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Sombor Uphill Indices of Graphs

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Abstract: *In this study, we introduce the uphill Sombor and modified uphill Sombor indices and their corresponding exponentials of a graph. Furthermore, we compute these indices for some standard graphs, wheel graphs, gear graphs, helm graphs.*

Keywords: *uphill Sombor index, modified uphill Sombor index, graph.*

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I. Introduction

In this paper, G denotes a finite, simple, connected graph, $V(G)$ and $E(G)$ denote the vertex set and edge set of G. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. Any undefined terminologies and notations may be found in [1].

A u-v path P in G is a sequence of vertices in G, starting with u and ending at V, such that consecutive vertices in P are adjacent, and no vertex is repeated. A path $\pi = v_1, v_2, \ldots v_{k+1}$ in G is a downhill path if for every i, $1 \le i \le k$, $d_G(v_i) \ge d_G(v_{i+1})$.

A vertex v is downhill dominates a vertex u if there exists a dowmhill path originated from u to v. The downhill neighborhood of a vertex v is denoted by $N_{dn}(v)$ and defined as: $N_{dn}(v) = \{u: v$ downhill dominates $u\}$. The downhill degree $d_{dn}(v)$ of a vertex v is the number of downhill neighbors of v [2].

Recently, some downhill indices were studied in [3-11].

The uphill domination is introduced by Deering in [12].

A u-v path P in G is a sequence of vertices in G, starting with u and ending at v, such that consecutive vertices in P are adjacent, and no vertex is repeated. A path $\pi = v_1, v_2, \ldots v_{k+1}$ in G is a uphill path if for every i, $1 \le i \le k$, $d_G(v_i) \le d_G(v_{i+1})$.

A vertex v is uphill dominates a vertex u if there exists an uphill path originated from u to v. The uphill neighborhood of a vertex v is denoted by $N_{up}(v)$ and defined as: $N_{up}(v) = \{u: v$ uphill dominates $u\}$. The uphill degree $d_{up}(v)$ of a vertex v is the number of uphill neighbors of v, see [13].

Recently, the F-hill index was studied in [14, 15].

The Sombor index was introduced in [16] and it is defined as

$$SO(G) = \sum_{uv \in E(G)} \sqrt{d_G(u)^2 + d_G(v)^2}.$$

Motivated by the definition of Sombor index, we introduce the uphill Sombor index of a graph and it is defined as

$$USO(G) = \sum_{uv \in E(G)} \sqrt{d_{up}(u)^2 + d_{up}(v)^2}.$$

Considering the uphill Sombor index, we introduce the uphill Sombor exponential of a graph G and defined it as

$$USO(G,x) = \sum_{uv \in E(G)} x^{\sqrt{d_{up}(u)^2 + d_{up}(v)^2}}.$$

We define the modified uphill Sombor index of a graph G as

$$^{m}USO(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_{up}(u)^2 + d_{up}(v)^2}}.$$

Considering the modified uphill Sombor index, we introduce the modified uphill Sombor exponential of a graph G and defined it as

$$^{m}USO(G,x) = \sum_{uv \in E(G)} x^{\frac{1}{\sqrt{d_{up}(u)^2 + d_{up}(v)^2}}}.$$

Recently, some Sombor indices were studied in [17-30].

In this paper, the uphill Sombor index, modified uphill Sombor index and their corresponding exponentials of certain graphs, honeycomb networks are computed.

