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Bulk Transportation Problem with Multi-Objectives: A Modern Approach in Fuzzy Environment

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***ABSTRACT**: Many researchers considered fuzzy parameters in the transportation problem but we dealt with intuitionistic fuzzy parameters in trapezoidal form. Seeking the most suitable solution for all objectives simultaneously in a multi-objective transportation problem is extremely challenging. Therefore, several contemporary techniques are applied to simultaneously identify the best compromise solution for all objectives. The "Bulk Transportation Problem" is a distinct kind of multi-objective transportation problem that we covered in this work. In this problem, each destination's demand is satisfied by a single source, but the source may supply to any number of destinations. First, we transformed fuzzy data into crisp data using a ranking function. Then, we suggested an alternative analytical approach for the bulk multi-objective transportation problem, which allows us to arrive at a workable solution. To demonstrate the effectiveness and potential of our suggested method for the bulk multi-objective transportation problem, we provided a numerical example.*

***KEYWORDS**: Multi-objective transportation problem, Bulk transportation problem, Fuzzy transportation problem, Fuzzy Logic approach.*

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I. INTRODUCTION

Hitchcock [1] initially addressed the transportation problem (TP), which is the problem in which we determine the quantity of goods to be transported at the lowest possible cost from any number of origins to any number of destinations. Although TP is a unique kind of linear programming problem, therefore, the simplex method is not suitable to solve it due to its unique structure. Firstly, Maio and Roveda [2] discovered the unique kind of transportation problem (bulk transportation problem) in which each destination's demand is satisfied by a single source, but the source may supply to any number of destinations. In real world there are many situations where more than one objective are to be considered and these objectives are optimized at the same time which is multi-objective transportation problem. With the use of the branch and bound technique, Prakash and Ram [3] have solved this case of a multi-objective transportation problem [4]. Moreover the parameters of transportation problem is not exact due to real situations i.e. cost or source and demand may be uncertain. To handle such situations Bellman and Zadeh [5] were the first to use the fuzzy set theory, which was presented by Zadeh [6], to decision-making problems. Oheigeartaigh [7] gave an algorithm to solve the fuzzy transportation problem. Zimmermann [8] gave the optimal solution of fuzzy transportation problem by fuzzy linear programming approach with several objectives [9]. The multi-objective bulk transportation problem was successfully resolved by Sukhveer et al. [10], while the bi-objective problem was solved by Sukhveer Singh and Singh [11] by transforming it into a single objective TP and identifying the fuzzy optimal solution for both fuzzy cost and fuzzy time [12,13]. The efficient solution for the multi-objective fuzzy transportation problem was also covered by Vidhya and Ganesan [14]. The concept of intuitionistic fuzzy sets, an extension of fuzzy sets that are most helpful in handling vagueness, was first presented by Atanassov [15]. An intuitionistic set deals with membership as well as non-membership with a hesitating part, which distinguishes it apart from fuzzy sets. The multi-objective transportation problem in an intuitionistic fuzzy environment was solved by Annie Christi [16].

In this chapter, we find the cost-time trade-off pair for intuitionistic fuzzy cost and intuitionistic fuzzy time, and we also compare our solution with the Vogel's approximation method's solution. We discuss the efficient solution for the multi-objective transportation problem under intuitionistic fuzzy environment. This chapter has been divided up into eight sections. Basic definitions and arithmetic operations on intuitionistic trapezoidal fuzzy numbers are presented in section second and third respectively. The mathematical framework of this topic is presented in section fourth. The suggested algorithm is presented in Section fifth and the numerical example it solves is shown in Section sixth. Examine the differences between the results produced by the suggested algorithm and Vogel's approximation method in section seventh. We talk about the conclusion in the last part.

