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Two- Echelon Trade Credit Financing in a Supply Chain with zeibull Distribution and Exponentially Increasing Holding Cost

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Abstract
In the present day, competitive marketplace, offering delay payments, has becomeacommonlyadoptedmethod.Previousinventorymodelsunderpermissibledelayinpaymentsusuallyassumedthat thedemandoftheitemswaseither constantormerelydependentontheretailingprice. Inthispaper,deterministic inventory model for deteriorating items having stock and time dependent demand undertheeffectsofdeteriorationhasbeenstudied.AtwoparameterWeibulldistributionhasbeenusedtorepresentthedeterioration rate.Thepresentmodelhas beensolvedanalyticallytominimizethetotalcostofthesystem. Thenecessary andsufficientconditionsfortheexistenceanduniquenessoftheoptimalsolutions whichcouldminimizetheretailer'stotalcostperunittimehasbeendiscussed. Anumericalexampleisincludedtodemonstratethedevelopedmodelandthe solutionprocedure.Toinvestigatetheeffectofchangesinsomemainparameters valuesontheoptimalsolution,weconductasensitivityanalysisanddiscusssome important managerial insights.
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I. Introduction

Bothindeterministicandprobabilisticinventorymodelsofclassicaltype,itisobserved that payment is made to the supplier immediately after receiving the items.In practice, the supplier will offer he retailer a delay period in paying for the amount of purchaseto increase the demand known as trade credit period.Offering such a credit period to theretailerwillencouragethesupplier'ssellingandreduceon-handstocklevel.Simultaneously, without a primary payment, the retailer can take the advantages of a credit period to reduce cost and increase profit.Thus,the delay in the payment offered bythe supplier is a kind of price discount it encourages the retailer to increase their order quantity. Moreover, during this credit period, the retailer can start to accumulate revenues on the sales and earn interest on that revenue.But the higher interest is chargedif the payment is not settled by the end of the credit period.Hence, trade credit canplayanimportant roleininventory modelforboth thesuppliersas wellasthe retailers.

To manage the inventory level successfully,the retailer needs to find a balance betweenthecostsandbenefitsofholdingstock.Thecostsofholdingstockincludethemoney has been spent buying the stock as well as storage.The benefits include having enough stock on hand to meet the demand of customers.Having too much stock equals extra expense for the retailer as it can lead to a shortfall in cash flow and incur excess storage costs.Andhavingtoolittlestockequalslostincomeintheformoflostsales,while also undermining customer confidence in retailer's ability to supply the products theretailerclaimstosell.Hencekeepingtherightstockandbeingabletosellitcanlead toincreasedsales,newcustomers,increasedcustomerconfidence,improvedcashflow.

Another class of inventory models have been developed with time-dependent deteriorationrate.TheWeibulldistributionisfrequentlyusedtorepresentthedistribution of time to deterioration of the item.In practice, the deterioration rate of items like fashionablegoods,fooditems,electronics

components,radioactivesubstance,chemicals and drugs is suitable to express in terms of Weibull distribution.In this connection manyresearcherattractedtheirattentiontowardstheWeibulldistributiontomodel the inventory problem.[14]Sharmila and Uthayakumar presented a two parameter Weibull distribution is used to represent the distribution of the time to deterioration.[2] Annadurai and Uthayakumar formulated an inventory model under two-levels of credit policy for deteriorating itemsbyassumingthedemandisafunctionofcreditperiodofferedbytheretailertothe customers.[16]Thangam and Uthayakumar implemented two different payment meth- ods for the retailer to pay off the loan to the supplier under two - echelon trade credit scenario.[15]SundaraRajan.R. and Uthayakumar,R.(2015),developed EOQ model for time varying demand and variable holding cost under permissible delay with short- ages.[9]Mary Latha and Uthayakumar developed a time dependent quadratic inventory model for deteriorating items with permissible delay in payments.And offering certain credit period without interest enthuse the consumers to order more quantities,as delayed payment indirectly reduces the purchase cost.[12]Sharmila and Uthayakumar presented fuzzy inventory model for deteriorating items with shortages under fully backloggedcondition.[7]KrishnaandBaniproposedamathematicalmodelofaninventory system in which demand depending upon stock level and time with various degree β, gives more flexibility of the demand pattern and more general to the study done so far with the condition to minimize the total average cost of the system.[3]Annadurai and Uthayakumar formulated EOQ model for deteriorating items with stock dependent demandunderpermissibledelayinpayments.Alsoobjectivetodeterminetheretailer’s optimal policy by finding the optimal length of inventory interval with positive inventory and the optimal length of order cycle for minimizing the cost.[13]Sharmila and Uthayakumar perceived that failure rate and life expectancy of many itemscanbeexpressedintermsofWeibulldistribution.[11]SharmilaandUthayakumar examinedthepartialtradecreditfinancinginasupplychainbyEOQ-basedmodel for decomposing items together with shortages.[5]Fredy and Artur presented an application of the proposed Γ-EW distribution to real data for illustrative purposes.Also considered some sub-models of the new four-parameter Γ-EW distribution to

fit this real data set for the sake of comparison:Weibull distribution, EE distribution, gamma Weibull ($\Gamma - W$) distribution, gamma exponentiated exponential ($\Gamma - EE$) distribution, and EW distribution.[10]Nita and Ankit derived under assumption of linearly increasingorexponentiallyincreasingdemand.However,inmarketofcommoditieslikefoodgrains,fashionapparels,electronicequipment’sdecreaseswithtimeduringthe end of season.[8]Madhavilata et al.discussed made so far is mainly meant for deter- mining order level inventory for an infinite horizon model when the demand increases exponentially.[6]Karmakar and Choudhury modified the demand rate is any functionof time up to the time-point of its stabilization (general ramp-type demand rate), andthe backlogging rate is any non increasing function of waiting time, up to the next replenishment.[4]Avikar et al.presented a production inventory model for deteriorating items with Exponentially increasing demand over a fixed time horizon.[17]Vijay et al.developed a model for the producer by assuming production to be stock dependent andexponentiallyincreasinganddemandratetobelinearandquadraticsimultaneously withconstantdeteriorationrateandholdingcost.[1]AjayandAnupamdevelopedasingle item inventory model with constant replenishment rate, exponential demand rate, infinite time horizon, with exponential partial back ordered rate, linearly increasing holding cost in both the warehouses and with the objective of maximizing the present worth of the total system profit.

1 NotationsandAssumptions

Thefollowingnotationsandassumptionsareusedthroughoutthispaper

1.1 Notations

A costofplacingoneorder

Q theorderquantity

D annualdemand

T Thecycletimeinyears

t_1 lengthoftimeinwhichtheinventoryhasnoshortage

θ thepositivenumberrepresentingthedeterioratingrate,where$0 \leq \theta \leq 1$

δ the fraction of the demand during the stock - out period that will be back ordered,where $0 \leq \delta \leq 1$

C unitpurchasingprice
C_1 shortagecostforbacklogged item
C_2 unitcostoflostsales
M theretailer'stradecreditperiodofferedbythesupplierinyears
N the customer's trade credit period offered by the retailer's in years h the inventory holding rate per unit time excluding interest charges I_e interest which can be earned per $ per year
I_p interestchargesper$instockperyearbythesupplier
$I(t)$ theinventorylevelattimet
TC thetotalannualcost

1.2 Assumptions

1. Demandrateisdefinedasthefunctionofstockandtimeas$D(q,t) = a + bt^{\beta-1}I(t)$.
2. Shortagesareallowedandcompletelybacklogged.
3. Leadtimeisnegligible.
4. Replenishmentoccursinstantaneouslyatinfiniterate.
5. Theitemsconsideredinthismodelaredeterioratingwithtime.
6. ThedeterioratingrateisdefinedastwoparameterWeibulldistribution$\theta(t) = \alpha\beta t^{\beta-1}$,where$0 < \alpha < 1$.
7. Theholdingcostistimedependentand$h(t) = f exp(dt)$wherefanddarepositive constant.
8. When $T \geq M$ the account is settled at $t = M$and the retailer would pay for theinterestchargesonitemsinstockwithrate$I_p$overtheinterval$[M,T]$.When $T \leq M$theaccountisalsosettledat$t = M$andtheretailerdoesnotneedtopay anyinterestchargeofitemsduringthewholecycle.
9. The retailer can accumulate revenue and earn interest from the very beginning thathiscustomerpaysfortheamountofpurchasingcosttotheretaileruntil the end of trade credit period offered by the supplier.That is the retailer can accumulaterevenueandearninterest

duringtheperiodfrom$t = N$to$t = M$ with rate $I_e$under the condition of trade credit.

10. There is no repair or replacement of deteriorated units during the planning horizon.The item will be withdrawn from warehouse immediately as they become deteriorated.

