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Some Integral Properties of Aleph Function, General Class of Polynomials Associated With Feynman Integrals

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ABSTRACT: *The aim of the present paper is to discuss certain integral properties of Aleph function and general class of polynomials. The exact partition of a Gaussian model in statistical mechanics and several other functions as its particular cases. During the course of finding, we establish certain new double integral relations pertaining to a product involving a general class of polynomials and the Aleph function.*

KEY WORDS AND PHRASES*: Feynman integrals, Aleph function, General Class of Polynomials, Hermite Polynomials, Laguerre Polynomials.*

I. INTRODUCTION

The conventional formulation may fail pertaining to the domain of quantum cosmology but Feynman path integrals apply interesting by Feynman path integrals are reformulation of quantum mechanics on more fundamental than the conventional formulation in term of operator. Feynman integral are useful in the study and development of simple and multiple variable hypergeometric series which in turn are useful in statistics mechanics.

The Aleph (χ)-functions, introduced by Südland et al.[9], however the notation and complete definition is presented here in the following manner in terms of the Mellin-Barnes type integrals

$$\chi[z] = \chi^{m,n}_{y_i,\tau_i;r}[z] = \chi^{m,n}_{x_i,y_i,\tau_i;r}\left[z \left| \begin{matrix} (a_j,A_j)_{1,n}\,[\tau_i(a_{ji},A_{ji})]_{n+1,x_i;r} \\ (b_j,B_j)_{1,m}\,[\tau_i(b_{ji},B_{ji})]_{m+1,y_i;r} \end{matrix} \right. \right]$$

$$= \frac{1}{2\pi\omega}\int_L \Omega^{m,n}_{x_i,y_i,\tau_i;r}(-s)\, z^{(s)}\, ds \qquad \ldots(1)$$

For all $z \neq 0$ where $\omega = \sqrt{-1}$ and

$$\Omega^{m,n}_{x_i,y_i,\tau_i;r}(s) = \frac{\prod_{j=1}^{m} \Gamma(b_j + B_j s) \prod_{j=1}^{n} \Gamma 1 - a_j - A_j s}{\sum_{i=1}^{r} \tau_i \prod_{j=n+1}^{x_i} \Gamma(a_{ji} + A_{ji} s) \prod_{j=m+1}^{y_i} \Gamma 1 - b_{ji} - B_{ji} s} \qquad \ldots(2)$$

the integration path $L = L_{i\gamma\infty}, \gamma \in R$ extends from $\gamma - i\infty$ to $\gamma + i\infty$, and is such that the poles, assumed to be simple, of $\Gamma(1 - a_j - A_j s)$, $j = 1,\ldots,n$ do not coincide with the pole of $\Gamma(b_j + B_j s)$, $j= 1,\ldots,m$ the parameter x_i, y_i are non-negative integers satisfying:

$$0 \le n \le x_i,\ 1 < m \le y_i,\ \tau_i > 0 \quad \text{for} \quad i = 1,\ldots, r$$

The parameters $A_j, B_j, A_{ji}, B_{ji} > 0$ and $a_j, b_j, a_{ji}, b_{ji} \in C$. The empty product in (2) is interpreted as unity. The existence conditions for the defining integral (1) are given below

$$\phi_\ell > 0, |\arg (z)| < \frac{\pi}{2}\phi_\ell \qquad \ell = 1,\ldots, r \qquad \ldots(3)$$

$$\phi_\ell \geq 0, |\arg (z)| < \frac{\pi}{2}\phi_\ell \text{ and } R\{\xi_\ell\} + 1 < 0 \qquad \ldots(4)$$

where

$$\phi_\ell = \sum_{j=1}^{n} A_j + \sum_{j=1}^{m} B_j - \tau_\ell \left(\sum_{j=n+1}^{x_\ell} A_{j\ell} + \sum_{j=m+1}^{y_\ell} B_{j\ell} \right) \qquad \ldots(5)$$

$$\xi_\ell = \sum_{j=1}^{m} b_j - \sum_{j=1}^{n} a_j + \tau_\ell \left(\sum_{j=m+1}^{y_\ell} b_{j\ell} - \sum_{j=n+1}^{x_\ell} a_{j\ell} \right) + \frac{1}{2}(x_\ell - y_\ell), \ell = 1,\ldots, r \qquad \ldots(6)$$

