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A Theoretical Approach for Measuring the Technical Efficiency of Decision Making Units

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ABSTRACT : *The data envelopment analysis (DEA) as formulated by Bankar,Cooper and Charnes (BCC) gives input and output targets to the decision making units. A major deficiency of DEA is the DMU_0 under evaluation chooses multiplier weights most advantageous to it. To get out of this problem the multi-objective DEA problems have to be solved. In this approach we minimize the maximum deviation or sum of all the deviations. This approach identifies not as much of numbers of Decision Making Units (DMUs) as efficient. The efficiencies are evaluated exposing all the DMUs to the same light. Any entrepreneur suffers from input technical inefficiency. The potential inputs of an inefficient decision making unit can be determined estimating his input distance shall be set as an empirical approach.*

KEYWORDS: *Measuring Efficiency of Production, Output distance function, Measurement of Technical Efficiency.*

I. INTRODUCTION

Efficiency estimation dates back to M.J. Farrel, who introduced the notions of technical, allocate and cost efficiencies. He decomposed the cost efficiency into the product of technical and allocates efficiencies. His measures were conditioned on scale efficiency of production units. Shephard, R.W. proposed an axiomatic approach to measure productive efficiency. The fundamental concepts of his study were,

[1] input level set : L(u)
[2] output level set : P(x)
[3] graph : G(x, u)

The input level set L (u) is the collection of all input vectors which can product the output vector u.

$L(u) = \{x : x \text{ produces } u\}$

The output level set P(x) is the collection of all output vectors u which can be produced by the input vector x.

$P(x) = \{u: u \text{ produced by } u\}$

The graph of the production technology is defined as,

$G(x,u) = \{(x, u) : x \in L(u)\}$
$= \{(x, u): u \in P(x)\}$

The technology is assumed to be piece wise linear.

Empirical implementation of productive efficiency estimation is due to Charnes, Cooper and Rhodes (CCR), who proposed a fractional programming problem to measure input technical efficiency. However, the fractional programming problem can be reduced to a linear programming problem by Charnes, Cooper transformation. Since this problem is always feasible, optimal solution exists. The optimal value of the objective function is the estimate of input technical efficiency. Historically, many modifications were made to this problem. Economic data is frequently subjected to returns to scale. To identify the nature of returns to scale such as, increasing or decreasing or constant a constant is added to the objective function. Depending on the sign and magnitude of these constant returns to scale are determined as increasing or decreasing or constant. For example, if this constant is zero in all the alternative optimal solutions of CCR linear programming, then returns to scale are constant.The dual of CCR problem is due to Banker, Charness and Cooper (BCC), called the envelopment problem. Variable returns to scale can be modeled into the BCC envelopment problem by means of the convexity constraint on the intensity parameters.Choice of inputs and outputs of a production technology is crucial in efficiency estimation. Inclusion of more and more inputs and outputs lead to the loss of degrees of freedom. Consequently, the efficient DMUs will increase in number. In the scenario of choice of several inputs and outputs, by including more production units the reliability of efficiency estimates can be improved.

If the production units are not adequately large in number, sensitivity analysis is desirable. The data envelopment analysis as formulated by BCC gives input and output targets to the decision making units. These targets are pro rata. For an inefficient DMU inputs are decreased and outputs are increased pro rata. Pro rata input targets reveal input losses and output targets yield output losses of a decision making unit whose efficiency is under evaluation. For a DMU it is desirable to reduce one input to its minimum possible level at the expense of other inputs, or to increase an output to its maximum possible level at the expense of other outputs. These pre-emptive priorities are introduced in input reduction and / or output augmentation. The BCC problem and its dual CCR problem fail to accommodate the above variation. Thus, a modification to the BCC problem is desirable.The multiplier weights of the CCR problem are such that in the optimal solution some of them may emerge to be zero, undermining the importance of the corresponding inputs and outputs in determining the efficiency. The flexibility of the weights can be reduced by the 'Assurance Region Approach'. For example, when two inputs are combined to produce one output and if the input markets are perfectly competitive, the producer attains equilibrium when the ratios of marginal products are equal to the ratio of prices. By constraining the marginal products to fall between two positive numbers not specified a priori, we augment additional constraints on CCR multiplier weights, which will yield subsequently positive weights of inputs and outputs.