2 ProposedModel

In this section, a mathematical model is developed to determine the optimal replenishment cycle time that minimizes the total annual relevant cost in an inventor system for deterioratingitemsunderpartiallypermissibledelayinpaymentsincludingshortages.
Duetoboththedemandanddeteriorationofitem,theinventoryleveldecreasesduring theperiod$[0,t_1]$andultimatelyfallstozeroat$t=t_1$.Thereafter,shortagesareallowed tooccurandthedemandduringtheperiod$[t_1,T]$ispartiallybacklogged.Thebehavior of inventory system at any time is depicted in Fig.1

3 MathematicalFormulation

As described above, the inventory level decreases owing to demand as well as deteriorationduring the time interval $[0,t_1]$.Hence, the differential equation representing the inventory status is given by

$$\frac{dI_1(t)}{dt} = -\theta(t) - D(q,t); 0 \leq t \leq t_1 \tag{1}$$

withboundarycondition $I_1(t_1) = 0, I_1(0) = I_{max}$

$$\frac{dI_2(t)}{dt} = -a\delta; t_1 \leq t \leq T \tag{2}$$

withboundarycondition$I_2(t_1) = 0$
Using the assumption that $D(q,t)$, the demand function and $\theta(t)$, the deterioration rate as a non linear function

$D(q,t) = a + bt^{\beta-1}I(t)and\theta(t) = \alpha\beta t^{\beta-1}$then(1)and(2)reduces to

$$I_1(t) = ae^{kt^\beta}\left[(t_1 - t) + \frac{k}{\beta+1}\left(t_1^{\beta+1} - t^{\beta+1}\right)\right]; \text{where } k = \frac{\alpha\beta + b}{\beta} \qquad (3)$$

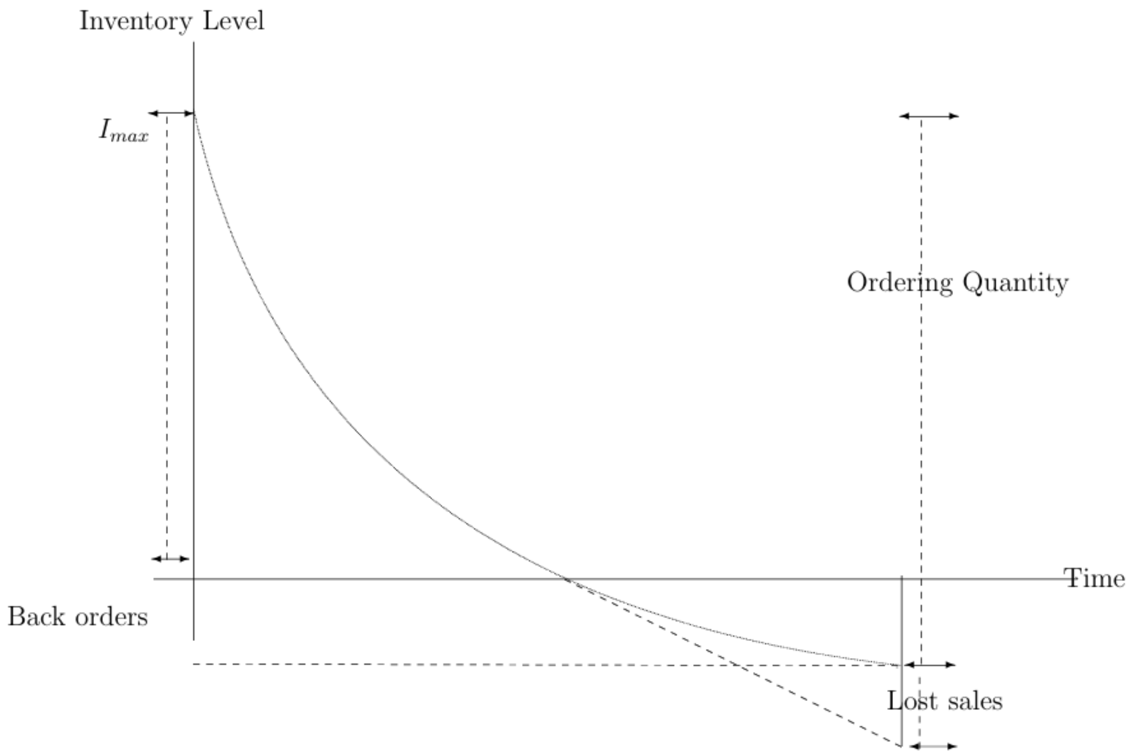


Figure 1: Graphical representation of inventory system

During the shortage interval $[t_1, T]$the demand at time t is partially backlogged at the fraction.Thus, the differential equation governing the amount of demand backlogged is as below

$$I_2(t) = -a\delta\left(t_1 - t\right) \qquad (4)$$

by letting *t*= *T*in (4) we can obtain the maximum amount of demand backlogged per cycle as

$$S = -I_2(T) = a\delta\left(T - t_1\right) \qquad (5)$$

Hence, the order quantity per cycle is given by

$$Q = I_{\max} + S$$
$$= a\left[t_1 + \frac{k}{\beta+1}\left(t_1^{\beta+1}\right)\right] + a\delta(T - t_1)$$

Thus, we have

$$I(t) = \begin{cases} I_1(t) & \text{if} \quad 0 \le t \le t_1 \\ I_2(t) & \text{if} \quad t_1 \le t \le T \end{cases}$$

Next, the expected annual cost of the inventory system for the retailer is computed using the following various components.

1. ordering cost OC = A
2. Holding cost (excluding interest charges) HC $= \int_0^{t_1} h(t) I_1(t) dt$

$$= fa\left[\left(t_1^2 - \frac{t_1^2}{2} + \frac{dt_1^3}{2} - \frac{dt_1^3}{3} + \frac{kt_1^{\beta+1}}{\beta+1} - \frac{kt_1^{\beta+2}}{\beta+2}\right) + \frac{k}{\beta+1}\left(t_1^{\beta+1} t_1 - \frac{\beta+2}{\beta+2} + \frac{dt_1^2 t_1^{\beta+1}}{2} - \frac{dt_1^{\beta+3}}{\beta+3} + \frac{kt_1^{\beta+1} t_1^{\beta+1}}{\beta+1} - \frac{kt_1^{\beta+3}}{\beta+3}\right)\right]$$

3. Deteriorating cost DC $= C\theta \int_0^{t_1} I_1(t) dt$

$$= aC\theta\left[t_1^2 - \frac{t_1^2}{2} + \frac{k}{\beta+1}\left(t_1^{\beta+2} - \frac{t_1^{\beta+2}}{\beta+2} big\right) + k\left(\frac{t_1^{\beta+2}}{\beta+1} - \frac{t_1^{\beta+2}}{\beta+2}\right) + \frac{k^2}{\beta+1}\left(t_1^{\beta+1} t_1^{\beta+1} - \frac{t_1^{2\beta+2}}{2\beta+2}\right)\right]$$

4. Shortages due to backlogging SC $= C_1 \int_{t_1}^{T} - I_2(t) dt$

$$= C_1 a\delta\left(t_1 T - \frac{T^2}{2} - \frac{t_1^2}{2}\right)$$

5. Opportunity cost due to lost sales OP$= C_2 \int_{t_1}^{T} (1-\delta) dt$

$$= C_2(1-\delta)(T - t_1)$$

According to the above assumptions, there are three possible cases to occur in the interest charged and earned in each order cycle and we discuss each case in detail as follows

4.1 Payment Method

In this method, at the end of trade credit(M), the retailer settles the account for all units sold and keeps the profits for other use and starts paying interest charges on the unpaid balance. To calculate interest payable and interest earned by the retailer, we consider the cases

i) $M \le T$
ii) $N \le T \le M$
iii) $T \le N$

Case(i) $M \le T$

Annual interest payable $= cI_p \int_M^T I_1(t) dt$

$$= cI_p a\left[\left(t_1 T - \frac{T^2}{2} + \frac{k}{\beta+1}\left(t_1^{\beta+1} T - \frac{T^{\beta+2}}{\beta+2}\right) + k\left(\frac{t_1 T^{\beta+1}}{\beta+1} - \frac{T^{\beta+2}}{\beta+2}\right) + \frac{k^2}{\beta+1}\left(t_1^{\beta+1}\frac{T^{\beta+1}}{\beta+1} - \frac{T^{\beta+3}}{\beta+3}\right)\right.\right.$$
$$\left.\left. - \left(t_1 M - \frac{M^2}{2} + \frac{k}{\beta+1}\left(t_1^{\beta+1} M - \frac{M^{\beta+2}}{\beta+2}\right) + k\left(\frac{t_1 M^{\beta+1}}{\beta+1} - \frac{M^{\beta+2}}{\beta+2}\right) + \frac{k^2}{\beta+1}\left(t_1^{\beta+1}\frac{M^{\beta+1}}{\beta+1} - \frac{M^{\beta+3}}{\beta+3}\right)\right)\right]$$

Case(ii) $N \le T \le M$

Annual interest payable = 0

Case(iii) $T \le N$

Annual interest payable = 0

Similar to interest payable, there are three cases that occur in costs of interest earned per year.
Case(i) $M \leq T$
Interest earned

$$= sI_e\left[\alpha\int_0^N \left(a + bt^{\beta-1}I(t)dt\right) + \int_N^M \left(a + bt^{\beta-1}I(t)dt\right)\right]$$

$$= sI_e\left[\alpha\left(a\left(\frac{kN^{\beta+1}}{\beta+1}\right) - b\left(\frac{N^{\beta+1}}{\beta(\beta+1)}\right) + \left[a\frac{kM^{\beta+1}}{\beta+1} - b\frac{M^{\beta+1}}{\beta(\beta+1)}\right]\left[a\frac{kN^{\beta+1}}{\beta+1} - b\frac{N^{\beta+1}}{\beta+1}\right]\right)\right]$$

Case(ii) $N \le T \le M$

Interest earned

$$= sI_e\left[\int_0^N \left(a + bt^{\beta-1}I(t)dt\right) + \int_N^T \left(a + bt^{\beta-1}I(t)dt\right)\right]$$

$$= sI_e\left[\alpha\left(a\frac{kN^{\beta+1}}{\beta+1} - b\frac{N^{\beta+1}}{\beta(\beta+1)}\right)\frac{N^2}{2} + \left(a\frac{kT^{\beta+1}}{\beta+1} - b\frac{T^{\beta+1}}{\beta+1}\right) - \left(a\frac{kN^{\beta+1}}{\beta+1} - b\frac{N^{\beta+1}}{\beta(\beta+1)}\right)N\right.$$

$$\left.\left[\left(a\left[\frac{kM^{\beta+1}}{\beta+1} - b\frac{M^{\beta+1}}{\beta(\beta+1)}\right]\left(-a\frac{kT^{\beta+1}}{\beta+1} - b\frac{T^{\beta+1}}{\beta(\beta+1)}\right)T\right)\right]\right]$$

