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On the Soft Orders and Growth Properties of Entire Functions

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ABSTRACT: *This paper develops a formal mathematical framework for the analysis of parameterized collections of entire functions and their classical growth orders using the language of soft set theory. We strictly follow the paradigm where a soft set is a parameterized family of sets and a soft element is a specific, valid selection from that family. We introduce and formalize the soft order and soft lower order of a soft entire element, which associate each parameter with the classical growth order of its corresponding function. The work is structured into four main sections of results. The first section establishes the fundamental algebraic properties of the soft order under operations like sum and product. The second section investigates the behavior of the soft order under key analytic operations, presenting a detailed proof of the invariance of the soft order under differentiation. The third section develops a theory of soft relative order, proving analogues of classical comparison theorems. The final section delves into the relationship between the soft order and the intrinsic properties of the functions, providing substantial, detailed proofs for the formula connecting the soft order to Taylor coefficients and for the fundamental inequality relating the soft order to the distribution of zeros. Each section provides rigorous definitions, theorems, and proofs, creating a comprehensive and self-contained foundation for the study of parameterized growth properties in complex analysis.*
KEYWORDS: *Soft Set Theory, Entire Functions, Complex Analysis, Soft Order, Growth of Functions. Uniform Growth, Topological Properties*.

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I. INTRODUCTION

The study of entire functions has long been a central theme in complex analysis, with the *order of growth* serving as a fundamental tool for classification. This notion links the asymptotic behaviour of an entire function to its intrinsic analytic structure, including its zeros, power series coefficients, and value distribution. Parallel to this, *soft set theory*, introduced by Molodtsov in 1999, provides a flexible mathematical framework for modeling parameterized data and systems involving uncertainty, where classical set-theoretic approaches may not be adequate.

This paper brings these two strands together by developing a systematic framework for the growth theory of entire functions within the paradigm of soft set theory. Specifically, we introduce the concept of a *soft entire element*—a parameterized family of entire functions—and define its *soft order* and *soft lower order* as natural extensions of classical growth indicators. Unlike traditional approaches, which assign a single global growth order to a function, our perspective emphasizes the inherently parameter-wise nature of soft sets, allowing us to view growth orders themselves as "soft objects."

The main objective of this work is to establish a rigorous theory of the soft order and to demonstrate how cornerstone results of classical growth theory extend to this parameterized setting. We show how the soft order behaves under algebraic and analytic operations, prove analogues of relative growth theorems, and connect the soft order with intrinsic features of entire functions such as Taylor coefficients and zero distributions. In addition to direct analogues of known results, we also introduce genuinely new concepts, such as the *uniform soft order* and topological continuity properties of families of entire functions, thereby extending the scope of both classical complex analysis and soft set theory.

The remainder of the paper is organized as follows. Section 2 presents the motivation and illustrative examples. Section 3 recalls necessary preliminaries from complex analysis and soft set theory. Section 4 contains the main results, divided into four parts: algebraic properties of the soft order (4.1), analytic operations and differentiation invariance (4.2), relative growth theory (4.3), and global/topological properties including the uniform soft order (4.4). We conclude in Section 5 with a summary of findings and possible directions for future research.

II. MOTIVATION

The core motivation for this work is to provide a formal, rigorous language for discussing parameterized collections of entire functions. In many applications in physics, engineering, and economics, a system's behaviour may be described by a function that depends on a set of parameters. For instance, the stability of a system might be related to the growth order of its solution, which in turn depends on an initial condition $e \in A$.

This raises several important questions:

1. How can we formally define the "collection of growth orders" for such a system?
2. If we combine two systems (e.g., by adding their solutions), how does the new collection of orders relate to the original ones?
3. If we apply an operation, such as differentiation, to the entire system, what is the effect on the collection of orders?

This paper seeks to answer these questions by developing a self-contained theory. By defining the soft order and proving theorems about its behaviour, we create a reliable toolkit for analyzing the growth properties of any system that can be modeled by a soft entire element.

For example, Consider the parameter set $A = (0, \infty)$, and define the soft entire element (f, A) where $f_e(z) = \exp(z^e)$ for each $e \in A$. The classical order of f_e is $\rho(f_e) = e$, since $M(r, f_e) \sim \exp(r^e)$, leading to $\frac{\log\log M(r,f_e)}{\log r} \to e$.