II. Results for Some Standard Graphs

Proposition 1. Let G be r-regular with n vertices and $r \geq 2$. Then

$$USO(G) = \frac{nr(n-1)}{\sqrt{2}}.$$

Proof: Let G be an r-regular graph with n vertices and $r \geq 2$ and $\frac{nr}{2}$ edges. Then $d_{up}(v) = n-1$ for every v in G.

$$USO(G) = \sum_{uv \in E(G)} \sqrt{d_{up}(u)^2 + d_{up}(v)^2}$$

$$= \frac{nr}{2}\sqrt{(n-1)^2 + (n-1)^2}$$

$$= \frac{nr(n-1)}{\sqrt{2}}.$$

Corollary 1.1. Let C_n be a cycle with $n \geq 3$ vertices. Then

$$USO(C_n) = \sqrt{2}n(n-1).$$

Corollary 1.2. Let K_n be a complete graph with $n \geq 3$ vertices. Then

$$USO(K_n) = \frac{n(n-1)^2}{\sqrt{2}}.$$

Proposition 2. Let G be r-regular with n vertices and $r \geq 2$. Then

$$USO(G,x)=\frac{nr}{2}x^{\sqrt{2}(n-1)}.$$

Proof: Let G be an r-regular graph with n vertices and $r \geq 2$ and $\frac{nr}{2}$ edges. Then $d_{up}(v)=n-1$ for every v in G.

$$USO(G,x)=\sum_{uv\in E(G)} x^{\sqrt{d_{up}(u)^2+d_{up}(v)^2}}$$

$$=\frac{nr}{2}x^{\sqrt{(n-1)^2+(n-1)^2}}$$

$$=\frac{nr}{2}x^{\sqrt{2}(n-1)}.$$

Corollary 2.1. Let C_n be a cycle with $n \geq 3$ vertices. Then

$$USO(C_n,x)=nx^{\sqrt{2}(n-1)}.$$

Corollary 2.2. Let K_n be a complete graph with $n \geq 3$ vertices. Then

$$USO(K_n,x)=\frac{n(n-1)}{2}x^{\sqrt{2}(n-1)}.$$

Proposition 3. Let G be r-regular with n vertices and $r \geq 2$. Then

$$^{m}USO(G)=\frac{nr}{2\sqrt{2}(n-1)}.$$

Proof: Let G be an r-regular graph with n vertices and $r \geq 2$ and $\frac{nr}{2}$ edges. Then $d_{up}(v)=n-1$ for every v in G.

$$^{m}USO(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{d_{up}(u)^2+d_{up}(v)^2}}$$

$$=\frac{nr}{2}\frac{1}{\sqrt{(n-1)^2+(n-1)^2}}$$

$$=\frac{nr}{2\sqrt{2}(n-1)}.$$

Corollary 3.1. Let C_n be a cycle with $n \geq 3$ vertices. Then

$$^{m}USO(C_n)=\frac{n}{\sqrt{2}(n-1)}=\sqrt{2}n(n-1).$$

Corollary 3.2. Let K_n be a complete graph with $n \geq 3$ vertices. Then

$$^{m}USO(K_n)=\frac{n}{2\sqrt{2}}.$$

Proposition 4. Let G be r-regular with n vertices and $r \geq 2$. Then

$$^{m}USO(G)=\frac{nr}{2}x^{\sqrt{2}(n-1)}.$$

Proof: Let G be an r-regular graph with n vertices and $r \geq 2$ and $\frac{nr}{2}$ edges. Then $d_{up}(v) = n-1$ for every v in G.

$$^{m}USO(G,x) = \sum_{uv \in E(G)} x^{\frac{1}{\sqrt{d_{up}(u)^2 + d_{up}(v)^2}}}$$

$$= \frac{nr}{2} x^{\frac{1}{\sqrt{(n-1)^2+(n-1)^2}}}$$

$$= \frac{nr}{2} x^{\frac{1}{\sqrt{2}(n-1)}}.$$

Corollary 4.1. Let C_n be a cycle with $n \geq 3$ vertices. Then

$$^{m}USO(C_n,x) = nx^{\frac{1}{\sqrt{2}(n-1)}}.$$

Corollary 4.2. Let K_n be a complete graph with $n \geq 3$ vertices. Then

$$^{m}USO(K_n,x) = \frac{n(n-1)}{2} x^{\frac{1}{\sqrt{2}(n-1)}}.$$

Proposition 5. Let P be a path with $n \geq 3$ vertices. Then

$$USO(P) = 2\sqrt{2n^2 - 10n + 13} + \sqrt{2}(n-3)^2.$$

Proof: Let P be a path with $n \geq 3$ vertices. Clearly, P has two types of edges based on the uphill degree of end vertices of each edge as follows:

$E_1 = \{uv \in E(P) \mid d_{up}(u) = n-2,\ d_{up}(v) = n-3\}$, $|E_1| = 2$.
$E_2 = \{uv \in E(P) \mid d_{up}(u) = d_{up}(v) = n-3\}$, $|E_2| = n-3$.

$$USO(P) = \sum_{uv \in E(P)} \sqrt{d_{up}(u)^2 + d_{up}(v)^2}$$

$$= 2\sqrt{(n-2)^2 + (n-3)^2} + (n-3)\sqrt{(n-3)^2 + (n-3)^2}$$

$$= 2\sqrt{2n^2 - 10n + 13} + \sqrt{2}(n-3)^2.$$

Proposition 6. Let P be a path with $n \geq 3$ vertices. Then

$$USO(P,x) = 2x^{\sqrt{2n^2-10n+13}} + (n-3)x^{(n-3)\sqrt{2}}.$$

Proof: We obtain

$$USO(P,x) = \sum_{uv \in E(P)} x^{\sqrt{d_{up}(u)^2 + d_{up}(v)^2}}$$

$$= 2x^{\sqrt{(n-2)^2+(n-3)^2}} + (n-3)x^{\sqrt{(n-3)^2+(n-3)^2}}$$

$$= 2x^{\sqrt{2n^2-10n+13}} + (n-3)x^{(n-3)\sqrt{2}}.$$

Proposition 7. Let P be a path with $n \geq 3$ vertices. Then

$$ {}^{m}USO(P)=\frac{2}{\sqrt{2n^2-10n+13}}+\frac{1}{\sqrt{2}}. $$

Proof: We obtain

$$ {}^{m}USO(P)=\sum_{uv\in E(P)}\frac{1}{\sqrt{d_{up}(u)^2+d_{up}(v)^2}} $$

$$ =\frac{2}{\sqrt{(n-2)^2+(n-3)^2}}+\frac{(n-3)}{\sqrt{(n-3)^2+(n-3)^2}} $$

$$ =\frac{2}{\sqrt{2n^2-10n+13}}+\frac{1}{\sqrt{2}}. $$

Proposition 8. Let P be a path with $n\geq 3$ vertices. Then

$$ {}^{m}USO(P,x)=2x^{\frac{1}{\sqrt{2n^2-10n+13}}}+(n-3)x^{\frac{1}{(n-3)\sqrt{2}}}. $$

Proof: We obtain

$$ {}^{m}USO(P,x)=\sum_{uv\in E(P)}x^{\frac{1}{\sqrt{d_{up}(u)^2+d_{up}(v)^2}}} $$

$$ =2x^{\frac{1}{\sqrt{(n-2)^2+(n-3)^2}}}+(n-3)x^{\frac{1}{\sqrt{(n-3)^2+(n-3)^2}}} $$

$$ =2x^{\frac{1}{\sqrt{2n^2-10n+13}}}+(n-3)x^{\frac{1}{(n-3)\sqrt{2}}}. $$

III. Results for Wheel Graphs

Let W_n be a wheel with $n+1$vertices and $2n$ edges, $n\geq 4$. Then there are two types of edges based on the uphill degree of end vertices of each edge as follows:

$E_1 = \{uv \in E(W_n) \mid d_{up}(u) = 0, d_{up}(v) = n\}$, $|E_1| = n$.
$E_2 = \{uv \in E(W_n) \mid d_{up}(u) = d_{up}(v) = n\}$, $|E_2| = n$.