Case(iii) $T \le N$

Interest earned

$$= sI_e\left[\alpha\left(\int_0^T \left(a + bt^{\beta-1}I(t)tdt\right)\right) + \alpha\int_T^N \left(a + bt^{\beta-1}I(t)Tdt\right) + \int_N^M \left(a + bt^{\beta-1}I(t)Tdt\right)\right]$$

$$= sI_e\left[\alpha\left(a\frac{kT^{\beta+1}}{\beta+1} - b\frac{T^{\beta+1}}{\beta(\beta+1)}\right)\frac{T^2}{2} + \alpha\left(a\frac{kN^{\beta+1}}{\beta+1} - b\frac{N^{\beta+1}}{\beta(\beta+1)}\right.\right.$$

$$\left.\left. - a\frac{kT^{\beta+1}}{\beta+1} + b\frac{T^{\beta+1}}{\beta(\beta+1)}\right)T + \left(a\frac{kM^{\beta+1}}{\beta+1} - b\frac{M^{\beta+1}}{\beta+1} - a\frac{kN^{\beta+1}}{\beta+1} + b\frac{N^{\beta+1}}{\beta(\beta+1)}\right)T\right]$$

From the above arguments, the annual total relevant cost for the retailer can be expressed as

$$TC(T) = \begin{cases} TC_1(T) & \text{if} \quad M \le T \\ TC_2(T) & \text{if} \quad N \le T \le M \\ TC_3(T) & \text{if} \quad T \le N \end{cases}$$

Where

$$TC_1(T) = \frac{1}{T}\left[OC + HC + DC + SC + OP + IP - IE\right]$$

$$TC_2(T) = \frac{1}{T}\left[OC + HC + DC + SC + OP + IP - IE\right] \qquad (9)$$

$$TC_3(T) = \frac{1}{T}\left[OC + HC + DC + SC + OP + IP - IE\right]$$

Since $TC_1(M) = TC_2(M)$ and $TC_2(N) = TC_3(N)$, $TC(T)$ is continuous and well defined. All $TC_1(T), TC_2(T), TC_3(T)$ and $TC(T)$ are defined on $T > 0$.

5. Solution Procedure For Payment Method

The optimal cycle time T_1^* can be obtained by solving the equation:

$$\frac{dTC_1}{dT} = 0$$

$$-\frac{1}{T^2}\left[A+fa\left[\left(t_1^2-\frac{t_1^2}{2}+\frac{dt_1^3}{2}-\frac{dt_1^3}{3}+\frac{kt_1^{\beta+1}}{\beta+1}-\frac{kt_1^{\beta+2}}{\beta+2}\right)\right.\right.$$
$$\left.\left.+\frac{k}{\beta+1}\left(t_1^{\beta+1}t_1-\frac{\beta+2}{\beta+2}+\frac{dt_1^2t_1^{\beta+1}}{2}-\frac{dt_1^{\beta+3}}{\beta+3}+\frac{kt_1^{\beta+1}t_1^{\beta+1}}{\beta+1}-\frac{kt_1^{\beta+3}}{\beta+3}\right)\right]+\right.$$
$$aC\theta\left[t_1^2-\frac{t_1^2}{2}+\frac{k}{\beta+1}\left(t_1^{\beta+2}-\frac{t_1^{\beta+2}}{\beta+2}big\right)+k\left(\frac{t_1^{\beta+2}}{\beta+1}-\frac{t_1^{\beta+2}}{\beta+2}\right)+\frac{k^2}{\beta+1}\left(t_1^{\beta+1}t_1^{\beta+1}-\frac{t_1^{2\beta+2}}{2\beta+2}\right)\right]+$$
$$C_1a\delta\left(t_1T-\frac{T^2}{2}-\frac{t_1^2}{2}\right)+C_2(1-\delta)(T-t_1)+cI_pa\left[\left(t_1T-\frac{T^2}{2}+\frac{k}{\beta+1}\left(t_1^{\beta+1}T-\frac{T^{\beta+2}}{\beta+2}\right)\right.\right.$$
$$+k\left(\frac{t_1T^{\beta+1}}{\beta+1}-\frac{T^{\beta+2}}{\beta+2}\right)+\frac{k^2}{\beta+1}\left(t_1^{\beta+1}\frac{T^{\beta+1}}{\beta+1}\right.$$
$$\left.\left.-\frac{T^{\beta+3}}{\beta+3}\right)\right)-\left(t_1M-\frac{M^2}{2}+\frac{k}{\beta+1}\left(t_1^{\beta+1}M-\frac{M^{\beta+2}}{\beta+2}\right)+k\left(\frac{t_1M^{\beta+1}}{\beta+1}-\frac{M^{\beta+2}}{\beta+2}\right)\right.$$
$$\left.\left.+\frac{k^2}{\beta+1}\left(t_1^{\beta+1}\frac{M^{\beta+1}}{\beta+1}-\frac{M^{\beta+3}}{\beta+3}\right)\right)\right]-sI_e\left[\alpha\left(a\left(\frac{kN^{\beta+1}}{\beta+1}\right)-b\left(\frac{N^{\beta+1}}{\beta(\beta+1)}\right)+\left[a\frac{kM^{\beta+1}}{\beta+1}-b\frac{M^{\beta+1}}{\beta(\beta+1)}\right]\right.\right.$$
$$\left.\left.\left.\left[a\frac{kN^{\beta+1}}{\beta+1}-b\frac{N^{\beta+1}}{\beta+1}\right]\right)\right]\right]+\frac{1}{T}\left[C_1a\delta(t_1-T)+C_2(1-\delta)+cI_pa(t_1-T)+\frac{k}{\beta+1}\left(t_1^{\beta+1}-T^{\beta+1}\right)+\right.$$
$$\left.k\left(t_1T^{\beta}-T^{\beta+1}\right)+T^{\beta+1}+\frac{k^2}{\beta+1}\left(t_1^{\beta+1}T^{\beta}-T^{\beta+2}\right)\right]=0$$

Let $f_1(t)$ be the function of T which is on the left hand side of equation (10). Let $\Delta_1=f_1(M)$.

Theorem 1: If $\Delta_1<0$, then $T*=T_1^*$ and $TC*(T)=TC_1(T*)$.

Proof: Taking the first derivative of $0036f_1(t)$ with respect to T , we get,

$$\frac{df_1(T)}{dT}==\frac{1}{T^3}\left[-\frac{1}{T^2}\left[A+fa\left[\left(t_1^2-\frac{t_1^2}{2}+\frac{dt_1^3}{2}-\frac{dt_1^3}{3}+\frac{kt_1^{\beta+1}}{\beta+1}-\frac{kt_1^{\beta+2}}{\beta+2}\right)\right.\right.\right.$$
$$\left.+\frac{k}{\beta+1}\left(t_1^{\beta+1}t_1-\frac{\beta+2}{\beta+2}+\frac{dt_1^2t_1^{\beta+1}}{2}-\frac{dt_1^{\beta+3}}{\beta+3}+\frac{kt_1^{\beta+1}t_1^{\beta+1}}{\beta+1}-\frac{kt_1^{\beta+3}}{\beta+3}\right)\right]+$$
$$aC\theta\left[t_1^2-\frac{t_1^2}{2}+\frac{k}{\beta+1}\left(t_1^{\beta+2}-\frac{t_1^{\beta+2}}{\beta+2}big\right)+k\left(\frac{t_1^{\beta+2}}{\beta+1}\right.\right.$$
$$\left.\left.-\frac{t_1^{\beta+2}}{\beta+2}\right)+\frac{k^2}{\beta+1}\left(t_1^{\beta+1}t_1^{\beta+1}-\frac{t_1^{2\beta+2}}{2\beta+2}\right)\right]+C_1a\delta\left(t_1T-\frac{T^2}{2}-\frac{t_1^2}{2}\right)+$$
$$C_2(1-\delta)(T-t_1)+cI_pa\left[\left(t_1T-\frac{T^2}{2}+\frac{k}{\beta+1}\left(t_1^{\beta+1}T-\frac{T^{\beta+2}}{\beta+2}\right)+k\left(\frac{t_1T^{\beta+1}}{\beta+1}-\frac{T^{\beta+2}}{\beta+2}\right)\right.\right.$$
$$\left.+\frac{k^2}{\beta+1}\left(t_1^{\beta+1}\frac{T^{\beta+1}}{\beta+1}-\frac{T^{\beta+3}}{\beta+3}\right)\right)-\left(t_1M-\frac{M^2}{2}+\frac{k}{\beta+1}\left(t_1^{\beta+1}M-\frac{M^{\beta+2}}{\beta+2}\right)\right.$$
$$\left.\left.+k\left(\frac{t_1M^{\beta+1}}{\beta+1}-\frac{M^{\beta+2}}{\beta+2}\right)+\frac{k^2}{\beta+1}\left(t_1^{\beta+1}\frac{M^{\beta+1}}{\beta+1}-\frac{M^{\beta+3}}{\beta+3}\right)\right)\right]-sI_e\left[\alpha\left(a\left(\frac{kN^{\beta+1}}{\beta+1}\right)\right.\right.$$
$$\left.\left.\left.-b\left(\frac{N^{\beta+1}}{\beta(\beta+1)}\right)+\left[a\frac{kM^{\beta+1}}{\beta+1}-b\frac{M^{\beta+1}}{\beta(\beta+1)}\right]\left[a\frac{kN^{\beta+1}}{\beta+1}-b\frac{N^{\beta+1}}{\beta+1}\right]\right)\right]\right]$$