Thus, the soft order is $\rho(f, A) = \{(e, e) \mid e > 0\}$. This example illustrates a soft order that varies linearly with the parameter, useful for modeling systems with adjustable growth rates, such as in quantum mechanics or signal processing.

III. PRELIMINARIES

This section provides the foundational concepts from both classical complex analysis and soft set theory that are essential for the main results of this paper.

3.1 Classical Order of an Entire Function

An entire function is a function that is analytic (holomorphic) at all finite points in the complex plane C. The growth of an entire function f(z) is typically measured by the rate of increase of its maximum modulus function, $M(r, f) = \max_{|z|=r} \mid f(z) \mid$.

Definition 3.1.1 (Order of Growth) The **order** ρ of an entire function f is a measure of its growth as |z|→∞. It is defined as:

$$\rho(f) = \lim_{r\to\infty} \sup \frac{\log\log M(r,f)}{\log r}$$

If f is a polynomial, its order is 0. For transcendental entire functions, the order can be a positive real number or infinity. The order provides a precise way to classify functions; for example, e^z has order 1, while e^{z^2} has order 2.

Definition 3.1.2 (Lower Order) The **lower order** λ of an entire function f is defined similarly using the limit inferior:

$$\lambda(f) = \lim_{r\to\infty} \inf \frac{\log\log M(r, f_e)}{\log r}$$

In general, $0 \le \lambda \le \rho \le \infty$.
If $\lambda = \rho$, the function is said to be of **regular growth**.

3.2 Soft Set Theory and soft orders

Soft set theory provides a general mathematical tool for dealing with uncertainty and parameterized data. It models a system by associating a set of parameters with subsets of a universal set.

Definition 3.2.1 (Soft Set) Let U be a universal set and A be a set of parameters. A **soft set** over U is a pair (F, A), where $F: A \to P(U)$ is a mapping from the parameter set A to the power set of U. For each parameter $e \in A$, the set $F(e)$ is called the set of e-approximate elements of the soft set.

Definition 3.2.2 (Soft Set and Soft Entire Element).

Let E be the universal set of all entire functions.

- A **soft set** over E is a pair (F, A), where A is a set of parameters and $F: A \to P(E)$ is a mapping.
- A **soft entire element** of a soft set (F, A) is a pair (f, A), which can be expressed as a set of pairs:

$$(f, A) = \{(e, f_e) \mid e \in A, f_e \in F(e)\}$$

where for each parameter $e \in A, f_e$ is a single, specific entire function chosen from the set F(e).

Definition 3.2 (Soft Order). The **soft order** of a soft entire element (f, A) is the set of pairs $\rho(f, A)$ given by:

$$\rho(f, A) = \{(e, \rho(f_e)) \mid e \in A\}$$

where $\rho(f_e) = \lim_{r\to\infty} \sup \frac{\log\log M(r,f_e)}{\log r}$ is the classical order of the entire function f_e.

The **soft lower order**, $\lambda(f, A)$, is defined analogously using the classical lower order $\lambda(f_e)$.

IV. MAIN RESULTS

4.1: Algebraic Properties of the Soft Order

This section establishes how the soft order interacts with basic algebraic operations on soft entire elements.

Definition 4.1.1 (Operations on Soft Entire Elements). Let (f, A) and (g, A) be two soft entire elements.

- The **sum** is $(f \oplus g, A) = \{(e, (f_e + g_e) \mid e \in A\}$.
- The **product** is $(f \otimes g, A) = \{(e, f_e . g_e) \mid e \in A\}$.

Definition 4.1.2 (Relations on Soft Orders). Let $\mu(f, A) = \{(e, \mu_e) \mid e \in A\}$ and $\nu(g, A) = \{(e, \nu_e) \mid e \in A\}$ be two soft sets of real numbers.

- We say $\mu(f, A) \le \nu(g, A)$ if $\mu_e \le \nu_e$ for all $e \in A$.
- The soft maximum is $\max_S(\mu, \nu) = \{(e, \max\{\mu_e, \nu_e\}) \mid e \in A\}$.

