №-bc-$\mathcal{CM}$ but not №-θ- $\mathcal{CM}$.

Theorem 4.6 Every №-θS-$\mathcal{CM}$ is №-bc-$\mathcal{CM}$.

Proof : Consider f : (Ũ, $N_R(x)$) → (Ṽ, N_R (y)) is №-θS-$\mathcal{CM}$ and H be a №- closed set in Ũ ,then f(H) is №- θS -closed in Ṽ. Since every №-θS-closed set is №- bc-closed, f(H) is №-bc-closed in Ṽ. Hence f is №-bc-$\mathcal{CM}$.

Example 4.7 Let Ũ = {ɷ1, ɷ2, ɷ3, ɷ4} with Ũ/R ={{ɷ2},{ɷ1, ɷ4,}, ɷ3}} and X ={ɷ2, ɷ4}. Then N.T

$N_R(x)$ = {Ũ,∅, {$ɷ_1$,$ɷ_3$,},{$ɷ_2$, $ɷ_4$},{$ɷ_1$, $ɷ_2$, $ɷ_3$}}.№- θS-CS ={ Ũ, ∅, {$ɷ_1$},{$ɷ_2$,$ɷ_4$ },{$ɷ_3$,$ɷ_4$,}. Then №-bc-C(x) = {Ũ, ∅, {$ɷ_1$, $ɷ_2$ },{$ɷ_3$},{$ɷ_4$,}. Let Ṽ = { $ʓ_1$, $ʓ_2$, $ʓ_3$, $ʓ_4$} with Ṽ /∅ ={{$ʓ_1$},{$ʓ_3$},{$ʓ_2$, $ʓ_4$}} and y ={ $ʓ_1$, $ʓ_2$}. Then N_R(y) = {Ṽ,∅,{$ʓ_1$},{$ʓ_1$, $ʓ_2$, $ʓ_4$},{$ʓ_2$, $ʓ_4$}}. №- θ-CS ={ Ũ, ∅, { $ʓ_1$,$ʓ_4$},{ $ʓ_1$}}. №- bc-C(y) ={Ṽ,∅, {$ʓ_1$},{$ʓ_1$, $ʓ_3$},{$ʓ_2$, $ʓ_4$}, { $ʓ_1$, $ʓ_2$, $ʓ_4$}, { $ʓ_2$, $ʓ_3$, $ʓ_4$ }}. Define η: (Ũ, $N_R(x)$) → (Ṽ, N_R (y)) by η($ɷ_1$) = $ʓ_2$, η($ɷ_2$) = $ʓ_1$, η($ɷ_3$) = $ʓ_4$, η($ɷ_4$) = $ʓ_3$. Then η is №-bc-$\mathcal{CM}$ but not №-θS-$\mathcal{CM}$.

Theorem 4.8 Every №-bc-$\mathcal{CM}$ is №-S-$\mathcal{CM}$.

Proof : Consider f : (Ũ, $N_R(x)$) → (Ṽ, N_R (y)) is №-bc-$\mathcal{CM}$ and H be a №-closed set in Ũ ,then f(H) is №- bc-closed in Ṽ. Since every №-bc-closed set is №- semi-closed, f(H) is №-semi-closed in Ṽ. Hence f is №-S-$\mathcal{CM}$.

