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Magic Squares with Specified Central Square

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ABSTRACT - A template for a magic square (MS) of order 4, which has four specified numbers located at the centre, and the sum (S) of the numbers of six 2×2 top 3 and bottom 3 squares and the four rows is the same, is derived. It has two independent variables which provides the flexibility to obtain different MSs. Interestingly, it also gives the same S for the four columns, two diagonals, four corners, broken diagonals, top and bottom middle squares, left and right middle rows. Conditions are derived so that all the cells have distinct non-negative numbers. An MS does not exist when S is less than 30. MSs with any desired sum can be derived from an MS of known S. This method can also be used to find MSs when only S is specified.

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I. INTRODUCTION

Magic squares (MSs) have intrigued mathematicians and hobbyists for centuries due to their harmonious numerical properties. An ordinary MS has the same sum of elements in every row, column, and the two main diagonals. Construction of such MSs of size 4×4 is well-documented when the elements are located in the *first row* [1]-[3]. In this paper, we deal with the MSs when the elements are located at the *central square*. Several MSs can be determined from this template by varying just two independent variables.

II. Template

2.1 Specified conditions

A general square matrix of order 4cnabe expressed as

$$M_S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}. \tag{1}$$

The four central cells are specified as

$$S_{22} = a;\ S_{23} = b;\ S_{32} = c;\ S_{33} = d, \tag{2}$$

where

$$a + b + c + d = S. \tag{3}$$

The S is known as magic number (MN). Let us assume

$$S_{11} = x \geq 0 \tag{4}$$

and

$$S_{12} = y \geq 0 \tag{5}$$

Then Equation (1) becomes

$$M_S = \begin{bmatrix} \boldsymbol{x} & \boldsymbol{y} & S_{13} & S_{14} \\ S_{21} & \boldsymbol{a} & \boldsymbol{b} & S_{24} \\ S_{31} & \boldsymbol{c} & \boldsymbol{d} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}. \tag{6}$$

Let the top 3 and bottom 3 horizontal 2×2 squares satisfy the conditions

$$x + y + S_{21} + a = S \tag{7}$$

$$y + S_{13} + b + a = S \tag{8}$$

$$S_{13} + S_{14} + b + S_{24} = S \tag{9}$$

$$S_{31} + c + S_{41} + S_{42} = \mathrm{S} \tag{10}$$

$$c + d + S_{42} + S_{43} = S \tag{11}$$

$$d + S_{34} + S_{43} + S_{44} = S \tag{12}$$

and the 4 rows satisfy the conditions

$$x + y + S_{13} + S_{14} = S \tag{13}$$

$$S_{21} + a + b + S_{24} = S \tag{14}$$

$$S_{31} + c + d + S_{34} = S \tag{15}$$

$$S_{41} + S_{42} + S_{43} + S_{44} = S. \tag{16}$$

2.2 Proposed Template

Solving Equations (7)-(16), we get

$$\begin{aligned} &\boldsymbol{S_{13}} = (c+d) - y,\ S_{14} = (a+b) - x,\ \boldsymbol{S_{21}} = (b+c+d) - (x+y) \\ &\boldsymbol{S_{24}} = (x+y) - b, \boldsymbol{S_{31}} = (a+b-d) - (x-y),\ \boldsymbol{S_{34}} = d + (x-y) \\ &\boldsymbol{S_{41}} = x - (b-d),\ \boldsymbol{S_{42}} = (b+d) - y,\ \boldsymbol{S_{43}} = y - (d-a),\ \boldsymbol{S_{44}} = (b+c) - x. \end{aligned} \tag{17}$$

Substituting these values in Equation (6), we get the template

$$T_S = \begin{bmatrix} x & y & (c+d) - y & (a+b) - x \\ (b+c+d) - (x+y) & a & b & (x+y) - b \\ (a+b-d) - (x-y) & c & d & d + (x-y) \\ x - (b-d) & (b+d) - y & y - (d-a) & (b+c) - x \end{bmatrix}. \tag{18}$$