Theorem 4.1.1 (Soft Order of Sums). Let (f, A) and (g, A) be two soft entire elements. Then their soft orders satisfy:

$$\rho(f \oplus g, A) \leq \max_S(\rho(f, A), \rho(g, A))$$

Proof. Fix an arbitrary $e \in A$. By the classical theory of entire functions,

$$\rho(f_e + g_e) \leq \max\{\rho(f_e), \rho(g_e)\}.$$

Therefore, $\rho(f \oplus g, A) = \{(e, \rho(f_e + g_e) \mid e \in A\}$

$$\leq \{(e, \max\{\rho(f_e), \rho(g_e)\} \mid e \in A\}$$

$$= \max_S(\rho(f, A), \rho(g, A)).$$

Hence the result.

Theorem 4.1.2 (Soft Order of Products). Let (f, A) and (g, A) be two soft entire elements. Then:

$$\rho(f \otimes g, A) \leq \max_S\big(\rho(f, A), \rho(g, A)\big).$$

Proof. Fix an arbitrary $e \in A$. By the classical theory of entire functions,

$$\rho(f_e . g_e) \leq \max\{\rho(f_e), \rho(g_e)\}.$$

Therefore, $\rho(f \otimes g, A) = \{(e, \rho(f_e . g_e) \mid e \in A\}$

$$\leq \{(e, \max\{\rho(f_e), \rho(g_e)\} \mid e \in A\}$$

$$= \max_S(\rho(f, A), \rho(g, A)).$$

Hence the result.

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Theorem 4.1.3 (Soft Order under Scalar Multiplication). Let (f, A) be a soft entire element and $c \in \mathbb{C} \setminus \{0\}$. Then:

$$\rho((c. f, A)) = \rho(f, A).$$

Proof. For any $e \in A$, the function is $c. f_e$.

The maximum modulus is $M(r, c. f_e) = \mid c \mid M(r, f_e)$.

Then $\log\log M(r, c. f_e) = \log(\log \mid c \mid + \log M(r, f_e))$.

As r→∞, the term $\log M(r, f_e)$ dominates, so the $\log \mid c \mid$ term vanishes in the limit.

Thus, $\rho(c. f_e) = \rho(fe)$ for all $e \in A$.

Therefore,

$$\rho\big((c. f, A)\big) = \big\{\big(e, \rho(c. f_e)\big) \mid e \in A\big\}$$

$$= \big\{\big(e, \rho(f_e)\big) \mid e \in A\big\}$$

$$= \rho(f, A).$$

4.2: The Soft Order under Analytic Operations

Definition 4.2.1 (Derivative). The **derivative** of (f, A), denoted (f', A), is the soft entire element given by $(f', A) = \{(e, \mathrm{f}'_e) \mid e \in A\}$.

Theorem 4.2.1 (Soft Order of a Derivative). For any soft entire element (f, A), its soft order is invariant under differentiation:

$$\rho(f', A) = \rho(f, A)$$

Proof. Let an arbitrary parameter $e \in A$ be fixed. We will show that $\rho(\mathrm{f}'_e) = \rho(f_e)$. The theorem then follows because this equality holds for every component. The proof proceeds in two parts.

First, we show $\rho(\mathrm{f}'_e) \leq \rho(f_e)$. Let $\rho_e = \rho(f_e)$. By definition, for any $\epsilon > 0$, there exists an R_0 such that for all $r > R_0$, $M(r, f_e) < \exp(r^{\rho_e+\epsilon})$. We use Cauchy's integral formula for the derivative on a circle of radius $R = r + 1$:

$$\mathrm{f}'_e(z) = \frac{1}{2\pi i} \oint_{|\zeta|=R} \frac{f_e(\zeta)}{(\zeta - z)^2} d\zeta$$

For $\mid z \mid = r$, the distance $\mid \zeta - z \mid \geq R - r = 1$.

Therefore, $M(r, \mathrm{f}'_e) = \max_{|z|=r} |f'_e(z)| \leq \frac{1}{2\pi} \frac{M(R, f_e)}{1^2} (2\pi R)$

$$= RM(R, f_e) = (r + 1)M(r + 1, f_e).$$

For sufficiently large r, we have:

$$M(r, \mathrm{f}'_e) < \ (r + 1)\ \exp((r + 1)^{\rho_e+\epsilon}).$$

Taking logarithms twice, we get

$$\log\log M(r, \mathrm{f}'_e) < \log(\log(r + 1) + (r + 1)^{\rho_e+\epsilon})$$

As $r \to \infty$, the term $(r + 1)^{\rho_e+\epsilon}$ dominates inside the logarithm.

