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Rayleigh Wave propagation in fibre-reinforced semi-infinite composite medium

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Abstract
Self-reinforced composite medium is the basic requirement for civil engineering construction because self-reinforced composite have become very important in many engineering applications due to their superiority over other structural materials requiring high strength and stiffness in lightweight components. Surface wave propagation encounters these types of constructions medium composites during earthquakes and other similar disturbances. Therefore, the study of surface wave propagation in such type of composite medium is of great importance. The present chapter aims to study the effect of propagation of Rayleigh wave in a fibre-reinforced semi-infinite composite medium. Numerical computation has been carried out to obtain secular equation for the propagation of Rayleigh type wave in the closed form.
Keywords: *Reinforced, Propagation, Composite, Secular Equation, Stresses*

I. Introduction

The study of earthquake effect on artificial structures is of prime importance to engineers and architects. During an earthquake and disturbances similar to it, a structure is excited into a more or less violent vibration, with resulting oscillatory stresses, which depend upon both physical properties of the structure and the ground vibration. Waves generated by earthquakes may travel thousands of miles. The study surface waves in an anisotropic medium with different boundaries are of great importance to seismologists as well as to geophysicists to understand and predict the seismic behaviour at the different margins of earth.

The study of mechanical behaviour of a self-reinforced material has great importance in geomechanics. Many elastic fibre-reinforced composite materials are strongly anisotropic in behaviour. It is desirable to study the shear wave propagation in anisotropic media, as the propagation of elastic waves in anisotropic media is fundamentally different from their propagation in isotropic media. Fibre-reinforced material has two main components, namely concrete and steel. They act together as a single anisotropic unit up to the elastic limit, i.e. the two components are bound together so that there can be no relative displacement between them. Reinforcement is a part of composite material that provides strength, stiffness and the ability to carry a load. The idea of continuous self-reinforcement at every point of an elastic solid was introduced by Belfield et al. (1986). When these types of rocks come in the way of seismic waves influence its propagation. So, in earthquake engineering, mining engineering, civil engineering, architecture and seismology the propagation of surface waves in anisotropic elastic media is of considerable importance. Sengupta and Nath (2001) investigated the surface waves in fibre-reinforced anisotropic elastic media. The reflection of quasi-P and quasi-SV waves at the free and rigid boundaries of a fibre-reinforced medium studied by Chattopadhyay et al. (2002). Chattopadhyay et al. (2013) studies the dispersion of horizontally polarized shear waves in an irregular non-homogeneous self-reinforced crustal layer over a semi-infinite self-reinforced medium. Baljeet (2005) discussed the wave propagation in thermally conducting linear fibre-reinforced composite materials. Baljeet (2007) studied the wave propagation in an incompressible transversely isotropic fibre-reinforced elastic media. Sapan and Ranjan (2011) investigated the surface wave propagation in fibre-reinforced anisotropic elastic layer between liquid saturated porous half space and uniform liquid layer. Sapan and Ranjan (2013) studied the propagation of Love waves in the fiber-reinforced layer over a gravitating porous half-space.

In this chapter we have studied the propagation of Rayleigh type surface wave in a fibre-reinforced half-space underlying an inviscid liquid layer has been studied. Secular equations for the propagation of Rayleigh type surface waves in fibre-reinforced media under the influence of liquid loading have been derived. The dispersion curves depicting the effects of fluid loading are presented graphically.

II. Formulation and solution of the problem

The dynamical equations of motion for a three-dimensional fibre-reinforced semi-infinite composite medium

$$\frac{\partial \tau_{xx}\big|_f}{\partial x}+\frac{\partial \tau_{xy}\big|_f}{\partial y}+\frac{\partial \tau_{xz}\big|_f}{\partial z}=\rho\frac{\partial^2 u_f}{\partial t^2}, \qquad (1)\ \frac{\partial \tau_{yx}\big|_f}{\partial x}+\frac{\partial \tau_{yy}\big|_f}{\partial y}+\frac{\partial \tau_{yz}\big|_f}{\partial z}=\rho\frac{\partial^2 v_f}{\partial t^2},$$

$$(2)\ \frac{\partial \tau_{zx}\big|_f}{\partial x}+\frac{\partial \tau_{zy}\big|_f}{\partial y}+\frac{\partial \tau_{zz}\big|_f}{\partial z}=\rho\frac{\partial^2 w_f}{\partial t^2}, \qquad (3)$$

where $\frac{\partial w_f}{\partial z}=-\left(\frac{\partial u_f}{\partial z}+\frac{\partial v_f}{\partial y}\right)$, ρ is the density of the material medium and $\tau_{ij}\big|_f=\tau_{ji}\big|_f$, are the stress components.

The constitutive equations used in a fibre-reinforced linearly elastic model are (Belfield, 1983)

$$\tau_{ij}\big|_f=\lambda e_{kk}\delta_{ij}+2\mu_T e_{ij}+\alpha\left(a_k a_m e_{km}\delta_{ij}+e_{kk}a_i a_j\right)+2\left(\mu_L-\mu_T\right)\left(a_i a_k e_{kj}+a_j a_k e_{ki}\right)+\beta\left(a_k a_m e e_{km}a_i a_j\right), \qquad (4)$$

where $\tau_{ij}\big|_f$ are components of stress and e_{ij} are components of infinitesimal strain and $2e_{ij}=u_{i,j}+u_{j,i}$, $a_i=(a_1,a_2,a_3)$, λ, μ_T are elastic constants and $\alpha,\beta,(\mu_L-\mu_T)$ are reinforced elastic constants. Here $a_1^2+a_2^2+a_3^2=1$.