For detailed account of the Aleph (χ)-function see (9) and (10).
The general class of polynomial introduced by Srivastava (9)

$$S_n^m[p] = \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} B_{n,k}\, p^k \quad n = 0,1,2,\ldots$$

Main Results:

We shall establish the following results:

(A)

$$\int_0^1 \int_0^1 \left(\frac{1-p}{1-pq}.q\right)^\sigma \left(\frac{1-q}{1-pq}\right)^\rho \left[\frac{1-pq}{(1-p)(1-q)}\right] S_n^m\left[\frac{1-p}{1-pq}wq\right] \chi^{m,n}_{x_i,y_i,\tau_i;r}\left[\frac{1-q}{1-pq}w\right] dp\, dq$$

$$= \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} B_{n,k}\, \Gamma(k+\sigma)\, w^k$$

$$\cdot\, \chi^{m,n+1}_{x_{i+1},y_{i+1},\tau_i;r}\left[w \,\middle|\, \begin{matrix} (1-\rho;1), (a_j,A_j)_{1,n} [\tau_i(a_{ji},A_{ji})]_{n+1,x_i;r} \\ (b_j,B_j)_{1,m}, [\tau_i(b_{ji},B_{ji})]_{m+1,y_i;r}\, \tau_i(1-k-\sigma-\rho),1 \end{matrix} \right] \qquad \ldots(2.1)$$

provided that

$$\text{Re}[\sigma + \rho + b_j/\rho_j] > 0, |\arg w| < \frac{T\pi}{2}$$

m is an arbitrary positive integer and the coefficient $B_{n,k}\,(n, k \geq 0)$ are arbitrary constant, real or complex.

Proof. We have

$$S_n^m\left[\frac{1-p}{1-pq}wq\right] \chi^{m,n}_{x_i,y_i\tau_i;r}\left[\frac{1-q}{1-pq}w\right] dp\, dq = \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} B_{n,k} \left(\frac{1-p}{1-pq}wq\right)^k$$

$$\times \frac{1}{2\pi\omega} \int_L \frac{\prod_{j=1}^{m} \Gamma b_j + B_j s \prod_{j=1}^{n} \Gamma 1 - a_j - A_j s}{\sum_{i=1}^{r} \tau_i \prod_{j=n+1}^{x_i} \Gamma a_{ji} + A_{ji} s \prod_{j=m+1}^{y_i} \Gamma 1 - b_{ji} - B_{ji} s} \left(\frac{1-q}{1-pq}w\right)^{-s} ds \qquad \ldots(2.2)$$

Multiplying both sides of (2.2) by

$\left[\frac{1-p}{1-pq}w\right]^\sigma \left[\frac{1-q}{1-pq}\right]^\rho \left[\frac{1-pq}{(1-p)(1-q)}\right]$ and integrating with respect to p and q between 0 and 1 for both the variable and we get the result

(B) $\int_0^\infty \int_0^\infty f(\eta + w)\,\eta^{\sigma-1} w^{\rho-1} S_n^m[\eta]\,\chi^{m,n}_{x_i,y_i,\tau_i;r}[w]\,d\eta\,dw$

$$= \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} B_{n,k} \int_0^\infty f(\xi)\,\xi^{\sigma+\rho+k-1}\,\Gamma(k+\sigma)$$

$$\cdot\, \chi^{m,n+1}_{x_{i+1},y_{i+1},\tau_i;r}\left[\xi \,\middle|\, \begin{matrix} (1-\rho;1),\ (a_j,A_j)_{1,n}\,[\tau_i(a_{ji},A_{ji})]_{n+1,x_i;r} \\ (b_j,B_j)_{1,m},[\tau_i(b_{ji},B_{ji})]_{m+1,y_i;r}\,[\tau_i(1-k-\sigma,1)] \end{matrix}\right] d\xi \qquad \ldots(2.3)$$

provided that

$\mathrm{Re}(\sigma + \rho + b_j/\rho_i) > 0$, m is an arbitrary positive integer and coefficients $B_{n,k}\,(n, k \ge 0)$ are arbitrary constants, real or complex.