It is interesting to note that this template also satisfies the sum properties for the columns, diagonals, corners, broken diagonals, middle numbers of top and bottom rows, middle numbers of left and right columns, irrespective of x and y. The last four satisfy the conditions

$$S_{21} + S_{12} + S_{43} + S_{34} = S \tag{19}$$

$$S_{31} + S_{42} + S_{13} + S_{24} = S \tag{20}$$

$$S_{12} + S_{13} + S_{42} + S_{43} = S \tag{21}$$

$$S_{21} + S_{24} + S_{31} + S_{34} = S. \tag{22}$$

2.3 Constraints

There is always a square possible for any $\{x, y\}$. However, the numbers may be negative and/or repetitive. In order to have all the cell values to be *non-negative*, the conditions from Equation (18) are

$$\max\{0, (b-d)\} \leq x \leq \min\{(a+b), (b+c)\} \tag{23}$$

$$\max\{0,\ (d-a)\} \leq y \leq \min\{(c+d), (b+d)\} \tag{24}$$

$$-d \leq (x-y) \leq (a+b-d), \tag{25}$$

$$b \leq (x+y) \leq (b+c+d) \tag{26}$$

Conditions for *distinct numbers* from Equation (18) are

$$S_{ij} \neq x, y, a, b, c, d, S_k, \qquad k \neq ij = 13,14,21,24,31,34,41,42,43,44. \tag{27}$$

2.4 Procedure

1. Insert the values of a, b, c, d in Equation (18).
2. Find the constraints on x and y from Equations (23)-(26). This will give all MSs with non-negative numbers.
3. Find all the permissible pairs $\{x, y\}$
4. Find MS for any one $\{x, y\}$, or for all $\{x, y\}$, if all valid MSs are required.

Example 1: Let $\{a, b, c, d\} = \{22, 12, 18, 87\}$. Substituting these values into Equation (18), we get

$$M_{139} = \begin{bmatrix} x & y & 105-y & 34-x \\ 117-(x+y) & 22 & 12 & (x+y)-12 \\ -53-(x-y) & 18 & 87 & 87+(x-y) \\ x+75 & 99-y & y-65 & 30-x \end{bmatrix}. \tag{28}$$

From Equations (23)-(26), constraints are

$$(x,y) \neq a,b,c,d \tag{29}$$

$$0 \leq x \leq 30 \tag{30}$$

$$\max\{65, x+53\} \leq y \leq \min\{99, 117-x,\ \ 87+x\} \tag{31}$$

$$\rightarrow\ 65 < y < P. \tag{32}$$

Here P varies from 87 to 99 as x takes the values from 0 to 30. For $\{x, y\} = \{0, 80\}, \{10, 90\}, \{10, 96\}$, MSs are

$$M_{139} = \begin{bmatrix} 00 & 80 & 25 & 34 \\ 37 & 22 & 12 & 68 \\ 27 & 18 & 87 & 07 \\ 75 & 19 & 15 & 30 \end{bmatrix}, \begin{bmatrix} 10 & 90 & 15 & 24 \\ 17 & 22 & 12 & 88 \\ 27 & 18 & 87 & 07 \\ 85 & 09 & 25 & 20 \end{bmatrix}, \begin{bmatrix} 10 & 96 & 09 & 24 \\ 11 & 22 & 12 & 94 \\ 33 & 18 & 87 & 01 \\ 85 & 03 & 31 & 20 \end{bmatrix}. \tag{33}$$

Note that these are alternative MSs to Ramanujan's birthday [4] where the birthdate is located in the first row instead of the central square.

To find all valid MSs, use the template for all possible $\{x, y\}$ and discard those which have duplicate numbers.

Example 2: Let $\{a, b, c, d\} = \{1, 2, 20, 0\}$. Since $S < 30$, the tuple cannot be realized with all DNN numbers. It can be verified that it does not satisfy the conditions of Equations (23)-(26).