$$+\frac{1}{T}\Big[C_1 a\delta(t_1-T)+C_2(1-\delta)+cI_p a(t_1-T)+\frac{k}{\beta+1}\big(t_1^{\beta+1}-T^{\beta+1}\big)+k\big(t_1T^{\beta}$$

$$-T^{\beta+1}\big)+T^{\beta+1}+\frac{k^2}{\beta+1}\big(t_1^{\beta+1}T^{\beta}-T^{\beta+2}\big)\Big]\Bigg]-\frac{1}{T^2}\big[\big[C_1 a\delta(t_1-T)+C_2(1-\delta)+$$

$$cI_p a(t_1-T)+\frac{k}{\beta+1}\big(t_1^{\beta+1}-T^{\beta+1}\big)+k\big(t_1T^{\beta}-T^{\beta+1}\big)+T^{\beta+1}+\frac{k^2}{\beta+1}\big(t_1^{\beta+1}T^{\beta}-T^{\beta+2}\big)\Big]$$

$$+\frac{1}{T}\left(C_1 a\delta+cI_p a\left(1+\frac{k}{\beta+1}\big(-\beta+1T^{\beta}\big)+k\big(t_1\beta T^{\beta-1}-\beta+1T^{\beta}\big)\right)\right.$$

$$\left.+\frac{k^2}{\beta+1}\big(\beta t_1^{\beta+1}T^{\beta-1}-(\beta+2)T^{\beta+1}\big)\right)\Bigg]>0 \text{ for } T>M$$

Thus, $f_1(T)$ is strictly increasing function of T in the interval $[M,\infty)$. Moreover, we know that

$$f_1(M)=\Delta_1 \text{ and } \lim_{T\to\infty} f_1(T)=\infty>0$$

Since $\Delta_1<0, f_1(M)<0$ and $\lim_{T\to\infty} f_1(T)>0$ by applying intermediate value theorem, there exists a unique $T_{1^*}\in[M,\infty)$ such that $f_1(T_{1^*})=0$. It is easy to verify that

$$\frac{d^2TC_1(T)}{dT^2}>0 \text{ at } T=T_{1^*}$$

Thus, $T_1*\in[M,\infty]$ is a unique optimum solution to $TC(T)$

After obtaining the first order derivatives of $TC_2(T)$ and $TC_3(T)$, the optimal replenishmenttimeT_2 andT_3 aretherootsoftheequations.

$$0=-\frac{1}{T^2}\left[A+fa\left[\left(t_1^2-\frac{t_1^2}{2}+\frac{dt_1^3}{2}-\frac{dt_1^3}{3}+\frac{kt_1^{\beta+1}}{\beta+1}-\frac{kt_1^{\beta+2}}{\beta+2}\right)\right.\right.$$

$$\left.+\frac{k}{\beta+1}\left(t_1^{\beta+1}t_1-\frac{\beta+2}{\beta+2}+\frac{dt_1^2t_1^{\beta+1}}{2}-\frac{dt_1^{\beta+3}}{\beta+3}+\frac{kt_1^{\beta+1}t_1^{\beta+1}}{\beta+1}-\frac{kt_1^{\beta+3}}{\beta+3}\right)\right]+$$

$$aC\theta\left[t_1^2-\frac{t_1^2}{2}+\frac{k}{\beta+1}\left(t_1^{\beta+2}-\frac{t_1^{\beta+2}}{\beta+2}big\right)+k\left(\frac{t_1^{\beta+2}}{\beta+1}\right.\right.$$

$$\left.\left.-\frac{t_1^{\beta+2}}{\beta+2}\right)+\frac{k^2}{\beta+1}\left(t_1^{\beta+1}t_1^{\beta+1}-\frac{t_1^{2\beta+2}}{2\beta+2}\right)\right]+C_1a\delta\left(t_1T-\frac{T^2}{2}-\frac{t_1^2}{2}\right)+C_2(1-\delta)(T-t_1)$$

$$-sI_e\left[\alpha\left(a\frac{kN^{\beta+1}}{\beta+1}-b\frac{N^{\beta+1}}{\beta(\beta+1)}\right)\frac{N^2}{2}+\left(a\frac{kT^{\beta+1}}{\beta+1}-b\frac{T^{\beta+1}}{\beta+1}\right)-\left(a\frac{kN^{\beta+1}}{\beta+1}-b\frac{N^{\beta+1}}{\beta(\beta+1)}\right)N-\right.$$

$$\left.\left[\left(a\left[\frac{kM^{\beta+1}}{\beta+1}-b\frac{M^{\beta+1}}{\beta(\beta+1)}\right]\left(-a\frac{kT^{\beta+1}}{\beta+1}-b\frac{T^{\beta+1}}{\beta(\beta+1)}\right)T\right)\right]\right]$$

$$+\frac{1}{T}\left(C_1a\delta(t_1-T)+C_2(1-\delta)-sI_e\left[\left(akT^{\beta}-\frac{bT^{\beta}}{\beta}\right)N-\left(akT^{\beta}-\frac{bT^{\beta}}{\beta}\right)T\right.\right.$$

$$\left.\left.-\left(\frac{akT^{\beta+1}}{\beta+1}-\frac{bT^{\beta+1}}{\beta(\beta+1)}\right)\right]\right)$$

(12)

$$0 =-\frac{1}{T^2}\Bigg[A + fa\Bigg[\Bigg(t_1^2 - \frac{t_1^2}{2} + \frac{dt_1^3}{2} - \frac{dt_1^3}{3} + \frac{kt_1^{\beta+1}}{\beta+1} - \frac{kt_1^{\beta+2}}{\beta+2}\Bigg)$$
$$+ \frac{k}{\beta+1}\Bigg(t_1^{\beta+1}t_1 - \frac{\beta+2}{\beta+2} + \frac{dt_1^2 t_1^{\beta+1}}{2} - \frac{dt_1^{\beta+3}}{\beta+3} + \frac{kt_1^{\beta+1}t_1^{\beta+1}}{\beta+1} - \frac{kt_1^{\beta+3}}{\beta+3}\Bigg)\Bigg] +$$
$$aC\theta\Bigg[t_1^2 - \frac{t_1^2}{2} + \frac{k}{\beta+1}\Bigg(t_1^{\beta+2} - \frac{t_1^{\beta+2}}{\beta+2} big\Bigg) + k\Bigg(\frac{t_1^{\beta+2}}{\beta+1}$$
$$- \frac{t_1^{\beta+2}}{\beta+2}\Bigg) + \frac{k^2}{\beta+1}\Bigg(t_1^{\beta+1}t_1^{\beta+1} - \frac{t_1^{2\beta+2}}{2\beta+2}\Bigg)\Bigg] + C_1 a\delta\Bigg(t_1 T - \frac{T^2}{2} - \frac{t_1^2}{2}\Bigg) + C_2(1-\delta)(T - t_1)$$
$$-sI_e\Bigg[\alpha\Bigg(a\frac{kT^{\beta+1}}{\beta+1} - b\frac{T^{\beta+1}}{\beta(\beta+1)}\Bigg)\frac{T^2}{2} + \alpha\Bigg(a\frac{kN^{\beta+1}}{\beta+1} - b\frac{N^{\beta+1}}{\beta(\beta+1)}$$
$$-a\frac{kT^{\beta+1}}{\beta+1} + b\frac{T^{\beta+1}}{\beta(\beta+1)}\Bigg)T + \Bigg(a\frac{kM^{\beta+1}}{\beta+1} - b\frac{M^{\beta+1}}{\beta+1} - a\frac{kN^{\beta+1}}{\beta+1} + b\frac{N^{\beta+1}}{\beta(\beta+1)}\Bigg)T\Bigg]$$
$$+\frac{1}{T}\Bigg(C_1 a\delta(t_1 - T) + C_2(1-\delta) - sI_e\Bigg(\Bigg[akT^{\beta} - \frac{bT^{\beta}}{\beta}\Bigg]\alpha\frac{T^2}{2} - \Bigg[\frac{akT^{\beta}}{\beta+1} - \frac{bT^{\beta+1}}{\beta(\beta+1)}\Bigg]\alpha T$$
$$-\Bigg[\alpha T akT^{\beta} - \frac{bT^{\beta}}{(\beta)} - \Bigg(\frac{akT^{\beta+1}}{\beta+1} - \frac{bT^{\beta+1}}{\beta(\beta+1)}\Bigg)\Bigg] + \alpha\Bigg(\frac{akN^{\beta+1}}{\beta+1} - \frac{bN^{\beta+1}}{\beta(\beta+1)}\Bigg) +$$
$$\Bigg(\frac{akM^{\beta+1}}{\beta+1} - \frac{bM^{\beta+1}}{\beta+1}\Bigg) - \Bigg(\frac{akN^{\beta+1}}{\beta+1} - \frac{bN^{\beta+1}}{\beta(\beta+1)}\Bigg)\Bigg)\Bigg)\Bigg]$$