Proof. We have

$$S_n^m[\eta]\,\chi^{m,n}_{x_i,y_i,\tau_i;r}(w) = \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} B_{n,k}(\eta)^k$$

$$\times \frac{1}{2\pi\omega}\int_L \frac{\prod_{j=1}^{m}\Gamma(b_j + B_j s)\prod_{j=1}^{n}\Gamma(1-a_j-A_j s)(w)^{-s}}{\sum_{i=1}^{r}\tau_i\prod_{j=n+1}^{x_i}\Gamma(a_{ji}+A_{ji}s)\prod_{j=m+1}^{y_i}\Gamma(1-b_{ji}-B_{ji}s)}\,ds \qquad \ldots(2.4)$$

Multiplying both side (2.4) by $f(\eta + w)\,\eta^{\sigma-1} w^{\rho-1}$ and integrating with respect to η and w between 0 and ∞ for both side the variable and get the result (2.3).

(C) $\int_0^1 \int_0^1 \psi(\eta w)(1-w)^{\rho-1}(1-\eta)^{\sigma-1} w^{\sigma} S_n^m[w(1-\eta)]\,\chi^{m,n}_{x_i,y_i,\tau_i;r}[1-w]\,d\eta\,dw$

$$= \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} B_{n,k}\,\Gamma(k+\sigma)\int_0^1 f(\xi)(1-\xi)^{k+\sigma+\rho-1}\,d\xi$$

$$\times\, \chi^{m,n+1}_{x_{i+1},y_{i+1},\tau_i;r}\left[1-\xi \,\middle|\, \begin{matrix} (1-\rho;1),\ (a_j,A_j)_{1,n}\,[\tau_i(a_{ji},A_{ji})]_{n+1,x_i;r} \\ (b_j,B_j)_{1,m},[\tau_i(b_{ji},B_{ji})]_{m+1,y_i;r}\,[\tau_i(1-k-\sigma-\rho),1] \end{matrix}\right] \qquad \ldots(2.5)$$

provided that Re(σ) >0, Re(ρ) >0, m is an arbitrary positive integer and coefficient $B_{n,k}$ (n,k ≥ 0) are arbitrary constant, real or complex.

Proof. We have

$$S_n^m[w(1-\eta)]\,\chi^{m,n}_{x_i,y_i,\tau_i;r}[1-w] = \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} B_{n,k}\,w^k(1-\eta)^k$$

$$\times \frac{1}{2\pi\omega}\int_L \frac{\prod_{j=1}^{m}\Gamma(b_j + B_j s)\prod_{j=1}^{n}\Gamma(1-a_j-A_j s)}{\sum_{i=1}^{r}\tau_i\prod_{j=n+1}^{x_i}\Gamma(a_{ji}+A_{ji}s)\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}-B_{ji}s)}(1-w)^{-s}\,ds \qquad \ldots(2.6)$$

Multiplying both side of (2.6) by $\psi(\eta w)(1-\eta)^{\sigma-1}(1-w)^{\rho-1} w^{\sigma}$ and integrating with respect to η and w between 0 and 1 for both the variable and get the result.

(D) $$\int_0^1 \int_0^1 \left[\frac{q(1-p)}{(1-pq)}\right]^{\alpha+\sigma} \left[\frac{1-q}{1-pq}\right]^{\rho} \frac{1}{(1-p)} S_n^m\left[\frac{q(1-p)}{(1-pq)}\right] \chi_{x_i,y_i,\tau_i;r}^{m,n}\left[\frac{wq(1-p)}{1-pq}\right] dp\, dq$$

$$= \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} B_{n,k} \frac{\Gamma(k+z+\sigma+\alpha)\Gamma\rho+1}{\Gamma(k+z+\sigma+\rho+\alpha+1)}$$

$$\cdot\, \chi_{x_{i+1},y_{i+1},\tau_i;r}^{m,n}\left[w \,\middle|\, \begin{matrix} (1-k-\sigma-\alpha;1),(a_j,A_j)_{1,n}[\tau_i(a_{ji},A_{ji})]_{n+1,x_i;r} \\ (b_j,B_j)_{1,m},[\tau_i(b_{ji},B_{ji})]_{m+1,y_i;r}[\tau_i(1-k-\sigma-\alpha-\rho,1)] \end{matrix}\right] \qquad \dots(2.7)$$

provided that $\mathrm{Re}(\sigma+\rho+\alpha+b_j/\rho_i)>0, |\arg w| < \frac{T}{2}\pi$, m is an arbitrary integer and the coefficient $B_{n,k}$ $(n,k \geq 0)$ are arbitrary constant, real or complex.