Example 3: Let $\{a, b, c, d\} = \{100, 200, 300, 400\}$. This is an example of 3-digit numbers including 0. Here, the constraints are

$$0 \leq x < 300,\ \ x \neq 100, 200, y \tag{34}$$

$$\max(300, x+100, 200-x) \leq y \min(600, x+400, 900-x), \qquad y \neq x \tag{35}$$

For $\{x, y\} = \{0, 399\}, \{10, 305\}$ MSs are

$$M_{1000} = \begin{bmatrix} 000 & 399 & 301 & 300 \\ 501 & 100 & 200 & 199 \\ 299 & 300 & 400 & 001 \\ 200 & 201 & 099 & 500 \end{bmatrix}, \begin{bmatrix} 010 & 305 & 395 & 290 \\ 585 & 100 & 200 & 115 \\ 195 & 300 & 400 & 105 \\ 210 & 295 & 005 & 490 \end{bmatrix}. \tag{36}$$

Example 4: Let $\{a, b, c, d\} = \{0, 15, 3, 12\}$. Constraints are

$$3 < x < 15, x \neq y, \tag{37}$$

$$12 < y < 15, y \neq x. \tag{38}$$

For $\{x, y\} = \{7, 13\}, \{7, 14\}, \{11, 14\}$, MSs are

$$M_{30} = \begin{bmatrix} 07 & 13 & 02 & 08 \\ 10 & 00 & 15 & 05 \\ 09 & 03 & 12 & 06 \\ 04 & 14 & 01 & 11 \end{bmatrix}, \begin{bmatrix} 07 & 14 & 01 & 08 \\ 09 & 00 & 15 & 06 \\ 10 & 03 & 12 & 05 \\ 04 & 13 & 02 & 11 \end{bmatrix}, \begin{bmatrix} 11 & 14 & 01 & 04 \\ 05 & 00 & 15 & 10 \\ 06 & 03 & 12 & 09 \\ 08 & 13 & 02 & 07 \end{bmatrix}. \tag{39}$$

2.5 Shift to Corner Theorem: If a, b, c, d are interchanged with $S_{11}, S_{14}, S_{41}, S_{44}$, respectively, in T_S of Equation (18), the new template T'_S will continue to satisfy all the sum properties of T_S if

$$x = (a+b+c-d)/2. \tag{40}$$

Proof: Let T_s be the original magic template and T'_s be a new template after exercising the shift to corner theorem (SCT). Using the template T_s, we get

$$T'_S = \begin{bmatrix} x' & y' & (c'+d')-y' & (a'+b')-x' \\ (b'+c'+d')-(x'+y') & a' & b' & (x'+y')-b' \\ (a'+b'-d')-(x'-y') & c' & d' & d'+(x'-y') \\ x'-(b'-d') & (b'+d')-y' & y'-(d'-a') & (b'+c')-x' \end{bmatrix}. \tag{41}$$

After shifting, we have

$$x' = a,\ y' = y \tag{42}$$

$$a' = x,\ b' = (a + b - x),\ c' = (x - b + d),\ d' = (b + c - x) \tag{43}$$

Substituting these values in Equation (41), we get

$$T'_S = \begin{bmatrix} a & y & c+d-y & b \\ b+c+d-(x+y) & x & (a+b)-x & x+y-b \\ x-c+y & x-(b-d) & (b+c)-x & a+b+c-x-y \\ c & a+2b+c-2x-y & y-b-c+2x & d \end{bmatrix}. \tag{44}$$

This template satisfies all the sum properties present in the original MS. However, S_{31}, S_{42}, S_{43}, S_{34} have assumed different values than those in T_s. Equating these values to the respective original values and solving, fortunately we get a single condition of Equation (40). Thus, the theorem is proved.

This value of x will be represented as x_c (critical value of x).