If $\vec{a}$ is chosen so that its components are $(1,0,0)$, so that the preferred direction is everywhere that of x - axis, then (4) becomes:

$$\begin{aligned}
\tau_{xx}\big|_f&=(\lambda+2\alpha+4\mu_L-2\mu_T+\beta)e_{xx}+(\lambda+\alpha)e_{yy}+(\lambda+\alpha)e_{zz},\\
\tau_{yy}\big|_f&=(\lambda+\alpha)e_{xx}+(\lambda+2\mu_T)e_{yy}+\lambda e_{zz},\\
\tau_{zz}\big|_f&=(\lambda+\alpha)e_{xx}+\lambda e_{yy}+(\lambda+2\mu_T)e_{zz},\\
\tau_{yz}\big|_f&=2\mu_T e_{yz},\\
\tau_{xz}\big|_f&=2\mu_L e_{xz},\\
\tau_{xy}\big|_f&=2\mu_L e_{xy}.
\end{aligned} \qquad (5)$$

Using eqns. (5) in eqns. (1), (2) and (3), the equation of motion for fibre-reinforced semi-infinite medium becomes

$$P_1\frac{\partial^2 u_f}{\partial x^2}+P_2\frac{\partial^2 w_f}{\partial x\partial z}+\mu_L\frac{\partial^2 u_f}{\partial z^2}=\rho\frac{\partial^2 u_f}{\partial t^2}, \qquad (6)$$

$$\mu_L\frac{\partial^2 w_f}{\partial x^2}+P_2\frac{\partial^2 u_f}{\partial x\partial z}+P_3\frac{\partial^2 w_f}{\partial z^2}=\rho\frac{\partial^2 v_f}{\partial t^2}. \qquad (7)$$

where $P_1=\left(\lambda+2\alpha+4\mu_L-2\mu_T+\beta\right)$, $P_2=\left(\alpha+\lambda+\mu_L\right)$, $P_3=\left(\lambda+2\mu_T\right)$.

We may assume the solution of eqns. (6) and (7) as

$$\begin{aligned}
u_f&=Be^{-skz+ik(x-Vt)},\\
w_f&=Ce^{-skz+ik(x-Vt)}.
\end{aligned} \qquad (8)$$

Using eqns. (8) in eqns. (6) and (7), we get

$$\left(s^2\mu_L+\rho V^2-P_1\right)B+\left(-sP_2\right)iC=0, \qquad (9)$$

$$\left(-sP_2\right)iB+\left(s^2P_3+\rho V^2-\mu_L\right)C=0. \qquad (10)$$

For the non-trivial solution of (9) and (10)

$$\mu_L P_2 s^4+M_1 s^2+N_1=0,$$

$$s_j^2 = \frac{-M_1 \pm \sqrt{M_1^2 - 4\mu_L P_2 N_1}}{2\mu_L P_2}, \left(j = 1,2\right) \tag{11}$$

where $M_1 = \mu_L\left(\rho V^2 - \mu_L\right) + P_3\left(\rho V^2 - P_1\right) - P_2^2$, $N_1 = \left(\rho V^2 - P_1\right)\left(\rho V^2 - \mu_L\right)$.

Eqns. (9) and (10) yield

$$\frac{w_f}{u_f} = \frac{s_j^2\mu_L + \rho V^2 - P_1}{i\left(s_j P_2\right)} = \eta_j \left(j = 1,2\right). \tag{12}$$

In view of eqns. (11) and (12) displacement components are given by

$$u_f = \left(B_1 e^{-s_1 kz} + B_2 e^{-s_2 kz}\right) e^{ik(x-Vt)}, \tag{13}$$

and

$$w_f = \left(\eta_1 B_1 e^{-s_1 kz} + \eta_2 B_2 e^{-s_2 kz}\right) e^{ik(x-Vt)}. \tag{14}$$

2.1 Boundary conditions

The boundary conditions for the problem are:

(i) The magnitude of normal component of the stress tensor of the solid should be zero. This leads to

$$\tau_{zz}\big|_f = 0, \text{ at } z = 0. \tag{15}$$

(ii) The tangential component of the stress tensor should be zero. This implies that $\tau_{zx}\big|_f = 0, \text{ at } z = 0.$ (16)

2.2 Secular equation

Using eqns. (13) and (14) on boundary conditions (15) and (16), we get following wave velocity equation for Rayleigh type surface waves in a fibre- semi-infinite composite medium:

$$\left(\xi_2\xi_5 - \xi_3\xi_4\right) = 0, \tag{17}$$

where

$$\beta_1^2 = \frac{\mu_L}{\rho}, \xi_2 = i\left(\lambda + \alpha\right) - \eta_2 s_2\left(\lambda + 2\mu_T\right), \xi_3 = i\left(\lambda + \alpha\right) - \eta_1 s_1\left(\lambda + 2\mu_T\right),$$

$$\xi_4 = \left(\eta_1 i - s_1\right)\mu_L, \xi_5 = \left(\eta_2 i - s_2\right)\mu_L.$$

III.Conclusion

Fibre-reinforced composite materials have become very attractive in many engineering applications recently due to their superiority over other structural materials in applications requiring high strength and stiffness in lightweight components. Consequently, the characterisation of their mechanical behaviour is of particular importance for structural design using these materials. As the Earth's crust and mantle are non-homogeneous and irregular in nature, so the findings may play a vital role to understand the cause of damage attributable to earthquake. Effects of earthquakes on artificial structures are of prime importance to engineers and architects. The considered problem can be used to model different problems of seismology, civil engineering, submarine engineering etc. It is more realistic model for earth crust.

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