(13)

respectively.Let$f_2(T)$and$f_3(T)$bethefunctionintheleftthandsideofequations(12)and (13), respectively. Then,

$$\frac{df_2(T)}{dT} = \frac{1}{T^3}\Bigg[A + fa\Bigg[\left(t_1^2 - \frac{t_1^2}{2} + \frac{dt_1^3}{2} - \frac{dt_1^3}{3} + \frac{kt_1^{\beta+1}}{\beta+1} - \frac{kt_1^{\beta+2}}{\beta+2}\right)$$

$$+\frac{k}{\beta+1}\left(t_1^{\beta+1}t_1 - \frac{\beta+2}{\beta+2} + \frac{dt_1^2 t_1^{\beta+1}}{2} - \frac{dt_1^{\beta+3}}{\beta+3} + \frac{kt_1^{\beta+1}t_1^{\beta+1}}{\beta+1} - \frac{kt_1^{\beta+3}}{\beta+3}\right)\Bigg] +$$

$$aC\theta\Bigg[t_1^2 - \frac{t_1^2}{2} + \frac{k}{\beta+1}\left(t_1^{\beta+2} - \frac{t_1^{\beta+2}}{\beta+2} big\right) + k\left(\frac{t_1^{\beta+2}}{\beta+1}\right.$$

$$\left.-\frac{t_1^{\beta+2}}{\beta+2}\right) + \frac{k^2}{\beta+1}\left(t_1^{\beta+1}t_1^{\beta+1} - \frac{t_1^{2\beta+2}}{2\beta+2}\right)\Bigg] + C_1 a\delta\left(t_1 T - \frac{T^2}{2} - \frac{t_1^2}{2}\right) + C_2(1-\delta)(T - t_1)$$

$$-sI_e\Bigg[\alpha\left(a\frac{kN^{\beta+1}}{\beta+1} - b\frac{N^{\beta+1}}{\beta(\beta+1)}\right)\frac{N^2}{2} + \left(a\frac{kT^{\beta+1}}{\beta+1} - b\frac{T^{\beta+1}}{\beta+1}\right)$$

$$-\left(a\frac{kN^{\beta+1}}{\beta+1} - b\frac{N^{\beta+1}}{\beta(\beta+1)}\right)N - \Bigg[\bigg(a[\frac{kM^{\beta+1}}{\beta+1} - b\frac{M^{\beta+1}}{\beta(\beta+1)}]\left(-a\frac{kT^{\beta+1}}{\beta+1}\right.$$

$$\left.-b\frac{T^{\beta+1}}{\beta(\beta+1)}\right)T\bigg)\Bigg]\Bigg] - \frac{1}{T^2}\Bigg(C_1 a\delta(t_1 - T) + C_2(1-\delta) - sI_e\Bigg[\left(akT^{\beta} - \frac{bT^{\beta}}{\beta}\right)N$$

$$-\left(akT^{\beta} - \frac{bT^{\beta}}{\beta}\right)T - \left(\frac{akT^{\beta+1}}{\beta+1} - \frac{bT^{\beta+1}}{\beta(\beta+1)}\right)\Bigg]\Bigg)\Bigg] + \frac{1}{T}(C_1 a\delta(t_1 - T) + C_2(1-\delta)$$

$$-sI_e\Bigg[\alpha\left(akT^{\beta} - \frac{bT^{\beta}}{\beta}\right)N - \left(akT^{\beta} - \frac{bT^{\beta}}{\beta}\right)T - \left(\frac{akT^{\beta+1}}{\beta+1} - \frac{bT^{\beta+1}}{\beta+1}\right)\Bigg]$$

$$\frac{1}{T}\Bigg[C_1 a\delta - sI_e\Bigg[\left(ak\beta T^{\beta-1} - bT^{\beta-1}\right)N - \left(ak\beta T^{\beta-1} - bT^{\beta-1}\right)T - \left(akT^{\beta} - \frac{bT^{\beta}}{\beta}\right)\Bigg]\Bigg]$$

$$\frac{df_3(T)}{dT} = \frac{1}{T^3}\Bigg[\Bigg[A + fa\Bigg[\left(t_1^2 - \frac{t_1^2}{2} + \frac{dt_1^3}{2} - \frac{dt_1^3}{3} + \frac{kt_1^{\beta+1}}{\beta+1} - \frac{kt_1^{\beta+2}}{\beta+2}\right)$$
$$+\frac{k}{\beta+1}\left(t_1^{\beta+1}t_1 - \frac{\beta+2}{\beta+2} + \frac{dt_1^2 t_1^{\beta+1}}{2} - \frac{dt_1^{\beta+3}}{\beta+3} + \frac{kt_1^{\beta+1}t_1^{\beta+1}}{\beta+1} - \frac{kt_1^{\beta+3}}{\beta+3}\right)\Bigg] + aC\theta\left[t_1^2 - \frac{t_1^2}{2} +\right.$$
$$\left.\frac{k}{\beta+1}\left(t_1^{\beta+2} - \frac{t_1^{\beta+2}}{\beta+2} big\right) + k\left(\frac{t_1^{\beta+2}}{\beta+1} - \frac{t_1^{\beta+2}}{\beta+2}\right) + \frac{k^2}{\beta+1}\left(t_1^{\beta+1}t_1^{\beta+1} - \frac{t_1^{2\beta+2}}{2\beta+2}\right)\right] +$$
$$C_1 a\delta\left(t_1 T - \frac{T^2}{2} - \frac{t_1^2}{2}\right) + C_2(1-\delta)(T - t_1) - sI_e\left[\alpha\left(a\frac{kT^{\beta+1}}{\beta+1} - b\frac{T^{\beta+1}}{\beta(\beta+1)}\right)\frac{T^2}{2} +\right.$$
$$\alpha\left(a\frac{kN^{\beta+1}}{\beta+1} - b\frac{N^{\beta+1}}{\beta(\beta+1)} - a\frac{kT^{\beta+1}}{\beta+1} + b\frac{T^{\beta+1}}{\beta(\beta+1)}\right)T + \left(a\frac{kM^{\beta+1}}{\beta+1} - b\frac{MM^{\beta+1}}{\beta+1} - a\frac{kN^{\beta+1}}{\beta+1}\right.$$
$$\left.\left.+b\frac{N^{\beta+1}}{\beta(\beta+1)}\right)T\right] - \frac{1}{T^2}\left(C_1 a\delta(t_1 - T) + C_2(1-\delta) - sI_e\left(\left[akT^\beta - \frac{bT^\beta}{\beta}\right]\alpha\frac{T^2}{2} -\right.\right.$$
$$\left[\frac{akT^\beta}{\beta+1} - \frac{bT^{\beta+1}}{\beta(\beta+1)}\right]\alpha T - \left[\alpha TakT^\beta - \frac{bT^\beta}{(\beta)} - \left(\frac{akT^{\beta+1}}{\beta+1} - \frac{bT^{\beta+1}}{\beta(\beta+1)}\right)\right] + \alpha\left(\frac{akN^{\beta+1}}{\beta+1}\right.$$
$$\left.\left.\left.-\frac{bN^{\beta+1}}{\beta(\beta+1)}\right) + \left(\frac{akM^{\beta+1}}{\beta+1} - \frac{bM^{\beta+1}}{\beta+1}\right) - \left(\frac{akN^{\beta+1}}{\beta+1} - \frac{bN^{\beta+1}}{\beta(\beta+1)}\right)\right)\right)\Bigg]\Bigg]$$
$$-\frac{1}{T^2}\left(C_1 a\delta(t_1 - T) + C_2(1-\delta) - sI_e\left(\left[akT^\beta - \frac{bT^\beta}{\beta}\right]\alpha\frac{T^2}{2} - \left[\frac{akT^\beta}{\beta+1} - \frac{bT^{\beta+1}}{\beta(\beta+1)}\right]\alpha T\right.\right.$$
$$-\left[\alpha TakT^\beta - \frac{bT^\beta}{(\beta)} - \left(\frac{akT^{\beta+1}}{\beta+1} - \frac{bT^{\beta+1}}{\beta(\beta+1)}\right)\right] + \alpha\left(\frac{akN^{\beta+1}}{\beta+1}\right.$$
$$\left.\left.\left.-\frac{bN^{\beta+1}}{\beta(\beta+1)}\right) + \left(\frac{akM^{\beta+1}}{\beta+1} - \frac{bM^{\beta+1}}{\beta+1}\right) - \left(\frac{akN^{\beta+1}}{\beta+1} - \frac{bN^{\beta+1}}{\beta(\beta+1)}\right)\right)\right)$$
$$+\frac{1}{T}\left(C_1 a\delta - sI_e\left(\alpha\left[ak\beta T^{\beta-1} - bT^{\beta-1}\right]\frac{T^2}{2}\right) + \alpha\left(akT^\beta - \frac{bT^\beta}{\beta}\right)T + \alpha\left[\frac{akT^{\beta+1}}{\beta+1} - \frac{bT^{\beta+1}}{\beta+1}\right]\right.$$
$$\left.+\alpha\left[akT^\beta - \frac{bT^\beta}{\beta}\right]T + \alpha\left[-akT^\beta - \frac{bT^\beta}{\beta}\right] + \left(ak\beta T^{\beta-1} - bT^{\beta-1}\right)T\alpha\left(akT^\beta - \frac{bT^\beta}{\beta}\right)\right)$$
$$-\frac{1}{T^2}\left(C_1 a\delta(t_1 - T) + C_2(1-\delta) - sI_e\left(\left[akT^\beta - \frac{bT^\beta}{\beta}\right]\alpha\frac{T^2}{2} - \left[\frac{akT^\beta}{\beta+1} - \frac{bT^{\beta+1}}{\beta(\beta+1)}\right]\alpha T\right.\right.$$
$$-\left[\alpha TakT^\beta - \frac{bT^\beta}{(\beta)} - \left(\frac{akT^{\beta+1}}{\beta+1} - \frac{bT^{\beta+1}}{\beta(\beta+1)}\right)\right] + \alpha\left(\frac{akN^{\beta+1}}{\beta+1} - \frac{bN^{\beta+1}}{\beta(\beta+1)}\right)$$
$$\left.\left.+\left(\frac{akM^{\beta+1}}{\beta+1} - \frac{bM^{\beta+1}}{\beta+1}\right) - \left(\frac{akN^{\beta+1}}{\beta+1} - \frac{bN^{\beta+1}}{\beta(\beta+1)}\right)\right)\right)$$