A major deficiency of DEA is the DMU_0 under evaluation chooses multiplier weights most advantageous to it. To get out of this problem the multi-objective DEA problems have to be solved. In this approach we minimize the maximum deviation or sum of all the deviations. This approach identifies fewer numbers of DMUs as efficient. The efficiencies are evaluated exposing all the DMUs to the same light.The constraint space of the CCR multiplier problem changes from one DMU to another DMU, as such the multiplier weights of one problem emerge to be different from the weights of another problem. It is desirable, sometimes, a common set of weights to all the decision making units. Under the hypothesis of 'benevolent attitude', 'aggressive attitude' of the DMU manager a common set of weights, hence global efficiency can be derived for the decision making units.

II. MEASURING EFFICIENCY OF PRODUCTION

To measure efficiency of production, three prominent approaches used are,

[1] the input approach, where output is held constant and inputs are reduced proportionately,

[2] the output approach where input vector is held constant and outputs are augmented pro rata.

[3] the graph approach, where possible input reduction and output augmentation are enquired simultaneously.

If the decision making units (DMU) cannot be adjust its inputs, it cannot be expand the plant size if a producer can asks. For further expansion is possible only in a short run of output approach. To estimate the efficiency of a production function on handling multi-inputs and multi-outputs with contented, the output technical efficiency can be measured with Cobb-Douglas, Zellnear-Revanker Frontier functions etc.,

III. OUTPUT DISTANCE FUNCTION

If a decision making unit (DMU) cannot adjust its inputs which is possible in short run as it cannot expand its plant size, the producer is asked if further output expansion is possible. An affirmative answer reveals that the producer is inefficient. To estimate his potential output vector the output distance function is used as a basic tool. Let P(x) is an output level set defined as, P(x) = {u: x producer u}. P(x) consists of the output vectors which can be producer by x. The set P(x) consists of efficient, weakly efficient and inefficient output vectors. The output distance function is defined as,

$$D_0(u_0, x_0) = \left[\max \ \{\theta : \theta u_0 \in p(x_0)\}\right]^{-1}$$

$$= \text{Min} \ \left\{\delta : \frac{u_0}{\delta} \varepsilon P(x_0)\right\}, \qquad 0 \leq D_0(u_0, x_0) \leq 1$$

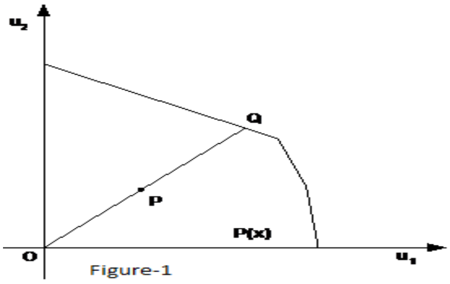

Figure-1

It can be shown that $D_0(u_0, x)$ satisfy certain structural properties.
The output distance function is linear homogeneous in outputs.

$$D_0(\lambda u_0, x_0) = \text{Min} \left\{ \delta : \frac{\lambda u_0}{\delta} \in P(x_0) \right\}$$

$$= \text{Min} \left\{ \delta : \frac{u_0}{\delta / \lambda} \in P(x_0) \right\}$$

$$= \lambda \text{ Min} \left\{ \frac{\delta}{\lambda} : \frac{u_0}{\delta / \lambda} \in P(x_0) \right\}$$

$$= \lambda D_0(u_0, x_0)$$

To estimate the output distance value of a decision making unit, one needs to assign an empirical status to the theoretical output level set $P(x_0)$. Following Banker, Charnes and Cooper, under the axioms of convexity, inefficiency and minimum extrapolation, we can express $P(x_0)$ as follows:

$$P(x_0) = \left\{ u : \sum_{j=1}^{n} \lambda_j x_j \le x_0 \sum_{j=1}^{n} \lambda_j u_j \ge u_0, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \ge 0 \right\}$$

$$D_0(x_0, u_0) = \text{Min } \delta$$

Subject to

$$\sum_{j=1}^{n} \lambda_j x_j \le x_0$$

$$\sum_{j=1}^{n} \lambda_j u_j \ge \frac{u_0}{\delta}$$

$$\sum_{j=1}^{n} \lambda_j = 1, \qquad \lambda_j \ge 0$$

The above problem appears to be a non-linear programming problem. By defining $\frac{1}{\delta} \ge \theta$ and maximization in being the objective the above problem takes a transformation,

$$[D_0(x_0, u_0)]^{-1} = \max \ \theta$$

Subject to

$$\sum_{j=1}^{n} \lambda_j x_j \le x_0$$

$$\sum_{j=1}^{n} \lambda_j u_j \ge \theta u_0$$

$$\sum_{j=1}^{n} \lambda_j = 1, \qquad \lambda_j \ge 0$$

Economic data are often subjected to returns to scale which are either increasing, or decreasing or constant. Again following Banker, Charnes and Cooper, the envelopment problem conditioned on the axioms of inefficiency, ray unbound ness and minimum extrapolation is used to estimate output technical efficiency under constant returns to scale. The output level set that admits constant returns to scale alone may be expressed as,

$$P^K(x_0) = \left\{u : \sum \lambda_j x_j \le x_0, \sum \lambda_j u_j \ge u, \lambda_j \ge 0\right\}$$