II. BASIC CONCEPTS

Fuzzy set:. A membership function that maps an element of a domain, space, or universe of discourse X to the unit interval [0, 1] characterizes a fuzzy set. In a discourse universe X, a fuzzy set $\widetilde{A}$ is defined as the pair set $\widetilde{A}$={(x,$f_{\widetilde{A}}$(x))}. Here: The membership function of the fuzzy set $\widetilde{A}$ is represented by the mapping $f_{\widetilde{A}}$: X→ [0,1], and the membership grade of x∈X on the fuzzy set $\widetilde{A}$ is denoted by $f_{\widetilde{A}}$(x). These membership grades $f_{\widetilde{A}}$(x) are often represented by real numbers ranging from [0,1].

Fuzzy number: A fuzzy set $\widetilde{A}$ defined on the set of real number r is said to be fuzzy number if fuzzy set $\widetilde{A}$ has the following characteristics-

i. $\widetilde{A}$ Is normal.
ii. $\widetilde{A}$ Is convex.
iii. The support of $\widetilde{A}$ is closed and bounded.

Trapezoidal fuzzy number: A fuzzy number $\widetilde{A}$= (a_1, a_2, a_3, a_4) is said to be trapezoidal fuzzy number if its membership function is represented by-

$$f_{\widetilde{A}}(X) = \begin{cases} 0; & x \le a_1 \; and \; x \ge a_4 \\ \frac{x-a_1}{a_2-a_1}; & a_1 \le x \le a_2 \\ 1; & a_2 \le x \le a_3 \\ \frac{a_4-x}{a_4-a_3}; & a_3 \le x \le a_4 \end{cases}$$

Trapezoidal intuitionistic fuzzy number: A trapezoidal intuitionistic fuzzy number is represented by $\widetilde{A}^{'}$ = $(a_1,a_2,a_3,a_4; a_1^{'},a_2^{'},a_3^{'},a_4^{'})$ whose membership and non-membership function is represented by as follows:

$$\mu_{\widetilde{A}}(X)= \begin{cases} 0; & x \le a_1 \; and \; x \ge a_4 \\ \frac{x-a_1}{a_2-a_1}; & a_1 \le x \le a_2 \\ 1; & a_2 \le x \le a_3 \\ \frac{a_4-x}{a_4-a_3}; & a_3 \le x \le a_4 \end{cases} \quad \text{(membership function)}$$

$$\vartheta_{\widetilde{A}}(X) = \begin{cases} 1; & x \le a_1^{'} \; and \; x \ge a_4^{'} \\ \frac{a_2^{'}-x}{a_2^{'}-a_1^{'}}; & a_1^{'} \le x \le a_2^{'} \\ 0; & a_2^{'} \le x \le a_3^{'} \\ \frac{x-a_3^{'}}{a_4^{'}-a_3^{'}}; & a_3^{'} \le x \le a_4^{'} \end{cases} \quad \text{(non-membership function)}$$

Where $a_1^{'} \le a_1 \le a_2^{'} \le a_2 \le a_3 \le a_3^{'} \le a_4 \le a_4^{'}$.

Ranking function on trapezoidal intuitionistic fuzzy number:

let $\widetilde{A}^{'}$= $(a_1,a_2,a_3,a_4; a_1^{'},a_2^{'},a_3^{'},a_4^{'})$ be trapezoidal intuitionistic fuzzy number then:

$$r(\widetilde{A}^{'})= \frac{a_1+a_2+a_3+a_4+a_1^{'}+a_2^{'}+a_3^{'}+a_4^{'}}{8}$$

If R($\widetilde{A}^{'}$)≤ R($\widetilde{B}^{'}$) then $\widetilde{A}^{'} \le \widetilde{B}^{'}$ and if R($\widetilde{A}^{'}$)≥ R($\widetilde{B}^{'}$) then $\widetilde{A}^{'} \ge \widetilde{B}^{'}$.

