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On Commutativity of Associative Rings

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ABSTRACT: *In this paper I have mainly focussed on some theorems related to commutativity of associative and non associative rings. I prove that if R is an associative ring with unity satisfying* $(x,y^2)-(y^2,x)$. $\forall x,y \in R$, *n* ≥ 2 *and* $xy^3=y^2xy$ $\forall x,y \in R$, *n* ≥ 2.*Then R is commutative ring and also I have mainly obtained two principles for a non associative ring to be a communtative ring..*

KEY WORDS: *Ring with unity, Associative ring, Non-associative ring.*

I. INTRODUCTION

The object of this note to investigate the commutativity of the associative and non associative rings satisfying condition '.' Such that $y(yx)=y(xy)$ $\forall$ x,y $\in$ R and $(yx)x = (xy)x$ $\forall$ x,y $\in$ R,

II. PRELIMINARIES

Definition:

(i) A non empty set R together with two binary operations + and . is said to be a ring (Associative ring) if (R,+) is an abelian group and (R,.) is a semi group satisfying distributive laws

(ii) In a ring R if there exists an element '1' in R such that a.1=1.a = a for all a $\in$R then R is said to be a ring with unity

Theorem 1. If R is an Associative Ring with unity 1 then R is Commutative

$(x,y^2)-(y^2,x)$ belongs to z(R) if $(x,y^2)-(y^2,x)$ belongs to z(R)

G.T. $xy^2 = y^2x$

$x=y+1 \Rightarrow x(y+1)^2=(y+1)^2x$

$X(y^2+2y+1)= (y^2+2y+1)x$

$Xy^2+2xy+x=y^2x+2yx+x$

$2xy=2yx$

$\boxed{xy=yx}$

Theorem 2. If R is an Associative Ring with unity then R is Commutative if $xy^3=y^2xy$

put $y=y+1 \Rightarrow \quad x(y+1)^3=(y+1)^2\, x\, (y+1)$

$x(y+1)(y^2+2y+1)= (y^2+2y+1)(xy+x)$

$(xy+x)(y^2+2y+1) = y^2xy+2yxy+xy+y^2x+2yx+x$

$xy^3+2xy^2+xy+xy^2+2xy+x = y^2xy+2yxy+xy+y^2x+2yx+x$

$2xy=2yx$ [from 1 , $xy^2=y^2x$]

$\boxed{xy=yx}$

Theorem 3. Let R be a prime ring with $yx^2y=xy^2x$ in Z(R) for every x,y in R. Then R is a commutative ring.

Proof : **Given that** $yx^2y=xy^2x$

Put x=x+1, $y(x+1)^2y=(x+1)(y^2)(x+1)$

$Y(x^2+2x+1)y=(xy^2+y^2)(x+1)$

$Yx^2y+2yxy+y^2=xy^2x+xy^2+y^2x+y^2$

$2yxy=xy^2+y^2x$ (**from the theorem** $y^nx=y^{n-1}xy$)

$\mathbf{Yxy=xy^2}$

put$Y=y+1$, $(y+1)(xy+x)=x(y^2+2y+1)$

$yxy+yx+xy+x=xy^2+2xy+x$

$\boxed{yx=xy}$

Theorem 4. If R is an Associative Ring with unity 1 then R is Commutative if and only if $x^3yx=x^4y$ for all x,y belongs to R.

Given that , $x^3yx=x^4y$

Put $x=x+1$,$(x+1)^3y(x+1)=(x+1)^4y$

$(x+1)(x+1)(x+1)\ y\ (x+1) = (x+1)^2(x+1)^2y$

$(x^2+2x+1)\ (xy+y)\ (x+1)= (x^2+2x+1)\ (x^2+2x+1)y$

$(x^3y+x^2y+2x^2y+2xy+xy+y)\ (x+1) = (x^2+2x+1)\ (x^2y+2xy+y)$

$3x^2yx+3xyx+yx=3x^2y+3x^3y+xy$ [by the theorem,$x^ny=x^{n-1}yx$]

$\boxed{yx=xy}$

Theorem 5. Let R be a non-associative ring with unity 1 satisfying $[(xy)^2-xy,x]=0$ then R is commutative.

Given that, $[(xy)^2-xy,x]=0 \Rightarrow [(xy)^2-xy]x = x[(xy)^2-xy]$

$\Rightarrow x=x+1$, $[(x+1)\ y]^2-(x+1)\ y](x+1) = (x+1)\ [\ ((x+1)y)^2-(x+1)y]$

$[(xy+y)^2-(xy+y)](x+1) = (x+1)\ [(xy+y)^2-(xy+y)\ ]$

$[(xy+y)\ (xy+y)-(xy+y)]\ (x+1) = (x+1)\ [(xy+y)\ (xy+y)-(xy+y)]$

$(xy)(yx) + y(xy)x-yx = x(xy)y + (xy)\ (xy)-xy$

$X=x+1$, $[\ (x+1)y]\ [y(x+1)]+y((x+1)yx+1)-y(x+1) = (x+1)\ ((x+1)y)y+((x+1)y)((x+1)y-(x+1)$

$(xy+y)(yx+y)+(yxy+y^2)(x+1)-yx-y = (X+1)\ (xy+y)y+(xy+y)\ (xy+y) - xy-y$

$Y(yx)+yxy = (xy)y+y(xy)$

$(yx)y = (xy)\ y$

$y=y+1 \Rightarrow yx=xy$

Theorem 6. If R be a non-associative ring with unity 1 satisfying $y(yx)=y(xy)$ **for all x,y belongs to R then R is commutative**.

Given that, $Y(yx)=y(xy)$

put $y=y+1$, $(y+1)\ [yx+x] = (y+1)\ [xy+x]$

$Y(yx)+yx+yx+x = y(xy)+yx+xy+x$

$\boxed{yx=xy}$

Theorem 7. If R be a non-associative ring with unity 1 satisfying $(yx)x = (xy)x$ **for all x,y belongs to R then R is commutative**.

Given that,$(yx)x = (xy)x$

put $x=x+1$, $[y(x+1)]\ (x+1) = [(x+1)y](x+1)$

$(yx+y)\ (x+1) = (xy+y)\ (x+1)$

$(yx)x+yx+yx+y = (xy)x+xy+yx+y$

$\boxed{yx=xy}$

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