consequently, $f_i(T)(i = 1,2,3)$ is also increasing on $(0, \infty)$. From $f_i(0) < 0, (i = 1,2,3)$ and $\lim_{T\to\infty} f(T) = \infty > 0, (i = 1,2,3)$byintermediate value theorem we see that $f_i(T)$ is decreasing on $[0, T_i *]$ and increasing on $(T_i *, \infty)$. Hence $TC_i(T); (i = 1,2,3)$ is convex on $T > 0$.Furthermore, we have $TC_1{}'(M) = TC_2{}'(M)$ and $TC_2{}'(N) = TC_3{}'(N)$. Hence $TC(T)$ is convex on $T > 0$.We, have,

$$TC_1'(M) = \frac{f_1(M)}{M^2}$$
$$TC_2'(M) = \frac{f_2(M)}{M^2}$$
$$TC_2'(N) = \frac{f_2(N)}{N^2}$$
$$TC_3'(N) = \frac{f_3(N)}{N^2}$$

Let $\Delta_2 = f_2(N) = f_3(N)$. Clearly $\Delta_1 \geq \Delta_2$.

6. Numericalexamplesandsensitivityanalysis

To illustrate the solution procedure and investigate the sensitivity analysis on optimal solution in our model, we consider the following examples

Given$A$ =\$200/*units,s* =\$2/*units,c* =\$0.50/*unit,*I_p=0.14,I_e=0.13,M= 2*years,*θ=0.2,α=0.05,N=2*years,a*=100*units,b*=0.30*units,f*=15,d= 8,δ = 0.23,C = 30,C_1= 0.021,C_2= 0.9785

TC_1=1019.5,TC_2=836.0634,TC_3=854.1909,Q =61.9049

6.1 Sensitivity Analysis

Let us consider the same data as in example.Here, we study the effects of changes in the values, on optimal cycle and minimum total cost.

Table 1: Sensitivitymodelforinventorymodelparameters

A	t_1	T	TC_1	TC_2	TC_3	Q
230	0.2921	0.3148	1981.7	1738.2	1631.0	47.6239
240	0.2922	0.3150	2013.2	1769.8	1662.8	47.6454
250	0.2925	0.3151	2047.2	1803.9	1697.0	47.6984
260	0.2941	0.3158	2089.7	1847.2	1740.8	47.9850
270	0.2982	0.3162	2158.9	1917.5	1811.4	48.6892

δ	t_1	T	TC_1	TC_2	TC_3	Q
0.55	0.2925	0.3150	1889.1	1645.7	1538.7	48.4161
0.65	0.2924	0.3150	1888.1	1644.8	1537.8	48.6284
0.75	0.2923	0.3151	1886.5	1643.2	1536.3	48.8502
0.85	0.2922	0.3152	1885.0	1641.7	1534.9	49.0760
0.95	0.2921	0.3151	1884.6	1641.3	1534.4	49.2868

θ	t_1	T	TC_1	TC_2	TC_3	Q
0.01	0.2925	0.3152	1657.6	1414.3	1307.5	47.7007
0.03	0.2924	0.3151	1681.5	1438.2	1331.3	47.6815
0.05	0.2923	0.3148	1706.6	1463.1	1355.9	47.6577
0.07	0.2918	0.3135	1733.8	1489.4	1381.3	47.5434
0.09	0.2905	0.3129	1750.4	1505.3	1396.7	47.3102

α	t_1	T	TC_1	TC_2	TC_3	Q
0.06	0.2934	0.3165	1897.7	1650.1	1545.7	48.0946
0.04	0.2931	0.3156	1882.3	1644.9	1536.9	48.5997
0.03	0.2924	0.3151	1869.8	1637.2n	1527.5	47.2599
0.02	0.2913	0.3142	1885.9	1627.8	1516.2	46.8462
0.01	0.2901	0.3139	1837.8	1614.5	1501.4	46.4325

β	t_1	T	TC_1	TC_2	TC_3	Q
0.10	0.2938	0.3166	10165	9613.5	9252.9	136.2395
0.15	0.2931	0.3157	5845.7	5431.9	5175.1	100.5021
0.20	0.2924	0.3151	4144.5	3795.1	3590.9	82.1613
0.25	0.2918	0.3145	3273.8	2961.5	2788.8	70.8794
0.30	0.2904	0.3137	2744.4	2455.2	2303.5	62.9757

M	t_1	T	TC_1	TC_2	TC_3	Q
0.05	0.2926	0.3154	1982.7	1727.3	1800.6	47.7222
0.07	0.2925	0.3152	1982.2	1727.2	1800.0	47.7007
0.09	0.2924	0.3151	1980.9	1726.5	1798.8	47.6815
0.11	0.2921	0.3148	1978.8	1724.9	1796.6	47.6239
0.13	0.2914	0.3139	1976.5	1722.5	1793.7	47.4851

N	t_1	T	TC_1	TC_2	TC_3	Q
0.02	0.2908	0.3131	1632.9	1664.9	1461.0	47.3654
0.03	0.2919	0.3142	1637.4	1669.7	1466.7	47.5764
0.04	0.2924	0.3151	1637.8	1669.7	1467.5	47.6815
0.05	0.2938	0.3167	1643.3	1674.8	1473.9	47.9550
0.06	0.2945	0.3175	1646.2	1677.4	1477.2	48.0918

I_e	t_1	T	TC_1	TC_2	TC_3	Q
0.10	0.2926	0.3154	1896.7	1664.6	1582.5	47.7222
0.11	0.2925	0.3152	1889.9	1658.0	1567.6	47.7007
0.12	0.2924	0.3151	1888.5	1650.9	1552.2	47.6815
0.13	0.2913	0.3134	1887.1	1642.5	1534.4	47.4567
0.14	0.2901	0.3102	1893.9	1641.4	1522.2	47.1807

I_p	t_1	T	TC_1	TC_2	TC_3	Q
0.13	0.2931	0.3155	1879.8	1649.0	1542.7	47.8090
0.14	0.2926	0.3152	1888.8	1645.6	1538.8	47.7176
0.15	0.2924	0.3151	1899.6	1644.2	1537.3	47.6815
0.16	0.2915	0.3148	1904.9	1637.0	1529.9	47.5226
0.17	0.2901	0.3142	1907.4	1626.5	1518.9	47.2727