Theorem 1. Let W_n be a wheel with $n+1$vertices and $2n$ edges, $n\geq 4$. Then

$$ USO(W_n)=(1+\sqrt{2})n^2. $$

Proof. We deduce

$$ USO(W_n)=\sum_{uv\in E(W_n)}\sqrt{d_{up}(u)^2+d_{up}(v)^2} $$

$$ = n\sqrt{0^2+n^2}+n\sqrt{n^2+n^2} $$

$$ =(1+\sqrt{2})n^2. $$

Theorem 2. Let W_n be a wheel with $n+1$vertices and $2n$ edges, $n\geq 4$. Then

$$ USO(W_n,x)=nx^n+nx^{\sqrt{2}n}. $$

Proof. We obtain

$$ USO(W_n,x)=\sum_{uv\in E(W_n)}x^{\sqrt{d_{dn}(u)^2+d_{dn}(v)^2}} $$

$$ = nx^{\sqrt{0^2+n^2}}+nx^{\sqrt{n^2+n^2}} $$

$$= nx^{n} + nx^{\sqrt{2}n}.$$

Theorem 3. Let W_n be a wheel with n+1vertices and 2n edges , n≥4. Then

$${}^{m}USO(W_n) = 1 + \frac{1}{\sqrt{2}}.$$

Proof. We deduce

$${}^{m}USO(W_n) = \sum_{uv \in E(W_n)} \frac{1}{\sqrt{d_{up}(u)^2 + d_{up}(v)^2}}$$

$$= \frac{n}{\sqrt{0^2 + n^2}} + \frac{n}{\sqrt{n^2 + n^2}}$$

$$= 1 + \frac{1}{\sqrt{2}}.$$

Theorem 4. Let W_n be a wheel with n+1vertices and 2n edges, n≥4. Then

$${}^{m}USO(W_n, x) = nx^{\frac{1}{n}} + nx^{\frac{1}{\sqrt{2}n}}.$$

Proof. We obtain

$${}^{m}USO(W_n, x) = \sum_{uv \in E(W_n)} x^{\frac{1}{\sqrt{d_{up}(u)^2 + d_{up}(v)^2}}}$$

$$= nx^{\frac{1}{\sqrt{0^2+n^2}}} + nx^{\frac{1}{\sqrt{n^2+n^2}}}$$

$$= nx^{\frac{1}{n}} + nx^{\frac{1}{\sqrt{2}n}}.$$

IV. Results for Gear Graphs

A bipartite wheel graph is a graph obtained from W_n with n+1 vertices adding a vertex between each pair of adjacent rim vertices and this graph is denoted by G_n and also called as a gear graph. Clearly, $|V(G_n)| = 2n+1$ and $|E(G_n)| = 3n$. A gear graph G_n is depicted in Figure 1.

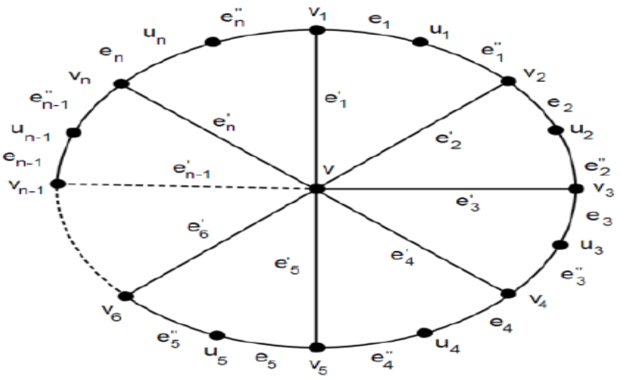

Figure 1. Gear graph G_n

Let G_n be a gear graph with 3n edges, n≥4. Then there are two types of edges based on the uphill degree of end vertices of each edge as follows:

$E_1 = \{u \in E(G_n) \mid d_{up}(u) = 1, d_{up}(v) = 0\}$, $\quad |E_1| = n$.