Example 4.9 Let Ũ = {$ɷ_1$, $ɷ_2$, $ɷ_3$, $ɷ_4$} with Ũ/R ={{$ɷ_1$},{$ɷ_1$, $ɷ_3$,}, $ɷ_2$}} and X ={$ɷ_1$, $ɷ_4$}.Then $N_R(x)$ = {Ũ,∅, {$ɷ_1$,$ɷ_2$,},{$ɷ_2$, $ɷ_3$},{$ɷ_1$, $ɷ_3$, $ɷ_4$}}.№- S-CS ={ Ũ, ∅, {$ɷ_2$, $ɷ_4$},{$ɷ_1$},{ $ɷ_3$, $ɷ_4$,}. Then №-bc-C(x) = {Ũ, ∅, { $ɷ_3$, $ɷ_4$ },{$ɷ_1$, $ɷ_4$},{ $ɷ_2$,}. Let Ṽ = { $ʓ_1$, $ʓ_2$, $ʓ_3$, $ʓ_4$} with Ṽ/∅ ={{$ʓ_1$},{$ʓ_3$},{$ʓ_2$, $ʓ_4$}} and y ={ $ʓ_1$, $ʓ_2$}. Then N_R(y) ={Ṽ,∅, {$ʓ_1$},{$ʓ_1$, $ʓ_2$, $ʓ_4$},{$ʓ_2$, $ʓ_4$}}. №- S-CS ={ Ũ, ∅, { $ʓ_1$,$ʓ_4$},{ $ʓ_1$}}. №-bc -C(y) ={Ṽ,∅, {$ʓ_1$},{$ʓ_1$, $ʓ_3$},{$ʓ_2$, $ʓ_4$}, { $ʓ_1$, $ʓ_2$, $ʓ_4$}, { $ʓ_2$, $ʓ_3$, $ʓ_4$ }}. Define η: (Ũ, $N_R(x)$) → (Ṽ, N_R (y)) by η($ɷ_1$) = $ʓ_2$, η($ɷ_2$) = $ʓ_1$, η($ɷ_3$) = $ʓ_4$, η($ɷ_4$) = $ʓ_3$. Then η is №-S-$\mathcal{CM}$ but not №- bc-$\mathcal{CM}$.

Theorem 4.10 Every №-bc-$\mathcal{CM}$ is №-b-$\mathcal{CM}$.

Proof : Consider f : (Ũ, $N_R(x)$) → (Ṽ, N_R (y)) is №-bc-$\mathcal{CM}$ and H be a №- closed set in Ũ ,then f(H) is №- bc –closed in Ṽ. Since every №-bc-closed set is №- b-closed, f(H) is №-b-closed in Ṽ. Hence f is №-b-$\mathcal{CM}$.

Example 4.11 Let Ũ = {$ɷ_1$, $ɷ_2$, $ɷ_3$, $ɷ_4$} with Ũ/R ={{$ɷ_1$,$ɷ_2$},{$ɷ_4$}, {$ɷ_3$,}} and X ={$ɷ_2$, $ɷ_3$}. Then N.T $N_R(x)$ = {Ũ,∅, {$ɷ_3$, $ɷ_1$,},{$ɷ_1$, $ɷ_4$},{$ɷ_1$, $ɷ_3$, $ɷ_4$}}.№- b-CS ={ Ũ, ∅, {$ɷ_2$, $ɷ_4$},{$ɷ_1$},{ $ɷ_3$, $ɷ_4$,}. Then №-bc-C(x) = {Ũ, ∅, { $ɷ_3$, $ɷ_4$ },{$ɷ_1$, $ɷ_4$},{ $ɷ_2$,}. Let Ṽ = { $ʓ_1$, $ʓ_2$, $ʓ_3$, $ʓ_4$} with Ṽ /∅ ={{$ʓ_1$},{$ʓ_3$},{$ʓ_2$, $ʓ_4$}} and y ={ $ʓ_1$, $ʓ_2$}. Then N_R(y) = {Ṽ,∅, {$ʓ_1$},{$ʓ_1$, $ʓ_2$, $ʓ_4$},{$ʓ_2$, $ʓ_4$}}. №- b-CS ={ Ũ, ∅, { $ʓ_1$,$ʓ_4$},{ $ʓ_1$}}. №- bcC(y) ={Ṽ,∅, {$ʓ_2$},{$ʓ_1$, $ʓ_4$},{$ʓ_1$, $ʓ_3$}, { $ʓ_1$, $ʓ_2$, $ʓ_4$}, { $ʓ_2$, $ʓ_3$, $ʓ_4$ }}. Define η: (Ũ, $N_R(x)$) → (Ṽ, N_R (y)) by η($ɷ_1$) = $ʓ_2$, η($ɷ_2$) = $ʓ_1$, η($ɷ_3$) = $ʓ_4$, η($ɷ_4$) = $ʓ_3$. Then η is №-b-$\mathcal{CM}$ but not №- bc-$\mathcal{CM}$.

Theorem 4.12 Every №-bc-$\mathcal{CM}$ is №- β -$\mathcal{CM}$.