The output level set,

$$P(x_0) = \left\{u : \sum \lambda_j x_j \le x_0, \sum \lambda_j u_j \ge u, \sum \lambda_j = 1, \lambda_j \ge 0\right\}$$

admits variable returns to scale. As $P(x_0)$ is more constrained than $P^K(x_0)$, we have,

$$P(x_0) \le P^K(x_0)$$

$$\text{Max}\ \{\theta : \theta u_0 \in P(x_0)\} \le \text{Max}\ \{\theta : \theta u_0 \in P^K(x_0)\}$$

$$D_0(u_0, x_0) \ge D_0^K(u_0, x_0)$$

The output technical efficiency implied by constant returns to scale can be decomposed into the product of pure output technical and scale efficiency.

$$\frac{D_o^K(u_0, x_0)}{D_0(u_0, x_0)} \text{ measures outputscal e efficiency}$$

$$D_0(u_0, x_0) = \frac{OP}{OQ}$$

$$D_o^K(u_0, x_0) = \frac{OP}{OR}$$

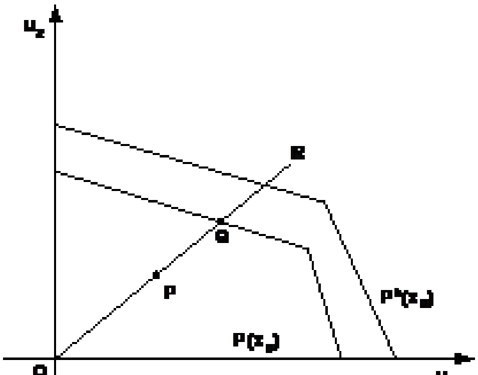

Figure-2

IV. MEASUREMENT OF TECHNICAL EFFICIENCY

The output approach discussed above can handle multi-inputs and multi-outputs with comfortable ease. Output technical efficiency can be measured apart from empirical production functions for parametric productions also. Such frontier production functions are the Cobb-Douglas, Zellnear – Revanker. Variable Returns to Scale and one-output multi-input Trans log production frontiers. Consider a one output multi-input situation where we have, $u \le \phi(x)$, where $\phi(x)$ is a frontier parametric production function which measures potential output if the best practice technology is implemented by the decision making unit whose efficiency under evaluation.

u: observed output in terms of output distance function we have,

$$[D_0(x, u)]^{-1} u = \phi(x)$$

$$\Rightarrow [D_0(x, u)]^{-1} = \frac{\phi(x)}{u}; \qquad 0 \le D_0(x, u) = \frac{u}{\phi(x)} \le 1$$

An output level set, in this case, may be defined as

$$P(x_0) = \left\{ u : \frac{u}{\phi(x_0)} \le 1 \right\} = \{u : \phi(x_0) \ge u\}$$

$$= \{ u : D_0 (x, u) \le 1 \}$$

Armed with a parametric production frontier one may suggest a programming approach to measure output technical efficiency of decision making units. The constraints of this problem are

$$\phi(x_j, \beta) \ge u_j, \; j = 1, 2, \ldots, n$$

Where x_j is the input vector of j^{th} DMU

u_j is scalar output of j^{th} DMU

ϕ is the production frontier, which may or may not be transformed into linear form by a transformation.

The production envelopment frontier is brought down such that at least one observed output falls on the frontier and rest fall below frontier. This exercise is called 'minimum extrapolation', and the process involves minimization of the sum of the slacks of the constraints. Let us consider an entrepreneur who can adjust his inputs but not outputs that he produces. In his case we may ask him if he can further reduce his inputs (radially) without harming the outputs bundle. If the answer is 'Yes', he suffers from input technical inefficiency. The potential inputs of an inefficient decision making unit can be determined estimating his input distance. Let $L(u_0)$ be the input level set, where u_0 is the fixed output vector of the entrepreneur.

$$L(u_0) = \{x : x \text{ produces } u_0\}$$

$L(u_0)$ and $P(x_0)$ are dualistica lly related

$$L(u) = \{x : u \in P(x)\}$$

$$P(x) = \{u : x \in L(u)\}$$

The input distance function is defined as,

$$D_j(x_0, u_0) = [\text{Min } \{\lambda : \lambda x_0 \in L(u_0)\}]^{-1}$$

$$= \text{Max } \left\{ \eta : \frac{x_0}{\eta} \in L(u_0) \right\}, \qquad D_i(x_0, u_0) \ge 1$$

Farrell's input and output technical efficiency, scores are inversely related to the Shephard's input and output distance functions.