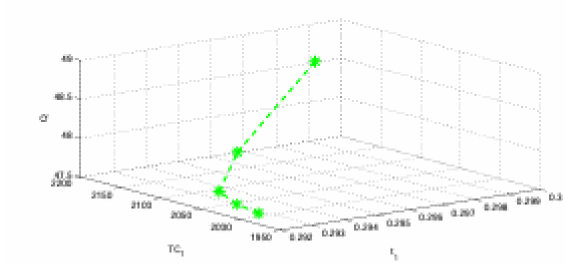


Figure 2: Variation of optimal cost TC_1,Q with respect to T, t_1

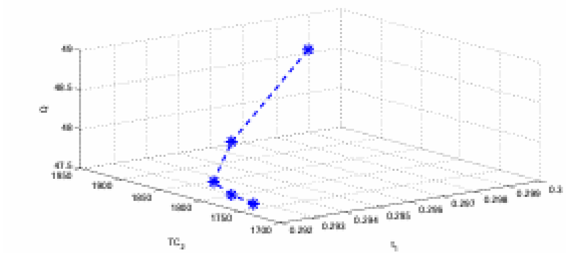


Figure 3: Variation of optimal cost TC_2,Q with respect to T, t_1

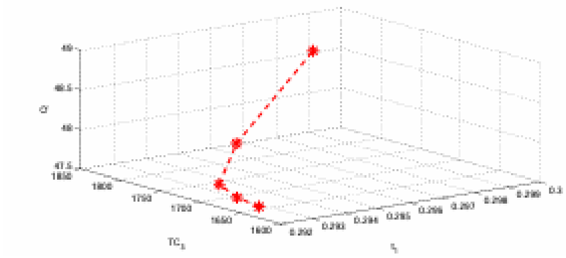


Figure 4: Variation of optimal cost TC_3,Q with respect to T, t_1

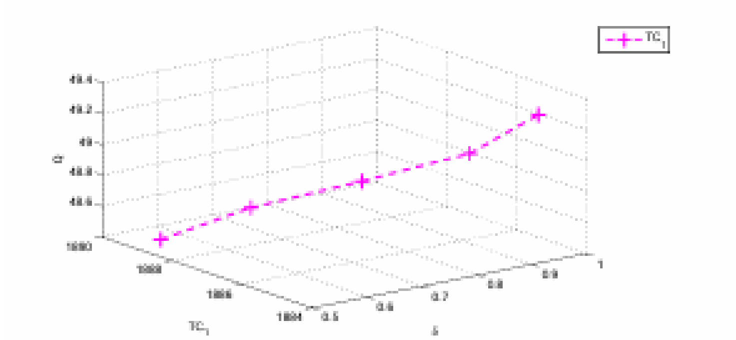


Figure 5: Variation of optimal cost TC_1,Q with respect to δ

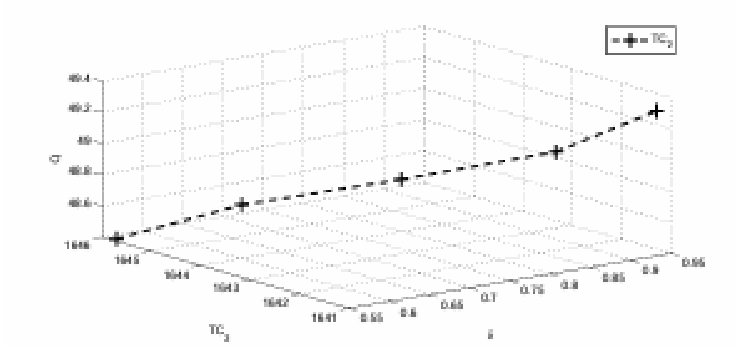


Figure 6: Variation of optimal cost TC_2,Q with respect to δ

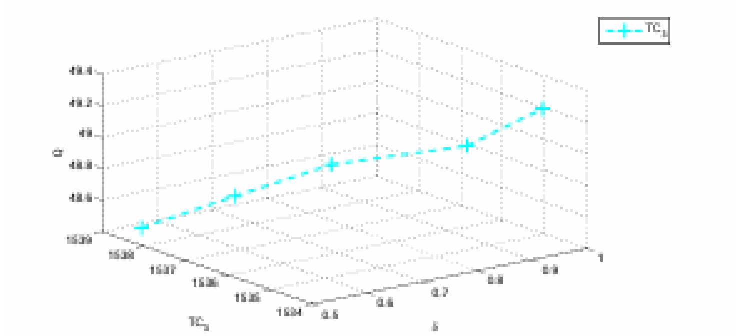


Figure 7: Variation of optimal cost TC_3,Q with respect to δ

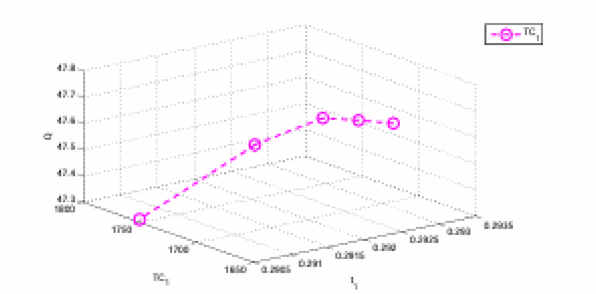


Figure 8: Variation of optimal cost TC_1, Q with respect to t_1

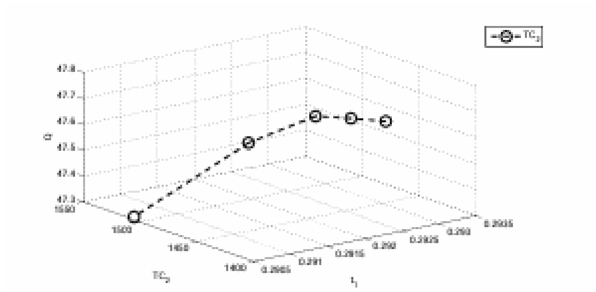


Figure 9: Variation of optimal cost TC_2, Q with respect to t_1

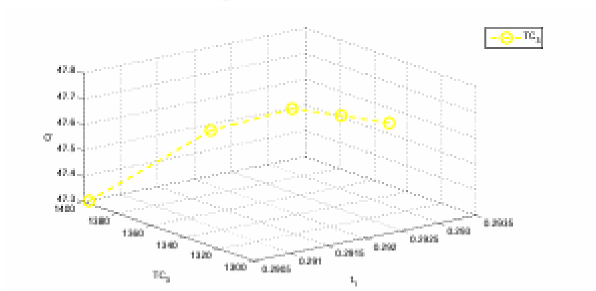


Figure 10: Variation of optimal cost TC_3, Q with respect to t_1

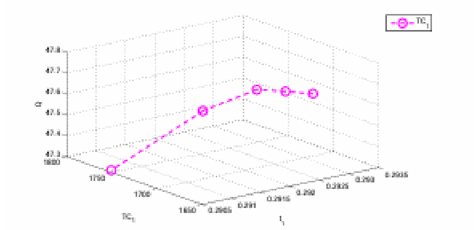


Figure 8: Variation of optimal cost TC_1,Q with respect to t_1

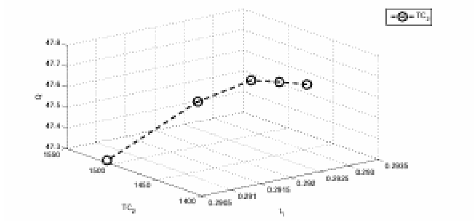


Figure 9: Variation of optimal cost TC_2,Q with respect to t_1

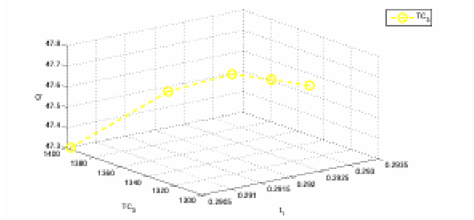


Figure 10: Variation of optimal cost TC_3,Q with respect to t_1

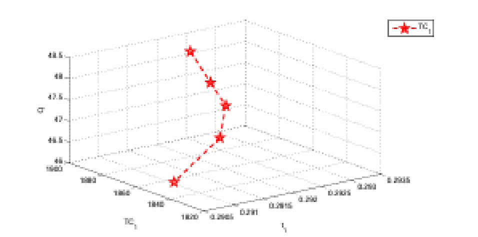


Figure 11: Variation of optimal cost TC_1,Q with respect to t_1

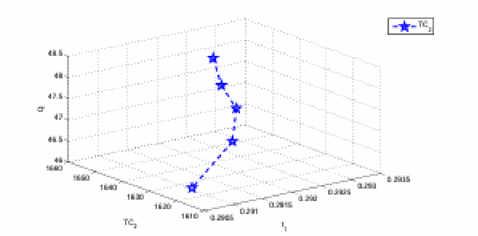


Figure 12: Variation of optimal cost TC_2,Q with respect to t_1

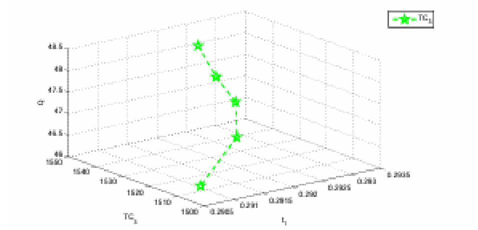


Figure 13: Variation of optimal cost TC_3,Q with respect to t_1

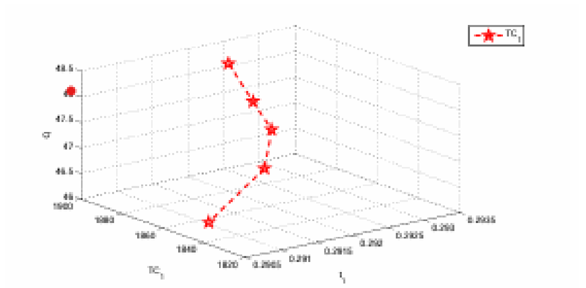
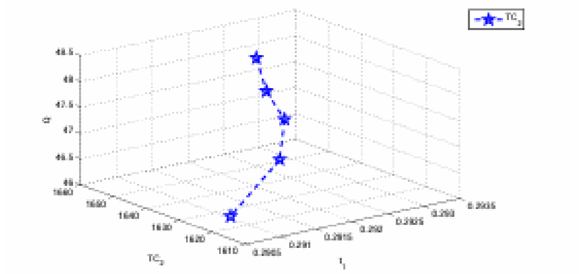


Figure 11: Variation of optimal cost TC_1,Q with respect to t_1



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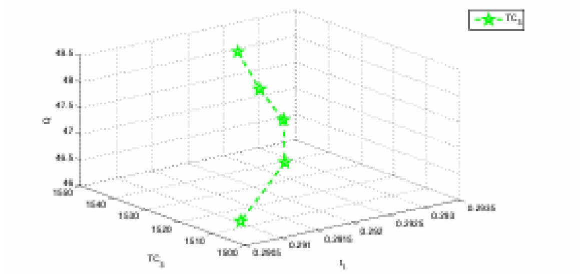

Figure 13: Variation of optimal cost TC_3,Q with respect to t_1

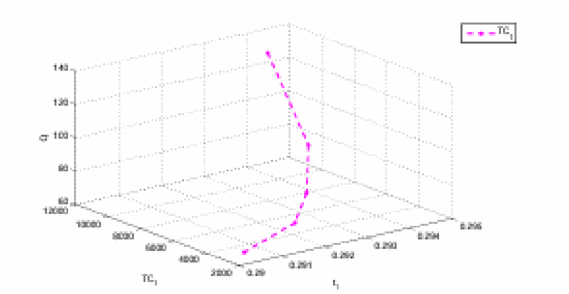
Figure 14: Variation of optimal cost TC_1,Q with respect to t_1

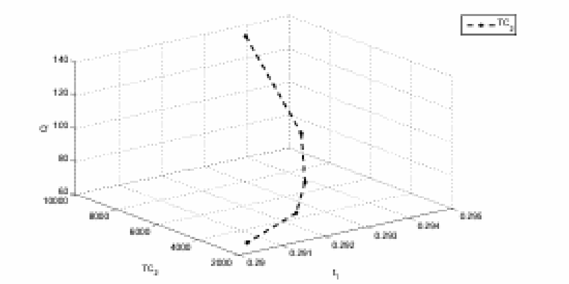
Figure 15: Variation of optimal cost TC_2,Q with respect to t_1