$E_2 = \{u \in E(G_n) \mid d_{up}(u) = 1, , d_{up}(v) = 3\}$, $\quad | E_2 | = 2n$.

Theorem 5. Let G_n be a gear graph with $2n$+1vertices, n≥3. Then the uphill Sombor index of G_n is

$$USO(G_n) = (1 + 2\sqrt{10})n.$$

Proof: We deduce

$$USO(G_n) = \sum_{uv \in E(G_n)} \sqrt{d_{up}(u)^2 + d_{up}(v)^2}$$
$$= n\sqrt{1^2 + 0^2} + 2n\sqrt{1^2 + 3^2}$$
$$= (1 + 2\sqrt{10})n.$$

Theorem 6. Let G_n be a gear graph with $2n$+1vertices, n≥3. Then the uphill Sombor exponential of G_n is

$$USO(G_n, x) = nx^1 + 2nx^{\sqrt{10}}.$$

Proof: We deduce

$$USO(G_n, x) = \sum_{uv \in E(G_n)} x^{\sqrt{d_{up}(u)^2 + d_{up}(v)^2}}$$
$$= nx^{\sqrt{1^2 + 0^2}} + 2nx^{\sqrt{1^2 + 3^2}}$$
$$= nx^1 + 2nx^{\sqrt{10}}.$$

Theorem 7. Let G_n be a gear graph with $2n$+1vertices, n≥3. Then the modified uphill Sombor index of G_n is

$$^{m}USO(G_n) = \left(1 + \frac{2}{\sqrt{10}}\right)n.$$

Proof: We deduce

$$^{m}USO(G_n) = \sum_{uv \in E(G_n)} \frac{1}{\sqrt{d_{up}(u)^2 + d_{up}(v)^2}}$$
$$= \frac{n}{\sqrt{1^2 + 0^2}} + \frac{2n}{\sqrt{1^2 + 3^2}}$$
$$= \left(1 + \frac{2}{\sqrt{10}}\right)n.$$

Theorem 8. Let G_n be a gear graph with $2n$+1vertices, n≥3. Then the modified uphill Sombor exponential of G_n is

$$^{m}USO(G_n, x) = nx^1 + 2nx^{\frac{1}{\sqrt{10}}}.$$

Proof: We deduce

$$^{m}USO(G_n, x) = \sum_{uv \in E(G_n)} x^{\frac{1}{\sqrt{d_{up}(u)^2 + d_{up}(v)^2}}}$$
$$= nx^{\frac{1}{\sqrt{1^2 + 0^2}}} + 2nx^{\frac{1}{\sqrt{1^2 + 3^2}}}$$
$$= nx^1 + 2nx^{\frac{1}{\sqrt{10}}}.$$

V. Results for Helm Graphs

The helm graph H_n is a graph obtained from W_n (with n+1 vertices) by attaching an end edge to each rim vertex of W_n. Clearly, $|V(H_n)| = 2n+1$ and $|E(H_n)| = 3n$. A graph H_n is shown in Figure 2.

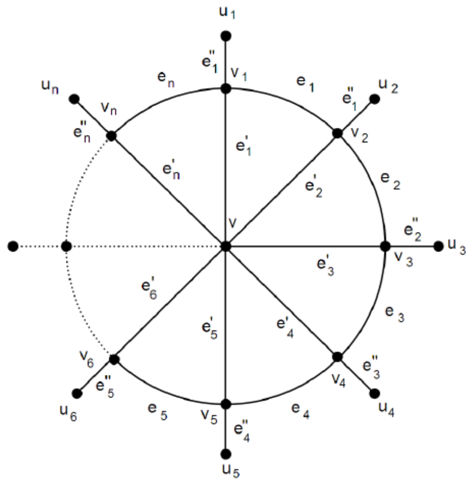

Figure 2. Helm graph H_n

Let H_n be a helm graph with $3n$ edges, $n \geq 3$. Then H_n has three types of the uphill degree of edges as follows:

$E_1 = \{uv \in E(H_n) \mid d_{up}(u) = n+1,\ d_{up}(v) = n\}$. $\quad |E_1| = n$.
$E_2 = \{uv \in E(H_n) \mid d_{up}(u) = d_{up}(v) = n\}$. $\quad |E_2| = n$.
$E_3 = \{uv \in E(H_n) \mid d_{up}(u) = n,\ d_{up}(v) = 0\}$. $\quad |E_3| = n$.