III. ARITHMETIC OPERATIONS ON TRAPEZOIDAL INTUITIONISTIC FUZZY NUMBER

Let $\widetilde{A}^{'}$= $(a_1,a_2,a_3,a_4; a_1^{'},a_2^{'},a_3^{'},a_4^{'})$ and $\widetilde{B}^{'}$= $(b_1,b_2,b_3,b_4; b_1^{'},b_2^{'},b_3^{'},b_4^{'})$ be two trapezoidal intuitionistic fuzzy number then:

Addition: $\widetilde{A}^{'}+ \widetilde{B}^{'}$= $(a_1,a_2,a_3,a_4; a_1^{'},a_2^{'},a_3^{'},a_4^{'})$+ $(b_1,b_2,b_3,b_4; b_1^{'},b_2^{'},b_3^{'},b_4^{'})$
=$(a_1+ b_1, a_2+ b_2, a_3+ b_3, a_4+ b_4; a_1^{'}+ b_1^{'}, a_2^{'}+ b_2^{'}, a_3^{'}+ b_3^{'}, a_4^{'}+ b_4^{'})$

Subtraction: $\widetilde{A}^{'}$- $B^{'}$= $(a_1,a_2,a_3,a_4; a_1^{'},a_2^{'},a_3^{'},a_4^{'})$ - $(b_1,b_2, b_3,b_4; b_1^{'},b_2^{'},b_3^{'},b_4^{'})$
=$(a_1- b_4, a_2- b_3, a_3- b_2, a_4- b_1; a_1^{'}- b_4^{'}, a_2^{'}- b_3^{'}, a_3^{'}- b_2^{'}, a_4^{'}- b_1^{'})$.

Multiplication: $\widetilde{A}^{'} \times B^{'}$= $(c_1,c_2,c_3,c_4; c_1^{'},c_2^{'},c_3^{'},c_4^{'})$

Where, c_1=min(a_1b_1,a_1b_4,a_4b_1, a_4b_4), c_2=min(a_2b_2,a_2b_3,a_3b_2, a_3b_3),
c_3=max(a_2b_2,a_2b_3,a_3b_2, a_3b_3), c_4=max(a_1b_1,a_1b_4,a_4b_1, a_4b_4),
$c_1^{'}$=min($a_1^{'}b_1^{'}$, $a_1^{'}b_4^{'}$, $a_4^{'}b_1^{'}$, $a_4^{'}b_4^{'}$), $c_2^{'}$=min($a_2^{'}b_2^{'}$, $a_2^{'}b_3^{'}$, $a_3^{'}b_2^{'}$, $a_3^{'}b_3^{'}$),
$c_3^{'}$ =max($a_2^{'}b_2^{'}$, $a_2^{'}b_3^{'}$, $a_3^{'}b_2^{'}$, $a_3^{'}b_3^{'}$), $c_4^{'}$ =max($a_1^{'}b_1^{'}$, $a_1^{'}b_4^{'}$, $a_4^{'}b_1^{'}$, $a_4^{'}b_4^{'}$)

IV. MATHEMATICAL FORMULATION

Let there be m source and n destination and each source has a_i (where i= 1,2,...M) available quantity of product respectively and each destination has b_j (where j= 1,2,…n) requirement of product respectively. Let C and T be the total intuitionistic fuzzy cost and total intuitionistic fuzzy time respectively.

The mathematical formulation of multi-objective transportation problem with total intuitionistic fuzzy cost and total intuitionistic fuzzy time is as follows:

Minimize
$C = \sum_{i=1}^{m}\sum_{j=1}^{n} c'_{ij}\, x'_{ij}$ and $T = \max\{t'_{ij}:\ x'_{ij}=1;\ i=1,2,\ldots m,\ j=1,2,\ldots n\}$
With the constraints
$\sum_{j=1}^{n} b_j\, x'_{ij} \le a_i$ (i=1,2,….M),
$\sum_{i=1}^{m} x'_{ij}=1$ (j=1,2,…n) and x'_{ij} = 1 or 0 (i=1,2,…m, j=1,2,…n).
Where a_i denotes the available quantity of product at i^{th} source and b_j denotes the demand of j^{th} destination.
c'_{ij} denotes the intuitionistic fuzzy cost of b_j units for transport from i^{th} source to j^{th} destination and t'_{ij} denotes time taken for transport b_j units from i^{th} source to j^{th} destination.
x'_{ij} is the decision variable with the assuming value 1 or 0 depending upon that i^{th} source fulfilled the requirement of j^{th} destination or not respectively.
Moreover c'_{ij} and t'_{ij} expressed as:
$c'_{ij}= (c_1,c_2,c_3,c_4;\ c'_1,c'_2,c'_3,c'_4)$ and $t'_{ij}= (t_1,t_2,t_3,t_4;\ t'_1,t'_2,t'_3,t'_4)$.