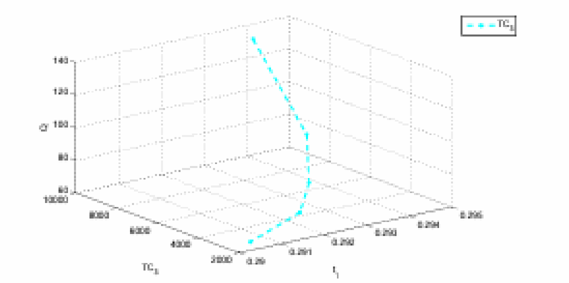
Figure 16: Variation of optimal cost TC_3,Q with respect to t_1

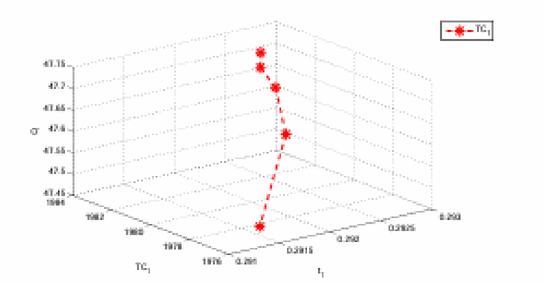

Figure 17: Variation of optimal cost TC_1,Q with respect to t_1

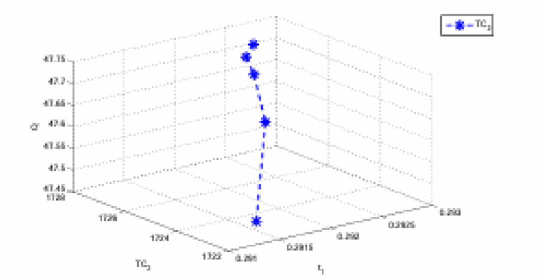

Figure 18: Variation of optimal cost TC_2,Q with respect to t_1

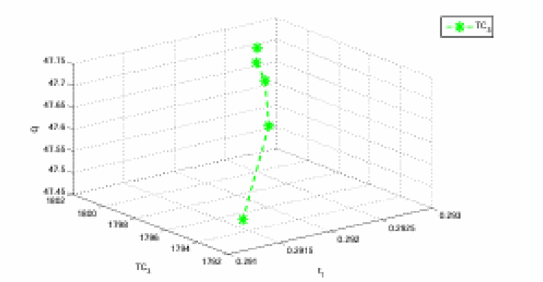

Figure 19: Variation of optimal cost TC_3,Q with respect to t_1

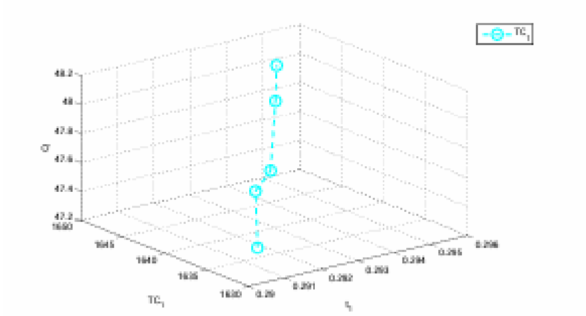

Figure 20: Variation of optimal cost TC_1,Q with respect to t_1

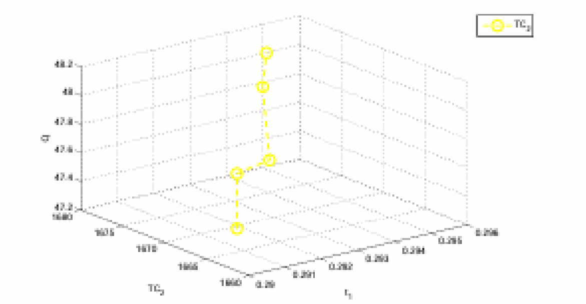

Figure 21: Variation of optimal cost TC_2,Q with respect to t_1

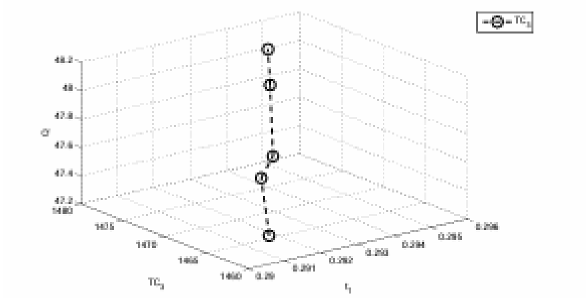

Figure 22: Variation of optimal cost TC_3,Q with respect to t_1

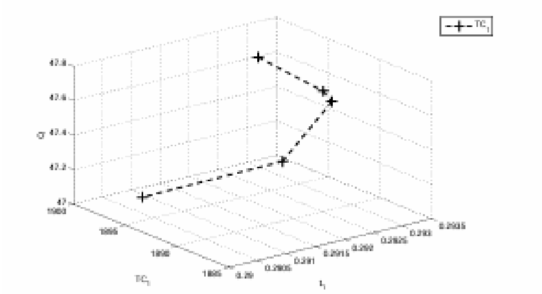

Figure 23: Variation of optimal cost TC_1,Q with respect to t_1

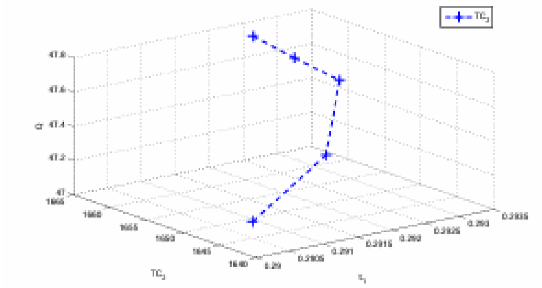

Figure 24: Variation of optimal cost TC_2,Q with respect to t_1

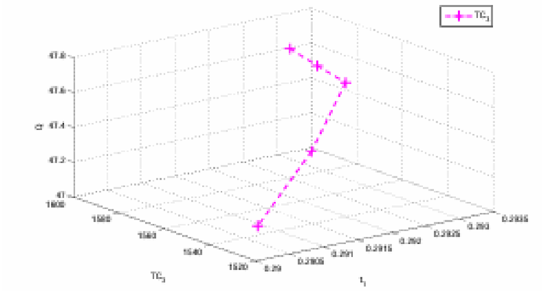

Figure 25: Variation of optimal cost TC_3,Q with respect to t_1

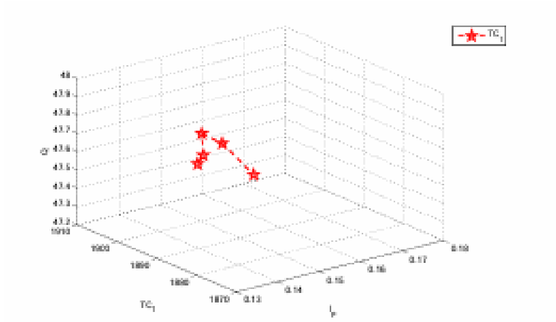

Figure 26: Variation of optimal cost TC_1,Q with respect to t_1

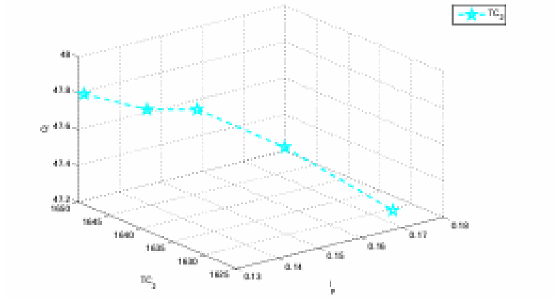

Figure 27: Variation of optimal cost TC_2,Q with respect to t_1

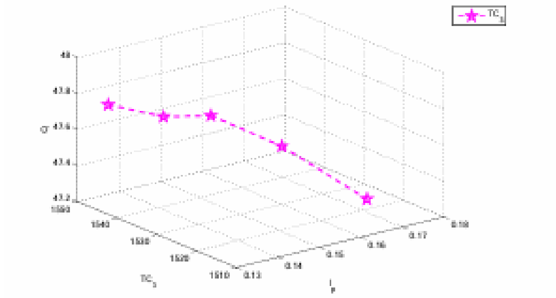

Figure 28: Variation of optimal cost TC_3,Q with respect to t_1

7. ManagerialImplication

1. Theretailershouldordermarginallylessorderquantitywhentradecreditperiod (M)increases.Actually,asMincreasesorderquantityshouldalsobeincreased.Buttheoptimalcycletimefallsinthe interval$M \leq T$.soafter,M, the retailer has to pay interest charges for the stock.Hence he wants to reduce marginally the ordering quantity.
2. Theretailerprefersnottoincreasethecustomer'stradecreditperiod.
3. Highervaluesofinterestpayableimplieshighertotalcost.
4. Highervaluesofinterestearnedimplieslowertotalcost.

8. Conclusion

The present paper developed two echelon trade credit financing in a supply chain with Weibulldistributionandexponentiallyincreasingholdingcost.Wedevelopeddifferent EOQinventory model with perishable items under the condition that $M \geq N$.Numericalexamplesaregiventoillustratethemodel.Sensitivityanalysisfortheeffectsofthe parametersonthedecisionsarealsooffered.Toarchiveoptimizedtradecreditpolicies, which is helpful for the supply chain, the supplier should share additional profits to encourage the retailer to cooperate.

In future research, our model can be extended in several ways.One can extend the model for two types of payment method, varying deterioration rate, selling price de- pendent demand or inventory level dependent demand, quantity discount, time-valueofmoneyandinflationsandsoon.Additionally,thisworkcanbeextendedfordemand as a function of price, quantity and time varying.

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