Example 5: Let $\{a, b, c, d\} = \{10, 30, 45, 15\}$, Here, $S = 100$. $x_c = 35$. Using the templates of T_s (Equation 18) and T'_s (Equation 44) with $x = 34, 35, 37$, and fixed $y = 12$, we get the transformed MSs

$$M_{100} = \begin{bmatrix} 34 & 12 & 48 & 06 \\ 44 & 10 & 30 & 16 \\ 03 & 45 & 15 & 37 \\ 19 & 33 & 07 & 41 \end{bmatrix} \rightarrow M'_{100} = \begin{bmatrix} 10 & 12 & 48 & 30 \\ 44 & 34 & 06 & 16 \\ 01 & 19 & 41 & 39 \\ 45 & 35 & 05 & 15 \end{bmatrix} \tag{45}$$

$$M_{100} = \begin{bmatrix} 35 & 12 & 48 & 05 \\ 43 & 10 & 30 & 17 \\ 02 & 45 & 15 & 38 \\ 20 & 33 & 07 & 40 \end{bmatrix} \rightarrow M'_{100} = \begin{bmatrix} 10 & 12 & 48 & 30 \\ 43 & 35 & 05 & 17 \\ 02 & 20 & 40 & 38 \\ 45 & 33 & 07 & 15 \end{bmatrix} \tag{46}$$

$$M_{100} = \begin{bmatrix} 37 & 12 & 48 & 03 \\ 41 & 10 & 30 & 19 \\ 00 & 45 & 15 & 40 \\ 22 & 33 & 07 & 38 \end{bmatrix} \rightarrow M'_{100} = \begin{bmatrix} 10 & 12 & 48 & 30 \\ 41 & 37 & 03 & 19 \\ 04 & 22 & 38 & 36 \\ 45 & 29 & 11 & 15 \end{bmatrix}. \tag{47}$$

Note that only when $x_c = 35$, all the elements (except corners and central square) remain the same.
The theorem is valid even if there is a repetition of numbers (See example 6).

Example 6: Let $\{a, b, c, d\} = \{13, 10, 15, 06\}$, Here, $S = 44$. $x_c = 16$.

$$M_{44} = \begin{bmatrix} 16 & \mathbf{12} & \mathbf{09} & 07 \\ 03 & \mathbf{13} & \mathbf{10} & 18 \\ \mathbf{13} & 15 & 06 & \mathbf{10} \\ \mathbf{12} & 04 & 19 & \mathbf{09} \end{bmatrix} \leftrightarrow M'_{44} = \begin{bmatrix} 13 & 12 & 09 & 10 \\ 03 & 16 & 07 & 18 \\ 13 & 12 & 09 & 10 \\ 15 & 04 & 19 & 06 \end{bmatrix}. \tag{48}$$

If we apply the SCT again to M'_S, we will get back the original M_S.

2.6 Minimum realizable value of *S*

There are 16 cells, and each has a DNN value; for minimum sum, these cell values must be from 0 to 15. (Unlike earlier authors, we have considered 0 also as one of the values. Therefore, their numbers were from 1 to 16, and $S = 34$). The total sum is $(15\times16)/2 = 120$. Since the sum of each row is S, therefore minimum value of $S = 120/4 = 30$. Thus, the realization of a tuple is only possible when $S \geq 30$.

2.7 Number of possible MSs

From eqn. (18), note that different $\{x, y\}$ yield different templates. However, it is a tedious procedure to find all valid squares analytically. It is recommended to find by hand calculations or use a computer programming.

III. Applications

3.1 Extensions of the method

The method is applicable to any four numbers, such as some date. Also, a similar method can be developed for the tuple numbers identified at different locations of the MS.

One can choose a fixed value for *x* (*y*), and get various MSs by varying *y* (*x*) alone. However, a MS is good enough, unless one has some particular requirement. These MSs can be used for decorating tiles, wallpaper, screens, sending greetings, etc.