Proof : Consider f : (Ũ, $N_R(x)$) → (Ṽ, N_R (y)) is №-bc-$\mathcal{CM}$ and H be a №-closed set in Ũ ,then f(H) is №- bc -closed in Ṽ. Since every №-bc-closed set is №- β -closed, f(H) is №- β -closed in Ṽ. Hence f is №- β –$\mathcal{CM}$.

Example 4.13 Let Ũ = {$ɷ_1$, $ɷ_2$, $ɷ_3$, $ɷ_4$} with Ũ/R ={{$ɷ_2$,$ɷ_3$},{$ɷ_1$}, {$ɷ_4$,}} and X ={$ɷ_1$, $ɷ_3$}. Then N.T $N_R(x)$ = {Ũ,∅, {$ɷ_2$, $ɷ_3$,},{$ɷ_4$},{$ɷ_2$, $ɷ_3$, $ɷ_4$}}. №- β -CS ={ Ũ, ∅, {$ɷ_2$, $ɷ_4$},{$ɷ_1$},{$ɷ_3$, $ɷ_4$,}. Then №-bcC(x) = {Ũ, ∅, { $ɷ_3$, $ɷ_4$ },{$ɷ_1$, $ɷ_4$},{ $ɷ_2$,}. Let Ṽ = { $ʓ_1$, $ʓ_2$, $ʓ_3$, $ʓ_4$} with Ṽ /∅ ={{$ʓ_1$},{$ʓ_3$},{$ʓ_2$, $ʓ_4$}} and y ={ $ʓ_1$, $ʓ_2$}. Then N_R(y) = {Ṽ,∅, {$ʓ_1$},{$ʓ_1$, $ʓ_2$, $ʓ_4$},{$ʓ_2$, $ʓ_4$}}. №- β-CS ={ Ũ, ∅, { $ʓ_1$,$ʓ_4$},{ $ʓ_1$}}. №- bcC(y) ={Ṽ,∅, {$ʓ_2$},{$ʓ_1$, $ʓ_4$},{$ʓ_1$, $ʓ_3$}, { $ʓ_1$, $ʓ_2$, $ʓ_4$}, { $ʓ_2$, $ʓ_3$, $ʓ_4$ }}. Define η: (Ũ, $N_R(x)$) → (Ṽ, N_R (y)) by η($ɷ_1$) = $ʓ_2$, η($ɷ_2$) = $ʓ_1$, η($ɷ_3$) = $ʓ_4$, η($ɷ_4$) = $ʓ_3$. Then η is №-b-$\mathcal{CM}$ but not №- bc-$\mathcal{CM}$

Theorem 4.14 A function f : (Ũ, $N_R(x)$) → (Ṽ, N_R (y)) is №-bc-$\mathcal{CM}$ if and only if №-bc-(cl-f(H)) ⊆ f (№-cl (H)) for any №-subset H of (Ũ, $N_R(x)$).

Proof : Let f : (Ũ, $N_R(x)$) → (Ṽ, N_R (y)) be a №-bc-$\mathcal{CM}$ and H ⊆ Ũ . Then №- cl (H) is №-closed in (Ũ, $N_R(x)$) and hence №-cl-f(H) is №-bc-closed mapping in (Ṽ, N_R (y)). Since №- cl(H) ⊆ H implies that №-cl-f(H) ⊆ f (H) and №-bc- cl(H) ⊆ f (№-cl(H)) is №-bc-CS containing f(H), it follows that №-bc- (cl-f (H)) ⊆ №-bc-(clf (№-cl-(H))) = f (№-cl (H)). Conversely let H be №-CS in (Ũ, $N_R(x)$). Then №- cl(H) = H and №-bc- (cl-f(H)) ⊆ f(№-cl(H)) = f(H) . Also, f(H) ⊆ (№-bc-(cl f(H)). Hence f(H) = №-bc-(cl-f(H)).Therefore f (H) is №-bc –CS . Hence f : (Ũ, $N_R(x)$) → (Ṽ, N_R (y)) is №-bc-$\mathcal{CM}$.

Theorem 4.15 A function f : (Ũ, $N_R(x)$) → (Ṽ, N_R (y)) is №-bc-closed if and only if for each subset H of (Ṽ, N_R (y)) and for each №-open set E of (Ũ, $N_R(x)$) containing f^{-1}(H), there is a№-bc-OS F of (Ṽ, N_R (y)) such that H ⊆ F and f^{-1}(F) ⊆ E.