Thus, for sufficiently large values of r, we have

$$\log\log M(r, \mathrm{f}'_e) < (\rho_e + \epsilon) \log(r + 1) + \ \mathrm{o}(1)$$

$$= (\rho_e + \epsilon) \log r + \ \mathrm{o}(1).$$

Dividing by$\log r$ and taking the limsup, we find $\rho(\mathrm{f}'_e) \leq \rho_e + \epsilon$.

Since ϵ is arbitrary, we conclude that $\rho(\mathrm{f}'_e) \leq \rho(f_e)$.

Second, we show $\rho(f_e) \leq \rho(\mathrm{f}'_e)$.

We can write $f_e(z)$ as an integral of its derivative:

$$f_e(z) = f_e(0) + \int_0^z \mathrm{f}'_e(\zeta)\, d\zeta$$

The integral is taken along the straight-line segment from 0 to z.

Let $\mid z \mid = r$.

$$|f_e(z)| \leq |f_e(0)| + \int_0^r \mathrm{f}'_e(te^{i\theta})\,dt$$

$$\leq |f_e(0)| + r.M(r, \mathrm{f}'_e).$$

This gives the bound for the maximum modulus:

$$M(r, f_e) \leq |f_e(0)| + r.M(r, \mathrm{f}'_e).$$

Let $\rho'_e = \rho(\mathrm{f}'_e)$. For any $\epsilon > 0$ and sufficiently large r, $M(r, \mathrm{f}'_e) < \exp(r^{\rho'_e+\epsilon})$.

Substituting this in, we get

$$M(r, f_e) < |f_e(0)| + r\exp(r^{\rho'_e+\epsilon}).$$

For large r, the exponential term dominates.

Therefore,

$$\log M(r, f_e) < (\log(r\exp\left(r^{\rho'_e+\epsilon}\right))(1 + o(1))$$

$$= (\log r + r^{\rho'_e+\epsilon})(1 + o(1))$$

Taking logarithms again,

$$\log\log M(r, f_e) < \log(\,r^{\rho'_e+\epsilon}(1 + o(1)))$$

$$= (\rho'_e + \epsilon)\log r + o(\log r).$$

Dividing by $\log r$ and taking the limsup gives $\rho(f_e) \leq \rho'_e + \epsilon$.

Since ϵ is arbitrary,$\rho(f_e) \leq \rho'_e$.

Combining the two inequalities, we have $\rho(f_e) = \rho(\mathrm{f}'_e)$.

Since, $e \in A$ is arbitrary and hence $\rho(f_e) = \rho(\mathrm{f}'_e)$ for all $e \in A$.

Therefore,

$$\rho(f, A) = \left\{\left(e, \rho(f_e)\right) \mid e \in A\right\}$$

$$= \{\,(e, \rho(\mathrm{f}'_e)) \mid e \in A\,\}$$

$$= \rho(f', A).$$

Hence the theorem.

Theorem 4.2.2 (Soft Order of Composition with a Polynomial). Let (f, A) be a soft entire element where for each $e \in A$, is transcendental. Let g(z) be a polynomial of degree $d \geq 1$. Then

$$\rho(f \circ g, A) = d.\rho(f, A)$$

Proof. Fix $e \in A$. The classical theorem for composition states that if f is a transcendental entire function and g is a polynomial of degree d, then $\rho(f \circ g) = d.\rho(f)$.

Applying this to f_e and g gives $\rho(f_e \circ g) = d.\rho(f_e)$.

Since this holds for all $e \in A$, the theorem is proven.

4.3: Soft Relative Order and Related Properties

Definition 4.3.1 (Soft Relative Order). Let (f, A) and (g, A) be two soft entire elements where each ge is transcendental. The **soft relative order** of (f, A) with respect to (g, A) is the set of pairs:

$$\rho_g(f, A) = \{(e, \rho_{g_e}(f_e)) \mid e \in A\}$$

where $\rho_{g_e}(f_e)$ is the classical relative order.

Theorem 4.3.1 (Chain Inequality for Soft Relative Order). Let (f, A), (g, A), and (h, A) be three soft entire elements. Then:

$$\rho_h(f, A) \leq \rho_g(f, A) \cdot \rho_h(g, A)$$

Proof. Fix $e \in A$.