Proof. We have

$$S_n^m\left[\frac{q(1-p)}{1-pq}\right] \chi_{x_i,y_i,\tau_i;r}^{m,n}\left[\frac{wq(1-p)}{(1-pq)}\right] = \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} B_{n,k} \left[\frac{q(1-p)}{1-pq}\right]^{k}$$

$$\times \frac{1}{2\pi\omega}\int_L \frac{\prod_{j=1}^{m}\Gamma(b_j+B_j s)\prod_{j=1}^{n}\Gamma(1-a_j-A_j s)}{\sum_{i=1}^{r}\tau_i \prod_{j=n+1}^{x_i}\Gamma(a_{ji}+A_{ji}s)\prod_{j=m+1}^{y_i}\Gamma(1-b_{ji}-B_{ji}s)} \left[\frac{wq(1-p)}{(1-pq)}\right]^{-s} ds$$

$$\dots(2.8)$$

Multiplying both side by $\left[\left[\frac{q(1-p)}{1-pq}\right]^{\sigma+\alpha}\left[\frac{1-q}{1-pq}\right]^{\rho}\frac{1}{(1-p)}\right]$ and integrating with respect to p and q between 0 and 1for both the variable and get the result (2.7).

3. Particular Cases

By applying our result given in (2.1), (2.3), (2.5), (2.7) to the case of Hermite polynomial (8) and (11) and by setting

$$S_n^2[p] = p^{n/2} H_n\left[\frac{1}{2\sqrt{p}}\right]$$

In which case m = 2, $B_{n,k} = (-1)^k$.

(A) $$\int_0^1 \int_0^1 \left(\frac{1-p}{1-pq}q\right)^{\sigma}\left(\frac{1-q}{1-pq}\right)^{\rho} \frac{(1-pq)}{(1-p)(1-q)}\left[\frac{1-p}{1-pq}qw\right]^{n/2}$$

$$\cdot\, \chi_{x_i,y_i,\tau_i;r}^{m,n}\left[\frac{1-q}{1-pq}w\right] dp\, dq\; H_n\left(\frac{1}{2\sqrt{\left(\frac{1-p}{1-pq}\right)wq}}\right)$$

$$= \sum_{k=0}^{[n/2]} \frac{(-n)_{2k}}{k!} (-1)^k \, w^k \, \Gamma(k+\sigma) \, \aleph^{m,\, n+1}_{x_i+1, y_i+1, \tau_i; r}$$

$$\cdot \left[w \left| \begin{matrix} (1-\rho,1), (a_j, A_j)_{1,n}, \tau_i (a_{ji}, A_{ji})_{n+1, x_i; r} \\ (b_j, B_j)_{1,m}, [\tau_i (b_{ji}, B_{ji})]_{m+1, y_i; r} [\tau_i (1-k-\sigma-\rho, 1)] \end{matrix} \right. \right]$$

valid under the same conditions as obtained from (A).

(B) $\int_0^{\infty} \int_0^{\infty} f(\eta + w) \, w^{p-1} \, \eta^{\sigma - 1 + n/2} \, H_n \left[\frac{1}{2\sqrt{n}} \right] \aleph^{m,n}_{x_i, y_i \tau_i; r} (w) \, dw \, d\eta$

$$= \sum_{k=0}^{[n/2]} \frac{(-n)_{2k}}{k!} (-1)^k \int_0^{\infty} f(\xi) \, \xi^{\sigma + \rho + k - 1} \, d\xi \, \Gamma(k+\sigma)$$

$$\cdot \aleph^{m,\, n+1}_{x_{i+1}, y_{i+1}, \tau_i; r} \left[\begin{matrix} (1-\rho,1), (a_j, A_j)_{1,n}, [\tau_i (a_{ji}, A_{ji})]_{n+1, x_i; r} \\ (b_j, B_j)_{1,m}, [\tau_i (b_{ji}, B_{ji})]_{m+1, y_i; r} [\tau_i (1-k-\sigma, 1)] \end{matrix} \right| \xi \left. \vphantom{\begin{matrix} a \\ b \end{matrix}} \right]$$

valid under the same conditions as required for (B).