To estimate the input technical efficiency of a DMU the input level set shall be given an empirical status.

$$L(u_0) = \left\{x : \sum \lambda_j x_j \le x, \sum \lambda_j u_j \ge u_0, \sum \lambda_j = 1, \lambda_j \ge 0\right\}$$

The input level set L (u_0) admits variable returns to scale

$$L^K(u_0) = \left\{x : \sum \lambda_j x_j \le x_0 \;\; \sum \lambda_j u_j \ge u_0, \lambda_j \ge 0\right\}$$

admits constant returns to scale

$$L(u_0) \subseteq L^K(u_0)$$

$$\frac{OP}{OQ} = D_i(x_0, u_0)$$

$$\frac{OP}{OQ} = D_i^K(x_0, u_0)$$

$$D_i^K(x_0, u_0) \ge D_i(x_0, u_0)$$

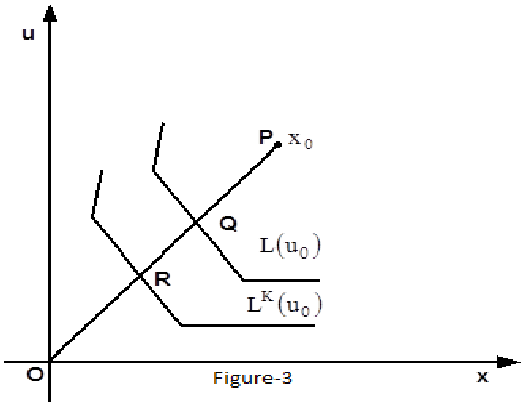

Figure-3

$\left[\dfrac{D_i(x_0, u_0)}{D_i^K(x_0, u_1)}\right]$ measures input scale efficiency of the decision making unit whose efficiency is under evaluation.

If a frontier parametric production function is used to measure technical efficiency, we have the constraints

$u \le \phi(x, \beta)$

Where β is the vector of parameters of the frontier production function $\phi(x_j, \beta)$.

$$u = \phi\left(D_i^{k-1}(u, x)\, x, \beta\right)$$

If returns to scale are assumed to be constant, then ϕ is linear homogeneous in x.

$$u = D_i^{k-1}(u, x)\ \ \phi(x, \beta)$$

$$D_i^K(u, x) = \frac{\phi(x, \beta)}{u} \ge 1$$

$$L(u) = \{x:\ \phi(x, \beta) \ge u\}$$

$$= \{x : D_i^K(u, x) \ge 1\}$$

If returns to scale are constant the input and output distance functions are inversely related.

$$D_0^K(u_0, x_0) = \left[\text{Max}\ \ \{\theta : \theta\ u_0 \in P^K(x_0)\}\right]^{-1}$$

$$= \left[\text{Max}\ \ \left\{\theta : u_0 \in P^K\left(\frac{x_0}{\theta}\right)\right\}\right]^{-1}$$

$$= \left[\text{Max}\ \ \left\{\theta : \frac{x_0}{\theta} \in L^K(u_0)\right\}\right]^{-1}$$

$$= \left[\text{Min}\ \ \left\{\frac{1}{\theta} : \frac{x_0}{\theta} \in L^K(u_0)\right\}\right]^{-1}$$

$$= \left[\text{Min}\ \ \{\lambda : \lambda\ x_0 \in L^K(u_0)\}\right]^{-1}$$

$$= \left[D_i^k(x_0, u_0)\right]^{-1}$$

Thus, $D_0^K(u_0, x_0)\ D_i^K(u_0, x_0) = 1$

If returns to scale are either increasing or decreasing we can relate $D_0(u_0, x_0)$ with $D_i(x_0, u_0)$

V. CONCLUSION:

In this paper, to estimate the producer's potential output vector the output distance function is used as a basic tool, output technical efficiency implied by the constant returns to scale can be decomposed in to the product of pure output technical and scale efficiency. If any entrepreneur can asks to reduce his inputs without harming the outputs bundle, the DMU's can be determined by the output distance instead of input distance with an empirical approach.

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