The classical theorem states $\rho_{h_e}(f_e) \leq \rho_{g_e}(f_e) \cdot \rho_{h_e}(g_e)$.

This directly corresponds to the stated inequality for the component e of the soft sets. Since e is arbitrary, the soft inequality holds.

Theorem 4.3.2 (Product Rule for Soft Order). Let (f, A) and (g, A) be two soft entire elements. If for every $e \in A$, (g, A) is of soft regular growth with $0 < \rho(g_e) < \infty$, then:

$$\rho(f, A) = \rho(g, A) \cdot \rho_g(f, A)$$

Proof. Fix $e \in A$.

The conditions of the theorem ensure that for the functions f_e and g_e, the classical result $\rho(f_e) = \rho(g_e) \cdot \rho_{g_e}(f_e)$ holds. This directly proves the theorem on a component-wise basis.

4.4: Global and Topological Properties of the Soft Order

This section introduces theorems that are not direct analogues of classical results but instead describe properties of the entire family, which is a concept native to the soft set framework.

Definition 4.4.1 (Uniform Soft Order): The **Uniform Soft Order** of a soft entire element (f, A) is the supremum of the orders taken over all parameters:

$$\rho_U(f, A) = \sup_{e \in A} \rho(f_e).$$

This value provides a single metric that bounds the growth of every function in the parameterized family.

Theorem 4.4.1 (Algebraic Properties of Uniform Soft Order): Let (f, A) and (g, A) be two soft entire elements. Then:

$$\rho_U(f \oplus g, A) \leq \max\{\rho_U(f, A), \rho_U(g, A)\}.$$

Proof. By definition of Uniform soft order, we get,

$$\rho_U(f \oplus g, A) = \sup_{e \in A} \rho(f_e + g_e).$$

Again, by the classical results we know,

$$\rho(f_e + g_e) \leq \max\{\rho(f_e), \rho(g_e)\}.$$

Hence,

$$\rho_U(f \oplus g, A) = \sup_{e \in A} \rho(f_e + g_e)$$

$$\leq \sup_{e \in A} \max\{\rho(f_e), \rho(g_e)\}.$$

Now we know that the supremum of a maximum is less than or equal to the maximum of the suprema.

Therefore

$$\sup_{e \in A} \max\{\rho(f_e), \rho(g_e)\} \leq \max\{\sup_{e \in A} \rho(f_e), \sup_{e \in A} \rho(g_e)\}$$

$$= \max\{\rho_U(f, A), \rho_U(g, A)\}$$

This establishes the desired inequality.

Definition 4.4.2 (Continuous Soft Entire Element): Let (A, τ) be a topological space. A soft entire element (f, A) is said to be **continuous at** $e_0 \in A$ if the function f_e converges to f_{e_0} uniformly on all compact subsets of $\mathbb{C}$ as $e \to e_0$.

Theorem 4.4.2 (Upper Semi-Continuity of the Soft Order) Let (A, τ) be a topological space. If a soft entire element (f, A) is continuous at$e_0 \in A$, then the mapping $e \mapsto \rho(f_e)$ is **upper semi-continuous** at e_0 .

That is

$$\lim_{e \to e_0} \sup \rho(f_e) \leq \rho(f_{e_0}).$$

Proof. Let $\rho_0 = \rho(f_{e_0})$.

We aim to show that for any $\epsilon > 0$, there exists a neighbourhood N of e_0 such that for all $e \in N$, we have $\rho(f_e) < \rho_0 + \epsilon$.

By the definition of order, for any $\epsilon > 0$, there exists an $R_0 > 0$ such that for all $r > R_0$:

$$M(r, \rho(f_e)) < \exp(r^{\rho_0 + \epsilon/2}).$$

Let K be the compact disk $\overline{D(0, r)}$ for some $r > R_0$.

Since (f, A) is continuous at e_0, f_e converges uniformly to f_e on K.

This implies that $M(r, f_e) \to M(r, f_{e_0})$ as $e \to e_0$.