V. PROPOSED ALGORITHM

Algorithm as follows:
Step 1: Construct the multi-objective transportation problem in intuitionistic form.
Step 2: Now remove that cell for which available quantity at i^{th} source is less than the required quantity for j^{th} destination in initial cost table.
Step 3: Now convert the cost into crisp form by ranking function. Then choose the minimum element from each row and subtract this cost from each cost in corresponding row. Do the same process for each column.
Step 4: Now choose the zero of the cell for which the number of zeros in corresponding row and column is minimum in the cost table and allocate 1 in this cell. If there is ties then choose that cell for which the sum of all entries in corresponding row and column is maximum. If again tie then choose that cell for which maximum demand is fulfilled.
Step 5: Now delete the destination which demand is fulfilled and source for which supply is less than demand for each destination. Repeat 2 to 4 step unless all the demands are fulfilled. Then first efficient solution of the problem is obtained.
Step 6: Now for find the next efficient solution leave the cell whose rank of time is greater than the rank of time for obtained solution for first cost time trade-off pair. Repeat same process and find all possible efficient solution for intuitionistic fuzzy cost time trade-off pair.
In the next multi-objective transportation problem with intuitionistic fuzzy cost and fuzzy time is considered such that in each cell upper entries are cost and lower entries are time as trapezoidal intuitionistic fuzzy number. Supply and demand are in crisp form.

VI. NUMERICAL EXAMPLE

1. Step 1:

Table: 1

	D_1	D_2	D_3	D_4	D_5	Supply
O_1	(0,1,2,5; 0,.5,1.5,5) (1,3,4,8; 0,2,5,9)	(1,2,3,6; 0,1,4,7) (1,3,4,8; 0,2,5,9)	(1,2,3,6; 0,1,4,7) (3,7,10,20; 2,6,11,21)	(2,5,7,14; 1,4,8,15) (3,5,8,16; 2,4,9,17)	(0,.5,1.5,2; 0,.5,1.5,2) (2,5,7,14; 1,4,8,15)	7
O_2	(1,3,4,8; 0,2,5,9) (1,3,4,8; 0,2,5,9)	(0,.5,1.5,2; 0,.5,1.5,2) (2,5,7,14; 1,4,8,15)	(0,.5,1.5,2; 0,.5,1.5,2) (5,7,12,24; 4,6,13,25)	(0,1,2,5; 0,.5,1.5,5) (5,9,14,28; 4,8,15,29)	(3,5,8,16; 2,4,9,17) (3,5,8,16; 2,4,9,17)	9
O_3	(0,.5,1.5,2; 0,.5,1.5,2) (3,5,8,16; 2,4,9,17)	(2,5,7,14; 1,4,8,15) (0,1,2,5; 0,.5,1.5,5)	(5,6,11,22; 4,5,12,23) (1,3,4,8; 0,2,5,9)	(0,.5,1.5,2; 0,.5,1.5,2) (1,3,4,8; 0,2,5,9)	(1,4,5,10; 0,3,6,11) (1,3,4,8; 0,2,5,9)	10
Demand	3	5	4	6	2	

2. After applying step 2 and 3 we get the initial cost:

Table 2:

	D_1	D_2	D_3	D_4	D_5	Supply
O_1	1	2	2	6	0	7
O_2	3	0	0	1	7	9
O_3	0	6	10	0	4	10
Demand	3	5	4	6	2	

By applying step 4 and 5, we get x_{15}=1. Then by repeat this process we get the first efficient solution of intuitionistic multi-objective fuzzy transportation problem. Solution is as follows $\{x_{15}, x_{34}, x_{31}, x_{12}, x_{23}\}$. By this the cost for transportation is C_1= (0,2.5,7.5,10; 0,2.5,7.5,10) and corresponding time T_1= (5,7,12,24;4,6,13,25), then first cost-time trade-off pair is (C_1, T_1).