Theorem 9. Let H_n be a helm graph with $2n+1$ vertices, $n \geq 3$. Then the uphill Sombor index of H_n is

$$USO(H_n) = n\sqrt{2n^2 + 2n + 1} + (\sqrt{2} + 1)n^2.$$

Proof: We obtain

$$USO(H_n) = \sum_{uv \in E(H_n)} \sqrt{d_{up}(u)^2 + d_{up}(v)^2}$$
$$= n\sqrt{(n+1)^2 + n^2} + n\sqrt{n^2 + n^2} + n\sqrt{n^2 + 0^2}$$
$$= n\sqrt{2n^2 + 2n + 1} + (\sqrt{2} + 1)n^2.$$

Theorem 10. Let H_n be a helm graph with $2n+1$vertices, $n \geq 3$. Then the uphill Sombor exponential of H_n is

$$USO(H_n, x) = nx^{\sqrt{2n^2 + 2n + 1}} + nx^{\sqrt{2}n} + nx^n.$$

Proof: We deduce

$$USO(H_n, x) = \sum_{uv \in E(H_n)} x^{\sqrt{d_{up}(u)^2 + d_{up}(v)^2}}$$
$$= nx^{\sqrt{(n+1)^2 + n^2}} + nx^{\sqrt{n^2 + n^2}} + nx^{\sqrt{n^2 + 0^2}}$$
$$= nx^{\sqrt{2n^2 + 2n + 1}} + nx^{\sqrt{2}n} + nx^n.$$

Theorem 11. Let H_n be a helm graph with $2n+1$vertices, $n \geq 3$. Then the modified uphill Sombor index of H_n is

$$^{m}USO(H_n) = \frac{n}{\sqrt{2n^2 + 2n + 1}} + \frac{1}{\sqrt{2}} + 1.$$

Proof: We deduce

$$^{m}USO(H_n) = \sum_{uv \in E(H_n)} \frac{1}{\sqrt{d_{up}(u)^2 + d_{up}(v)^2}}$$

$$= \frac{n}{\sqrt{(n+1)^2 + n^2}} + \frac{n}{\sqrt{n^2 + n^2}} + \frac{n}{\sqrt{n^2 + 0^2}}$$

$$= \frac{n}{\sqrt{2n^2 + 2n + 1}} + \frac{1}{\sqrt{2}} + 1.$$

Theorem 12. Let H_n be a helm graph with $2n+1$vertices, $n\geq3$. Then the modified uphill Sombor exponential of H_n is

$$^{m}USO(H_n, x) = nx^{\frac{1}{\sqrt{2n^2+2n+1}}} + nx^{\frac{1}{n\sqrt{2}}} + nx^{\frac{1}{n}}.$$

Proof: We deduce

$$^{m}USO(H_n) = \sum_{uv \in E(H_n)} \frac{1}{\sqrt{d_{up}(u)^2 + d_{up}(v)^2}}$$

$$= nx^{\frac{1}{\sqrt{(n+1)^2+n^2}}} + nx^{\frac{1}{\sqrt{n^2+n^2}}} + nx^{\frac{1}{\sqrt{n^2+0^2}}}$$

$$= nx^{\frac{1}{\sqrt{2n^2+2n+1}}} + nx^{\frac{1}{n\sqrt{2}}} + nx^{\frac{1}{n}}.$$

VI. Conclusion

In this paper, the uphill Sombor index, modified uphill Sombor index and their corresponding exponentials of certain graphs are determined.

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