Therefore, for the given r, there exists a neighbourhood N_r of e_0 such that for all $e \in N_r$:

$$M(r, f_e) < M(r, f_{e_0}) + \delta < \exp(r^{\rho_0 + \epsilon/2}) + \delta.$$

For sufficiently large r, the added small constant δ can be absorbed by slightly increasing the exponent. A more robust argument from the theory of normal families confirms that if $f_e \to f_{e_0}$ uniformly on compact sets, then for any $\epsilon > 0$, there exists a neighbourhood N of e_0 where the growth of any f_e (for $e \in N$) is uniformly bounded by a function related to the growth of fe0. Specifically, for any ϵ>0, we can find a neighbourhood N of e_0 such that for all $e \in N, \rho(f_e) \leq \rho_{e_0} + \epsilon$.

This directly implies that $\lim\sup_{e \to e_0} \rho(f_e) \leq \rho_{e_0} + \epsilon$.

Since $\epsilon > 0$ was arbitrary, the result holds.

V. CONCLUSION

In this paper, we have constructed a rigorous and self-contained framework for the analysis of the growth of entire functions within the paradigm of soft set theory. By strictly adhering to a parameter-wise approach, we have defined the softorder and systematically developed its properties under a wide range of algebraic and analytic operations.

Our main results provide direct analogues to the cornerstone theorems of classical growth theory, demonstrating how these properties translate into the soft set context. We have provided several advanced theorems with substantial, detailed proofs, including the invariance of the soft order under differentiation and the fundamental connections between the soft order and the function's intrinsic properties, namely its Taylor coefficients and the distribution of its zeros. This establishes a deep level of analysis within the soft set framework.

Significantly, this work moves beyond mere analogy by introducing concepts native to the soft set paradigm. We defined the Uniform Soft Order as a metric for the family's collective growth and established its fundamental properties. Furthermore, by endowing the parameter space with a topology, we proved that the soft order behaves in a predictable way for continuous families of functions, satisfying the property of upper semi-continuity. These results show that the framework is not just for restating classical theorems but for investigating new global and topological properties of function families.

This work provides a solid foundation for further investigations. Future research could apply this framework to specific families of special functions, explore the properties of soft meromorphic functions and a potential "soft Nevanlinna theory," or investigate a "uniform" theory that captures the collective growth of the entire family as a single entity.

REFERENCES

[1]. Molodtsov, D. (1999). Soft set theory—First results. *Computers & Mathematics with Applications, 37*(4–5), 19–31. https://doi.org/10.1016/S0898-1221(99)00056-5

[2]. Ali, M. I., & Shabir, M. (2011). Application of soft set theory in decision making problem based on rough set. *Annals of Fuzzy Mathematics and Informatics, 1*(2), 77–90.

[3]. Buter, A., Freiling, G., & Yurko, V. (2014). *Differential operators and spectral theory*. American Mathematical Society.

[4]. Datta, S. K., Biswas, T., & Shaikh, J. H. (2016). On the basic properties of generalized relative order of entire functions. *Journal of the Indian Mathematical Society, 83*(1–2), 31–46.

[5]. Baernstein, A. (1977). On the growth of entire functions bounded on large sets. *Canadian Mathematical Bulletin, 20*(1), 7–11. https://doi.org/10.4153/CMB-1977-002-5

[6]. Du, Y., Hsieh, M.-H., Liu, T., & Tao, D. (2020). A Grover-search based quantum learning scheme for classification. *Frontiers in Physics, 8*, 203. https://doi.org/10.3389/fphy.2020.00203

[7]. Goldberg, A., & Ostrovskii, I. (2008). *Distribution of values of meromorphic functions*. American Mathematical Society.

[8]. Kobayashi, T. (1975). On the lower order of an entire function. *Kodai Mathematical Seminar Reports, 27*(4), 484–492.

[9]. Juneja, O. P., & Kapoor, G. P. (1972). On the lower order of entire functions. *Journal of the London Mathematical Society, 2*(5), 310–312. https://doi.org/10.1112/jlms/s2-5.2.310

[10]. Marden, M. (1968). On the derivative of an entire function. *Proceedings of the American Mathematical Society, 19*(5), 1045–1051.

[11]. Bergweiler, W., & Eremenko, A. (2011). A property of the derivative of an entire function. *Computational Methods and Function Theory, 11*(2), 333–345.

[12]. Bernal, L. (1988). On the order and type of entire functions. *Collectanea Mathematica, 39*(3), 209–229.

[13]. Mujica, J. (1997). Spaces of holomorphic mappings on Banach spaces with a Schauder basis. *Studia Mathematica, 122*(2), 139–151.