Now by applying step 6 and then step 3 the initial cost table for second efficient solution of the problem:

	D_1	D_2	D_3	D_4	D_5	Supply
O_1	1	2	0	6	0	7
O_2	3	0	-	-	7	9
O_3	0	6	8	0	4	10
Demand	3	5	4	6	2	

Now by applying step 6, we get the next efficient solution for the cost time problem. Second efficient solution is as follows $\{x_{22}, x_{34}, x_{31}, x_{13}, x_{15}\}$. Then the cost for transportation is C_2 = (0,2.5,7.5,10; 0,2.5,7.5,10) and corresponding time T_2 =(3,7,10,20;2,6,11,21), then second cost-time trade-off pair is (C_2, T_2) . By using this algorithm we get all possible efficient solution and cost-time trade-off pair.
Now again by applying step 6 and step 3 the initial cost table for next efficient solution of the problem:

	D_1	D_2	D_3	D_4	D_5	Supply
O_1	1	2	-	6	0	7
O_2	3	0	-	-	7	9
O_3	0	6	0	0	4	10
Demand	3	5	4	6	2	

Then next efficient solution is as follows$\{x_{15}, x_{22}, x_{34}, x_{33}, x_{11}\}$ and the cost for transportation is C_3={5,8.5,17.5,33;4,7,18,34} and corresponding time T_3= {2,5,7,14;1,4,8,15} then second cost-time trade-off pair is (C_3, T_3).
By using this algorithm we get all possible efficient solution and cost-time trade-off pair which is in next table:

Optimal solution	Total intuitionistic fuzzy cost	Total intuitionistic fuzzy time
$X_1 = \{x_{15}, x_{34}, x_{31}, x_{22}, x_{23}\}$ $X_2 = \{x_{22}, x_{34}, x_{31}, x_{13}, x_{15}\}$ $X_3 = \{x_{15}, x_{22}, x_{34}, x_{33}, x_{11}\}$	C_1 = {0,2.5,7.5,10; 0,2.5,7.5,10} C_2= {1,4,9,14;0,3,10,15} C_3= { 5,8.5,17.5,33;4,7,18,34}	T_1 = {5,7,12,24; 4,6,13,25} T_2= {3,7,10,20;2,6,11,21} T_3= {2,5,7,14;1,4,8,15}

Now the efficient solution and cost time trade-off pair by using vogel's approximation method:

Optimal solution	Total intuitionistic fuzzy cost	Total intuitionistic fuzzy time
$X_1 = \{x_{34}, x_{31}, x_{22}x_{23}, x_{15}\}$ $X_2 = \{x_{13}, x_{22}, x_{34}, x_{31}, x_{15}\}$ $X_3 = \{x_{33}, x_{34}, x_{15}, x_{22}, x_{11}\}$	$C_1 = \{0,2.5,7.5,10; 0,2.5,7.5,10\}$ C_2= {1,4,9,14;0,3,10,15} C_3= { 5,8.5,17.5,33;4,7,18,34}	$T_1 = \{5,7,12,24; 4,6,13,25\}$ T_2= {3,7,10,20;2,6,11,21} T_3= {2,5,7,14;1,4,8,15}

VII.COMPARISON

We get that the cost time trade-off pair obtained by proposed algorithm is same as the cost time trade-off pair obtained by Vogel's Approximation method. That's means proposed approach is efficient to find the compromise optimal solution for solving such type multi-objective transportation problem under intuitionistic fuzzy environment.

VIII. CONCLUSION

In this paper, a modern approach is developed to find an efficient solution of multi-objective transportation problem by representing cost and time as trapezoidal intuitionistic fuzzy number and cost, time are converting into crisp form by using the ranking function of trapezoidal intuitionistic fuzzy number. This method gives the optimal solution of special type transportation problem as Vogel's approximation method (VAM). The obtained result by proposed algorithm is same as the VAM. By this method this special type problem is solved in less computational work and it is very easy to apply.

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