3.2 Deriving MSs for different *S*

Example 7: Let {*a, b, c, d*} = {0, 7, 10, 13}. In this case, the constraints are:

$$0 \le x \le 7 \tag{49}$$

$$13 \le y \le (x + 13), y \ne x \tag{50}$$

One possible MS is

$$M_{30} = \begin{bmatrix} 06 & 15 & 08 & 01 \\ 09 & 00 & 07 & 14 \\ 03 & 10 & 13 & 04 \\ 12 & 05 & 02 & 11 \end{bmatrix}. \tag{51}$$

This MS has same *S* = 30 for all rows, columns, diagonals, 4 corners, top (3) and bottom (3) horizontal 2×2 squares, broken diagonals, top and bottom middle squares, left and right middle rows. There are 4 sets of numbers: {0, 1, 2, 3}, {4, 5, 6, 7}, {8, 9, 10, 11}, {12, 13, 14, 15}. Each set has 4 numbers distributed in different rows, different columns, and one of them on a diagonal.

To get the MS for any integer 30 + *n*, add *n* to all the numbers in the set that has highest 4 numbers (12,13,14,15).

Example 8: From $M_{30,}$

$$M_{30} = \begin{bmatrix} 06 & \mathbf{15} & 08 & 01 \\ 09 & 00 & 07 & \mathbf{14} \\ 03 & 10 & \mathbf{13} & 04 \\ \mathbf{12} & 05 & 02 & 11 \end{bmatrix} \rightarrow M_{31} = \begin{bmatrix} 06 & \mathbf{16} & 08 & 01 \\ 09 & 00 & 07 & \mathbf{15} \\ 03 & 10 & \mathbf{14} & 04 \\ \mathbf{13} & 05 & 02 & 11 \end{bmatrix}. \tag{52}$$

This is also applicable to any MS. For example,

$$M_{139} = \begin{bmatrix} 10 & \mathbf{70} & 35 & 24 \\ 37 & 22 & 12 & \mathbf{68} \\ 07 & 18 & \mathbf{87} & 27 \\ \mathbf{85} & 29 & 05 & 20 \end{bmatrix} \rightarrow M_{149} = \begin{bmatrix} 10 & \mathbf{80} & 3\ 5 & 24 \\ 37 & 22 & 12 & \mathbf{78} \\ 07 & 18 & \mathbf{97} & 27 \\ \mathbf{95} & 29 & 05 & 20 \end{bmatrix}. \tag{53}$$

For lowest *S*, max value of *n* can be such that MS does not have a single negative value. For example, for above M_{139}, the maximum *n* could be -68. Thus, the lowest value of *S* will be 139 - 68 = 71. Thus, the MS with the lowest possible *S* = 71 is

$$M_{71} = \begin{bmatrix} 10 & \mathbf{02} & 35 & 24 \\ 37 & 22 & 12 & \mathbf{00} \\ 07 & 18 & \mathbf{19} & 27 \\ \mathbf{17} & 29 & 05 & 20 \end{bmatrix}. \tag{54}$$

If only a sum *S* is specified, it can be expressed as a sum of 4 DNN numbers. There are many such tuples possible. Three realizations for *S* = 257 are

$$\begin{bmatrix} \mathbf{007} & 197 & 042 & 011 \\ 012 & 041 & 200 & 004 \\ 201 & 009 & \mathbf{007} & 040 \\ 037 & 010 & 008 & 202 \end{bmatrix}, \begin{bmatrix} 023 & 033 & 051 & 150 \\ 058 & 143 & 030 & 026 \\ 123 & 024 & 060 & 050 \\ 053 & 057 & 116 & 031 \end{bmatrix}, \begin{bmatrix} 032 & 198 & 012 & 015 \\ 020 & 007 & 040 & 190 \\ 013 & 010 & 200 & 034 \\ 192 & 042 & 005 & 018 \end{bmatrix}. \tag{55}$$

The first one viral on social media has 007 repeated [4]. As demonstrated, it appears that *S* must have 3 digits, no one can be 0.

IV. Conclusion

A template for a MS of order 4, which has four specified numbers located at the centre, is derived. It has two independent variables which provide the flexibility to generate different squares. It also gives the same sum for all rows, columns, diagonals, 4 corners, top (3) and bottom (3) horizontal 2×2 squares, top and bottom middle squares, left and right middle rows and broken diagonals. It is shown how to obtain the MSs for any desired sum from a given MS. This method is applicable to find MSs also when only the sum is specified with any number of digits including 0.

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