(C) $\int_0^1 \int_0^1 \psi(\eta w)(1-\eta)^{\sigma - 1} (1-w)^{\rho - 1} \, w^{\sigma + n/2} \, (1-\eta)^{n/2}$

$$\cdot \, H_n \left[\frac{1}{2 \, w (1-\eta)} \right] \aleph^{m,n}_{x_i, y_i \tau_i; r} [1 - w] \, dw \, d\eta$$

$$= \sum_{k=0}^{[n/2]} \frac{(-n)_{2k}}{k!} (-1)^k \, \Gamma(k+\sigma) \int_0^1 \psi(\xi)(1-\xi)^{k+\sigma+\rho-1}$$

$$\cdot \aleph^{m,\, n+1}_{x_{i+1}, y_{i+1}, \tau_i; r} \left[\begin{matrix} (1-\rho,1), (a_j, A_j)_{1,n}, [\tau_i (a_{ji}, A_{ji})]_{n+1, x_i; r} \\ (b_j, B_j)_{1,m}, [\tau_i (b_{ji}, B_{ji})]_{n+1, y_i; r}, \tau_i [1-k-\sigma-\rho, 1] \end{matrix} \right| 1 - \xi \left. \vphantom{\begin{matrix} a \\ b \end{matrix}} \right]$$

valid under the same condition as required for (C).

(D)

$$\int_0^1 \int_0^1 \left[\frac{q(1-p)}{1-pq} \right]^{\sigma + \alpha} \left(\frac{1-q}{1-pq} \right)^{\rho} \frac{1}{(1-p)^{1-n/2}} \frac{q^{n/2}}{(1-pq)^{n/2}}$$

$$\cdot \, H_n \left[\frac{1}{2\sqrt{\frac{q(1-p)}{1-pq}}} \right] \aleph^{m,n}_{x_i, y_i \tau_i; r} \left(\frac{wq(1-p)}{1-pq} \right) dp \, dq$$

$$= \sum_{k=0}^{[n/2]} \frac{(-n)_{2k}}{k!} (-1)^k \, \Gamma(\rho + 1) \, \aleph^{m,\, n+1}_{x_{i+1}, y_{i+1}, \tau_i; r} \left[\begin{matrix} (1-k-\sigma-\alpha, 1), (a_j, A_j)_{1,n}, \tau_i (a_{ji}, A_{ji})_{n+1, x_i; r} \\ (b_j, B_j)_{1,m}, (\tau_i (b_{ji}, B_{ji})_{m+1, y_i; r} [\tau_i (1-k-\sigma-\rho, 1)] \end{matrix} \right| w \left. \vphantom{\begin{matrix} a \\ b \end{matrix}} \right]$$

valid under the same conditions as obtained from (D).

For the Laguerre polynomials ([8]and [11]) setting $S_n^{'}(x) \to L_n^{r}(x)$ in which case $B_{n,k} = \binom{n+r}{n} \frac{1}{(r+1)_k}$ the result (A), (B),(C) and D reduced to the following formulae

(1)

$$\int_0^1\int_0^1 \left(\frac{1-p}{1-pq}q\right)^{\sigma}\left(\frac{1-q}{1-pq}\right)^{\rho}\left(\frac{(1-pq)}{(1-p)(1-q)}\right) L_n^r\left[\frac{1-p}{1-pq}wq\right]\chi^{m,n}_{x_i,y_i\tau_i;r}\left(\frac{1-q}{1-pq}w\right)dp\ dq$$

$$=\sum_{k=0}^{[n/2]}\frac{(-n)_k}{k!}\binom{n+r}{n}\frac{1}{(r+1)_k}w^k\,\Gamma k+\sigma$$

$$\cdot\ \chi^{m,n+1}_{x_i+1,y_i+1,\tau_i;r}\left[\left.\begin{matrix}(1-\rho,1),(a_j,A_j)_{1,n},[\tau_i(a_{ji},A_{ji})]_{n+1,x_i;r}\\ (b_j,B_j)_{1,m},(\tau_i(b_{ji},B_{ji})_{m+1,y_i;r}[\tau_i(1-k-\sigma-\rho,1)]\end{matrix}\right|\ w\right]$$

valid under the same conditions as required for (A).