Proof : Let H be the №-subset of (Ṽ, N_R (y)) and E be a №-OS of (Ũ, $N_R(x)$) such that f^{-1}(H) ⊆ E. Now Ṽ- f (Ũ – E), say F is a №-bc- OS containing H in Ṽ such that f^{-1}(H) ⊆ F. Conversely, let W be a №-closed set of (Ũ, $N_R(x)$) , then Ṽ- f (W) and Ũ –W is №-open. Now, there is a №-bc-OS F of (Ṽ, N_R (y)) such that Ṽ- f (W) ⊆ F and f^{-1}(F) ⊆ Ũ –W. Hence W ⊆ Ũ – f^{-1}(F) and thus Ṽ- F ⊆ f (w) ⊆ f(Ũ –f^{-1}(F)) Ṽ- F .This implies that f(w) = Ṽ- F. Since Ṽ- F is №-bc-closed , f(w) is №-bc-closed set in (Ṽ, N_R (y)) for every №-C set W in (Ũ, $N_R(x)$). Hence f is №-bc-closed mapping.

Remark 4.16 The Composition of two №-bc-closed mappings need not be №-bc-closed as shown from the following example.

Example 4.17 Let Ũ = {$ɷ_1$, $ɷ_2$, $ɷ_3$, $ɷ_4$} with Ũ/R ={{ $ɷ_1$},{$ɷ_3$}, { $ɷ_2$, $ɷ_4$,}} and X ={$ɷ_1$, $ɷ_2$}and N.T

$N_R(x)$ = {Ũ,∅, {ω₂, ω₄,},{ω₁},{ω₁, ω₂, ω₄}}.№-CS = {Ũ,∅, {ω₁, ω₃,},{ω₁},{ω₂, ω₃, ω₄}} №- β -CS ={ Ũ, ∅, {ω₂, ω₄},{ω₁},{ω₃, ω₄,}. Then №-bcC(x) = {Ũ, ∅, { ω₃, ω₄ },{ω₁, ω₄},{ ω₂,}. Let Ṽ = { ʐ₁, ʐ₂, ʐ₃, ʐ₄} with Ṽ /∅ ={{ʐ₁},{ʐ₃},{ʐ₂, ʐ₄}} and y ={ ʐ₁, ʐ₂}. Then N_R(y) = {Ṽ,∅, {ʐ₁},{ʐ₁, ʐ₂, ʐ₄},{ʐ₂, ʐ₄}}. №--CS ={ Ũ, ∅, { ʐ₁,ʐ₄},{ ʐ₁}}. №- bcC(y) ={Ṽ,∅, {ʐ₂},{ʐ₁, ʐ₄},{ʐ₁, ʐ₃}, { ʐ₁, ʐ₂, ʐ₄}, { ʐ₂, ʐ₃, ʐ₄ }}. Define η: (Ũ, $N_R(x)$) → (Ṽ, N_R (y)) by η(ω₁) = ʐ₂, η(ω₂) = ʐ₁, η(ω₃) = ʐ₄, η(ω₄) = ʐ₃. Let W = {ω₁, ω₂, ω₃, ω₄} with W/R ={{ ω₂},{ω₄}, { ω₁, ω₃,}} and X ={ω₁,ω₂}and $N_R(x)$ = {Ũ,∅, {ω₂, ω₄,},{ω₁},{ω₁, ω₂, ω₄}}.№-CS = {Ũ,∅, {ω₁, ω₃,},{ω₁},{ω₂, ω₃, ω₄}} №- β -CS ={ Ũ, ∅, {ω₂, ω₄},{ω₁},{ω₃, ω₄,}. Then №-bcC(x) = {Ũ, ∅, { ω₃, ω₄ },{ω₁, ω₄},{ ω₂,}.Here we observe that let gof : Ũ → W be two composition of №-bc-closed mappings and (gof) (ω₃) is not №-bc-closed in W, Hence the Composition of two №-bc-closed mappings need not be №-bc-closed.

V.CONCLUSION

Many different forms of continuous functions have been introduced over the years. Its importance is significant in various areas of mathematics and related sciences. In this paper we presented №-bc-open mappings and №-bc-closed mappings and discussed some of their properties .Also we investigate the relationships between the other existing nano open and closed mappings.

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