(2) $$\int_0^{\infty}\int_0^{\infty} f(\eta+w)\,w^{\rho-1}\eta^{\sigma-1}L_n^r(\eta)\,\chi^{m,n}_{x_i,y_i\tau_i;r}[w]\,dw\ d\eta$$

$$=\sum_{k=0}^{n}\frac{(-n)_k}{k!}\binom{n+r}{n}\frac{1}{(r+1)_k}\int_0^{\infty} f(\xi)\,\xi^{\sigma+\rho+k-1}\,\Gamma(k+\sigma)$$

$$\cdot\ \chi^{m,n+1}_{x_i+1,y_i+1,\tau_i;r}\left[\begin{matrix}(1-\rho,1),(a_j,A_j)_{1,n},\tau_i(a_{ji},A_{ji})_{n+1,x_i;r}\\ (b_j,B_j)_{1,m},\tau_i[(b_{ji},B_{ji})]_{m+1,y_i;r}[\tau_i(1-k-\sigma,1)]\end{matrix}\right]$$

valid under the same condition as required for (B).

(3) $$\int_0^1\int_0^1 \psi(w,\eta)(1-\eta)^{\sigma-1}(1-w)^{\rho-1}w^{\sigma}L_n^r[w(1-\eta)]\,\chi^{m,n}_{x_i,y_i\tau_i;r}[1-w]\,dw\ d\eta$$

$$=\sum_{k=0}^{n}\frac{(-n)_k}{k!}\binom{n+r}{n}\frac{1}{(r+1)_{k'}}\Gamma(k+\sigma)\int_0^1 f(\xi)(1-\xi)^{\sigma+k+\rho-1}$$

$$\cdot\ \chi^{m,n+1}_{x_{i+1},y_{i+1},\tau_i;r}\left[\begin{matrix}(1-\rho,1),(a_j,A_j)_{1,n},\tau_i(a_{ji},A_{ji})]_{n+1,x_i;r}\\ (b_j,B_j)_{1,m},[\tau_i(b_{ji},B_{ji})]_{n+1,y_i;r},[\tau_i(1-k-\sigma-\rho,1)]\end{matrix}\right]$$

valid under the same condition as required for (C).

(4)

$$\int_0^1\int_0^1\left[\frac{q(1-p)}{1-pq}\right]^{\sigma+\alpha}\left(\frac{1-q}{1-pq}\right)^{\rho}\frac{1}{(1-p)}$$

$$\cdot\ L_n^r\left[\frac{q(1-p)}{(1-pq)}\right]\chi^{m,n}_{x_i,y_i\tau_i;r}\left(\frac{wq(1-p)}{1-pq}\right)dp\ dq$$

$$=\sum_{k=0}^{n}\frac{(-n)_k}{k!}\binom{n+r}{n}\frac{1}{(r+1)_{k'}}\Gamma\rho+1$$

$$\cdot\ \chi^{m,n+1}_{x_{i+1},y_{i+1},\tau_i;r}\left[\left.\begin{matrix}(1-k-\sigma-\alpha,1),(a_j,A_j)_{1,n},[\tau_i(a_{ji},A_{ji})]_{n+1,x_i;r}\\ (b_j,B_j)_{1,m},[\tau_i(b_{ji},B_{ji})]_{m+1,y_i;r}[\tau_i(1-k-\sigma-\rho,1)]\end{matrix}\right|\ w\right]$$

valid under the same condition as obtained for (D).

II. CONCLUSION

The results obtained here are basic in nature and are likely to find useful applications in the study of simple and multiple variable hypergeometric series which in turn are useful in statistical mechanics, electrical networks and probability theory. These integrals reformulation of quantum mechanics are more fundamental than the conventional formulation in terms of operators.

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